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A SOLAR SAIL INTEGRATED SIMULATION TOOLKIT

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This paper describes an integrated set of high fidelity software tools that are being developed for the design of solar sail missions and analysis of the guidance, navigation and control (GNC) of these missions. This integrated set of simulation tools will be able to predict, re-calibrate and optimize the trajectory, maneuvers and propulsive performance of a sail during a representative flight mission. This tool has capabilities for mission design and for simulation of the sail attitude determination and control, navigation based on ground based and on board tracking, feedback control performance evaluation, parameter estimation and covariance analysis and analysis of attitude control systems. It is capable of evaluating the sail performance accounting for structural dynamic effects, billowing, bending and material degradation effects..

INTRODUCTION

Solar sails have been studied for a variety of NASA missions to achieve orbits that cannot be reached with conventional propulsion techniques. The technology utilizes the solar radiation pressure force acting on a large reflective surface to provide continuous low-level thrust propulsion directed by the orientation of the sail. Because of the availability of a continuous thrust, unique mission concepts (Ref. 1) are enabled by solar sail spacecraft. These include a mission to the sun-Earth L1 libration point which uses the solar sail to maintain an orbit closer to the sun than the conventional L1, pole sitter missions that are capable of hovering above the Earth poles and a Solar Polar Imager mission that uses solar sail concepts to achieve a highly inclined heliocentric polar orbit at 0.5 AU.

Currently no single integrated software tool exists that can be used to design trajectories and evaluate guidance, navigation and control (GNC) strategies for solar sail missions. This paper describes an integrated set of high fidelity software tools that are being developed for the optimal design of solar sail trajectories, the determination of feasible solar sail attitude control strategies and for simulation and analysis of the guidance, navigation and control of these missions. This Solar Sail Integrated Simulation Toolkit (SSIST) will be able to predict, re-calibrate and optimize the trajectory, maneuvers and propulsive performance of a sail during a representative flight mission. The software (which is referred to as S5-*Solar Sail Spaceflight Simulation Software*) has capabilities for

- solar sail mission design
- simulation and analysis of the sail attitude determination and control
- navigation based on ground based and on board tracking
- evaluation of feedback control performance
- parameter estimation and covariance analysis
- computation of solar radiation pressure forces and torques
- evaluation of structural dynamic effects, billowing, bending and material degradation effects on solar sail performance

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The simulation tool, which will be made available to solar sail community, will initially be demonstrated by supporting a NRA prototype solar sail design that is currently being planned. This mission, which is referred as Geostorm Warning Mission utilizes solar sail technology to transfer a satellite from the L1 to the sub L-1 point and to maintain a halo orbit in the vicinity of this location.

REQUIREMENTS AND CAPABILITIES

The solar sail toolkit consists of a mission design capability and a GNC simulation capability which uses the optimal trajectory and controls output by the mission design. The GNC segment includes the attitude determination and control, trajectory optimization, orbit determination, trajectory prediction, and trajectory control modules that operate together to form the navigation feedback control loop. The feedback loop is cycled as time progresses in order to refine estimates, to reduce uncertainties and to return the spacecraft orbit to a desired condition (i.e., the nominal flight profile). To achieve the objectives of a comprehensive design and simulation tool, the toolkit is organized into five basic modules which embody the mission design capability, the GNC simulation and the modeling of the solar radiation force and torque. The modules include: a Mission Design Module (OPT), a Solar Pressure Module (SRP), Attitude Dynamics and Control Module (ADC), Trajectory Control Module (TCN), and a Navigation and Orbit Determination Module (DET).

The functions of the component modules are briefly described below along with their relationship. More detailed descriptions of characteristics and capabilities of the modules are presented in the Module Description Section.

- *OPT:* The Mission Design Module determines an optimal trajectory and sail control strategy which optimizes a given performance criteria (ie minimum time, control effort or sail dimension) subject to control and/or state constraints. A second order-gradient optimization algorithm is used to converge to the optimal trajectory. This module outputs the optimal control and control gain matrices which are used by the simulation and control modules.

- *SRP:* The Solar Radiation Pressure Module is the source of the solar radiation pressure thrusts and torques for the OPT, ADC and DET modules. Using sail and spacecraft characteristics and knowledge of the spacecraft state and orientation, this module computes the thrust and the total torque due to solar radiation pressure. Structural dynamic effects, billowing, beam bending and sail degradation are accounted for in the computation of forces and moments.

- *ADC*: The Attitude Dynamics and Control Module simulates the rotational dynamics of a sailcraft, including torque induced by solar radiation pressure, other environmental disturbance sources (e.g., gravity gradient, aerodynamic, and magnetic moment), and conventional spacecraft actuators (e.g., reaction wheel assembly and thrusters). It models sailcraft attitude control using articulated control vanes located at the sail periphery or mass displacement (i.e., mass on a gimbaled boom altering center of mass location relative to the center of pressure). Simulated sensor measurements are processed to estimate attitude and angular velocity.

- *TCN*: The Trajectory Control Module updates the thrust control profile based on the current estimate of the spacecraft state provided by the DET module. TCN uses the gain matrix from OPT in combination with previously designed control laws to update the control and predict the updated target conditions. The updated controls (thrusts) are input to the ADC module.

- *DET*: The Orbit Determination Module simulates the navigation performance. DET propagates the equations of motion, simulates ground based and on board (optical and accelerometer) observations and processes the observables with a Kalman type filter to estimate the current state and statistics. It is designed to be used for covariance analysis or Monte Carlo studies.

The toolkit may be operated in an integrated mode or individual modules can be run in a stand-alone mode. The integrated mode, in which all the modules are executed as a continuous feedback loop, is intended for a comprehensive design and GNC simulation case. Mission designers, navigation and guidance analysts, attitude control analysts and solar sail technologist can run the individual modules to independently evaluate sailcraft mission design, attitude control and GNC performance.

1. Mission designers and Solar Sail Technologists can use the OPT module for analysis and design of optimal trajectories and to conduct parametric trade-off studies on the dependency of solar sail properties and mission characteristics and support sailcraft sizing studies

2. Control and navigation specialists may use the TCN, DET and SRP modules for (a) navigation performance assessment and covariance analysis to determine observable requirements, evaluate the sensitivity to error sources and determine the ability to estimate solar pressure parameters; (b) Monte Carlo simulations to determine rate of convergence, sensitivity to nonlinear effects and mismodeling errors and (c) feedback control and guidance evaluation to investigate performance of control algorithms and rates of convergence.

3. Attitude Control Engineers can use ADC and SRP modules to simulate the attitude dynamics and control system; conduct trade-off studies of controller strategy and architecture; study the effect of structural dynamics and environmental effects on the attitude control system performance

FUNCTIONAL ARCHITECTURE

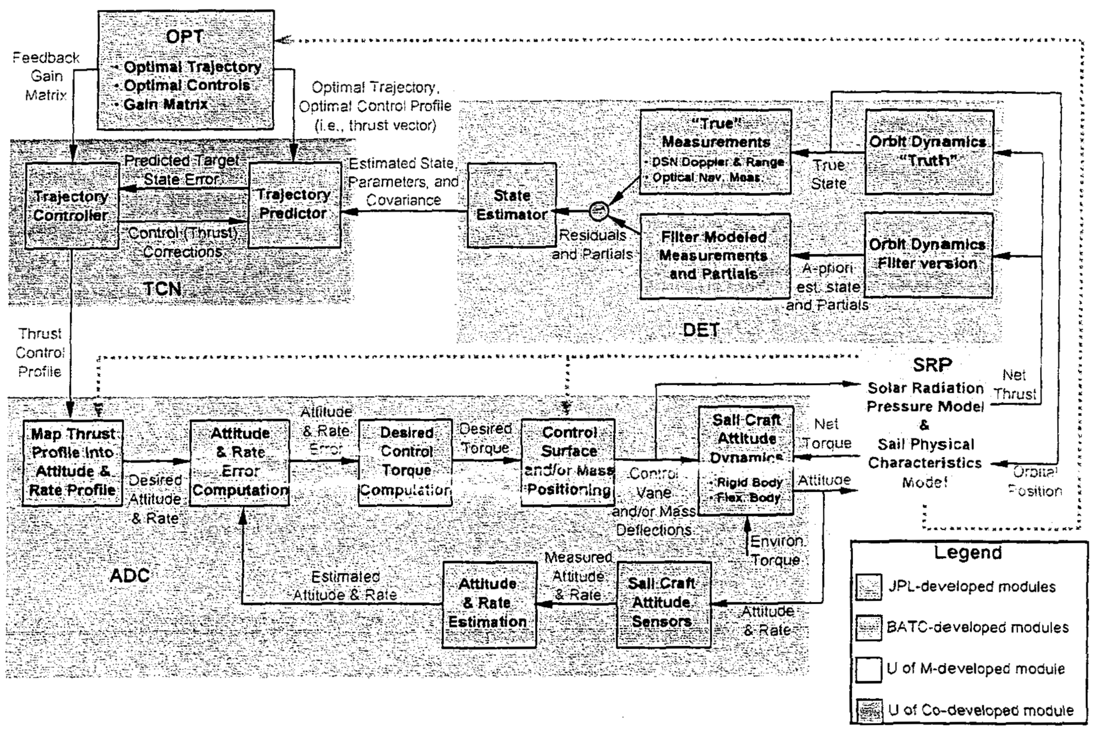

Figure 1 Functional Architecture

Figure 1 illustrates the modular architecture of the tool kit and the interfaces and data flow between the modules when used in the integrated mode. The integrated mode essentially starts with the design of a nominal solar sail trajectory which satisfies user optimality criteria. The optimal trajectory and thrusts are input to the GNC simulation to evaluate the actual sailcraft guidance, navigation and control behavior. Attitude determination and control performance are evaluated along with the navigation accuracy and the performance of the closed loop guidance that corrects the sail back onto an optimal flight path. The Solar Pressure Module is the primary source of solar pressure thrust and torque for all the modules. SRP uses the sail characteristics to transform sail attitude into solar radiation pressure force and torque-accounting for bending of the beams, billowing of the sail and degradation of the reflectivity.

The functional flow of the modules in the integrated mode is described below

1. The design of the solar sail mission is performed by *OPT* which computes the trajectory and controls to optimize a given performance criteria. OPT outputs the *optimal trajectory, optimal controls profile* consisting of direction of solar radiation pressure thrust vector and *gain matrices* for use in the feedback control loop. Assuming that any errors introduced by the GNC will still be within the region of linearity, the design need only be run once during the course of a simulation.

2. Initially, the optimal attitude profile and trajectory generated by the design is input to the Trajectory Control Module (TCN) which for the initial control segment formally passes the controls to the attitude control module (ADC).

3. The Attitude Control System (ADC) Module transforms thrust commands received from the Trajectory Control (TCN) Module into sailcraft attitude commands and then implements an attitude control feedback loop to point the thrust vector in the desired direction. The ADC Module simulates rigid- and flexible-body dynamics, enabling the evaluation of control-structure interaction. ADC enforces limits on actuator deflection angles, sailcraft attitude excursion relative to the sun line, and vehicle angular velocity. Simulated attitude sensor measurements are processed to estimate the attitude and determine the desired control torque. Output includes a profile of the attitude history for each of the sail components which is transformed to thrust and torques by the SRP module.

4. The thrusts based on the implemented attitude profile determined by ADC are input to the Navigation Module (DET) which propagates the solar sail trajectory and simulates the observations for the orbit determination. Errors in the dynamics and observations are accounted for by maintaining orbits and observations for two models-a planned or desired trajectory (which is based on the nominal trajectory from OPT) and a truth model based on a realistic assessment of dynamic parameters models, modeling errors and process noise uncertainty. For each control segment, DET estimates the sailcraft orbit and other unknown modeling parameters.

5. The TCN module uses the current orbit determination solution from DET to compute the deviations from the nominal or desired trajectory and predict the error in the target conditions. An updated thrust control profile is computed using the OPT computed gain matrices in combination with a user provided control law.

6. The TCN updated control thrusts are input to the ADC module- which repeats the control implementation process described in step 3 for the updated trajectory and control profile. Steps 3-5 are repeated for each control update segment until target or terminal conditions are satisfied.

Output to evaluate the performance is retained during the computation for display purposes. This consists of

- Optimal trajectories, control profiles and measures of performance
- Sailcraft attitude, angular velocity and angular acceleration
- Sailcraft control surface deflection angles for vanes systems
- Estimates and uncertainties of spacecraft position and velocity, solar radiation pressure parameters and other dynamic parameters determined by DET

- Updates to the optimal control (corrected commanded control) determined by TCN and error in targeting conditions

SOLAR RADIATION PRESSURE MODELING (SRP)

There are two crucial components needed to properly model the effect of solar radiation pressure on the sail vehicle. First is an accurate model of the solar photon flux, second is the model of the sail itself.
For modeling the pressure generated by the sun we use the following formulae derived in (Ref. 1):

$$P(r) = P^*(r)F(r)$$

$$P^*(r) = \frac{I_0 \pi}{c}\left(\frac{R_s}{r}\right)^2$$

$$F(r) = \frac{2}{3}\left(\frac{r}{R_s}\right)^2 \left\{1 - \left[1 - \left(\frac{R_s}{r}\right)^2\right]^{3/2}\right\}$$

where I_0 is the specific intensity of the sunlight integrated over all frequencies, c is the speed of light, Rs is the solar radius, and r is the distance of the sail from the sun center. This model includes a first order correction for the finite disk of the sun.

The local properties of the sail are of equal importance. The forces act on a differential area element of the sail along the normal $\hat{n}$ and transverse $\hat{t}$ directions and represent the contribution from radiation absorbed by the sail dF_a, reflected specularly dF_{rs} and diffusively dF_{ru} from it, and emitted by radiation from the sail dF_e respectively (Ref. 1).

$$dF_a = P(r) a \cos(\alpha)\left[-\cos(\alpha)\hat{n} + \sin(\alpha)\hat{t}\right] dA$$

$$dF_{rs} = P(r)\cos(\alpha)\rho s\left[-\cos(\alpha)\hat{n} - \sin(\alpha)\hat{t}\right] dA$$

$$dF_{ru} = -P(r)\cos(\alpha) B_f \rho(1 - s)\hat{n} dA$$

$$dF_e = -P(r)\cos(\alpha)(1 - \rho)\frac{\varepsilon_f B_f - \varepsilon_b B_b}{\varepsilon_f + \varepsilon_b}\hat{n} dA$$

decomposing the forces into the normal and tangential components yields:

$$dF_n = -P(r)\left[(1 + \rho s)\cos^2(\alpha) + B_f(1 - s)\rho\cos(\alpha) + (1 - \rho)\frac{\varepsilon_f B_f - \varepsilon_b B_b}{\varepsilon_f + \varepsilon_b}\cos(\alpha)\right] dA$$

$$dF_t = P(r)(1 - \rho s)\cos(\alpha)\sin(\alpha) dA$$

In the above, α is the angle between the local sail normal and the incident light, ρ is the reflectivity, a is the absorptivity, s is the fraction of light reflected specularly, B is a coefficient describing the deviation of the surface from a Lambertian surface, ε is the surface emissivity, and the f and b subscripts correspond to the front and back of the sail surface, respectively.

The total force and moment acting on the sail is then given by integrals over the entire surface of the sail, assuming that self-shadowing does not occur:

$$F = \int_A (\hat{n} dF_n + \hat{t} dF_t)$$

$$M = \int_A \rho \times dF)$$

where $\int_A$ denotes an integration over the sail surface area, ρ is the location of the sail element dA and M is the total moment acting on the vehicle about the coordinate origin defined in the sail-fixed frame.

At the heart of the solar radiation pressure (SRP) module are precise models of the sailcraft that predict the total force and moment acting on the vehicle. Two options are available for users to input these sail models. Fort he case in which the mission design has progressed to the point where a detailed hardware design is available, sail designers are expected to provide solar pressure forces and moments for the main sail and vanes, along with mass properties for systems utilizing center of mass-center of pressure offset for control. A standardized input format has been defined by Billy Derbes of L'Garde Inc (Ref. 2) for characterizing the sail which is based on the sail designer (in this L'Garde) providing a table of point values of forces and moments in the form of dimensionless coefficients. The computation of the forces and moments by the sail designer is assumed to account for sail and vane shape effects, billowing, beam deflections as well as surface reflectivity properties. The computed forces and moments are tabulated as a function of attitude relative to the sun and distance from the sun. This form of sail model is very intensive to define and compute, but would likely be the model of choice for flight of an actual sail vehicle.

An alternate model is available for case studies in which detailed results for the sail design are not available. This model analytically computes the forces and moments based on user specification of sailcraft components including sail shape, size, mass, reflectivity properties and body fixed location of the sail components. The toolkit will use a new methodology that reduces the force and moment representation to a series of precomputed constants independent of incident light pressure and direction (Ref.3). This method assumes the sail will not change shape as the attitude varies and that there is no self shadowing. Given these constants, the sail reflectivity properties and the incident light pressure and direction, the total force and moment acting on the sail is explicitly computed. Partial derivatives of the force with respect to reflectivity parameters are computed for use by DET to update the estimate of force and the solar radiation pressure parameters.

To compute the force and moment from the sail surface requires us to map the location of the sun into the sail-fixed frame. Given the position vector of the sail with respect to the sun r and the orientation of the sail normal $\hat{n}$ a unit vector can be defined as

$$\hat{r} = \bar{r} / \|\bar{r}\|$$

then the sun-line angle α and tangential vector $\hat{t}$ are found from:

$$\alpha = \arccos(-\hat{n} \cdot \hat{r})$$

$$\hat{t} = \frac{-\hat{n} \times \hat{n} \times \hat{r}}{|\hat{n} \times \hat{n} \times \hat{r}\ |}$$

The normal vector $\boldsymbol{n}$ must be known before α and $\hat{t}$ can be computed and is found from the sail attitude. Given the sail attitude in quaternion form, the rotation matrix is given by:

$$R = (q_4^2 - q^T q)I + 2qq^T - 2q_4 S(q)$$

where

$$q = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^T and \ldots S(q) = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}$$

Assuming that R transforms the coordinates from inertial to body-fixed frame and the normal vector is along the body fixed z-axis, the normal vector inertial coordinates are given by:

$$\hat{n} = R^T z_{bf}$$

where the subscript ***bf*** denotes the body-fixed frame. In this case, the normal vector inertial coordinates are contained in the third row of R.

MISSION DESIGN AND TRAJECTORY OPTIMIZATION MODULE (OPT)

OPT – the Trajectory Optimization Module – determines an optimal trajectory and sail control strategy that minimizes a given performance criteria which weights minimum transit time and minimum control effort, subject to control and state constraints. That is, the problem solved by OPT is:

Given the (n x 1) state vector X, and the (m x 1) control vector u, the performance index that is the basis for computing the optimal trajectory is given in a general form by:

$$J = \phi(X_f, t_f) + v^T \psi(X_f, t_f) + \int_{t_0}^{t_f} L(X, u, t) + \lambda^T(t)\{f(X, u, t) - \dot{X}\}dt \qquad (1)$$

in which:

$\phi(X_f, t_f)$ is a terminal index, i.e. a scalar function of the state to be minimized at the final time.
$\psi(X_f, t_f)$ is a (p x 1) vector of terminal constraints, $p \leq n$, which are functions of the final state X_f at the terminal time t_f to be satisfied by the optimal solution.
L(X, u, t) is a Lagrangian, i.e. a scalar function of state, control and time the integral of which is to be minimized over the time interval t_0 through t_f.
f(X, u, t) is an (n x 1) vector of dynamics constraints, equal to X, the time derivative of the state vector.
$\lambda(t)$ and ν are vectors, (n x 1) and (p x 1) respectively, of Lagrange multipliers.

Next, we discuss the formulation of the elements of J. The Lagrangian for a minimum time-of-flight solution is given simply as:

$$L(X, u, t) = 1.0 \tag{2}$$

A performance index which weights the goals of minimizing time of flight and also minimizing control effort is:

$$L(X, u, t) = W_t + u^T(t)W_c u(t) \tag{3}$$

Here, W_t and W_c are weights for minimum time and minimum control, respectively; W_t is a positive semi-definite scalar and W_c is a positive semi-definite (m x m) matrix. This performance index can be appended with function vectors C(u(t)) and S(X(t)) that impose inequality constraints on the control and state vectors:

$$L(X, u, t) = W_t + u^T(t)W_c u(t) + m^T C(u(t)) + r^T S(X(t)) \tag{4}$$

The elements of the multiplier vectors m and r are functions of the corresponding elements of C and S, respectively, such that if the constraints are satisfied at time t, the elements of m and r are set equal to 0 at time t. Conversely, if the constraints are violated at time t, yielding non-zero positive-valued elements of C and/or S, the elements of m and/or r corresponding to the violated constraint elements are set equal to 1 at time t. To provide a very general optimization framework, the Lagrangian to be employed in OPT is the one given in equation. (4). The terminal index ϕ is chosen for OPT to be zero, as there is no scalar function of the terminal state that is being minimized. However, terminal constraint vector ν will in general be non-zero, and is an arbitrary (user-provided or user-selected) function of the state vector at the final time. This module generates the optimal control and control gain matrices which are used by the simulation and control modules.

The OPT algorithm involves two optimization steps. A free-final time, first order gradient search algorithm is used to find a preliminary guess for the trajectory and control history. Then a free-final time, second order gradient optimization (i.e. neighboring-extremal) algorithm is used to converge to the optimal trajectory. These gradient method numerical algorithms are well-known and discussed in the literature, c.f. (Ref 4). The primary inputs provided by a user are:

- the start time
- sailcraft's design parameters (sail and payload masses, sail dimensions, etc.)
- initial six-degree-of-freedom state vector, consisting of position and velocity, an initial sailcraft attitude and body rate and a body-axis rotation rate vector
- an initial guess for the sailcraft control history, expressed as a time series of discrete sun-incidence
- and clock angles describing the orientation of the sail normal vector
- information about the desired target, to constrain the terminal state
- state constraints on the allowable attitudes and distances from bodies including the sun
- control constraints on body rotation rates, and also a limit on the magnitude of the torque that can be applied to the sailcraft

On this last point regarding the torques, the OPT module does not utilize explicit modeling of a specific attitude control systems, such as a sailcraft vane system or an articulated mass system [see discussion in (Ref. 1) or in papers by Wie (Ref 5.) or Mettler and Ploen (Ref 6.). Instead, OPT acts upon constant, averaged torques acting over a discrete control time segment. This treatment will be discussed in a future paper about the OPT mathematics and results.

The primary outputs, which are used by other modules or can be directly accessed by the user for plotting and analysis, are the state vector history versus time, the control history, and the gradients generated in the second-order method. The latter gradients are available for use in the TCN module, and the controls can be altered in TCN or used directly in the ADC and DET modules.

ATTITUDE DYNAMICS AND CONTROL MODULE (ADC)

Figure 2 depicts the functional aspects of the Attitude Dynamics and Control (ADC) Module. The ADC module converts trajectory control commands (i.e., desired thrust) into vehicle attitude commands and then implements an attitude control feedback loop. Key elements of the ADC module are the algorithms for mapping (i) commanded solar sail thrust into desired vehicle attitude and (ii) desired attitude control torque into actuator deflection commands. Both algorithms employ vehicle reflective surface characteristics captured in the solar radiation pressure model to execute their respective mapping processes. The ADC Module may be run as a stand-alone module to evaluate detailed attitude control system performance using control update periods smaller than standard navigation update time scales, or it may be run with the integrated SSIST simulation.

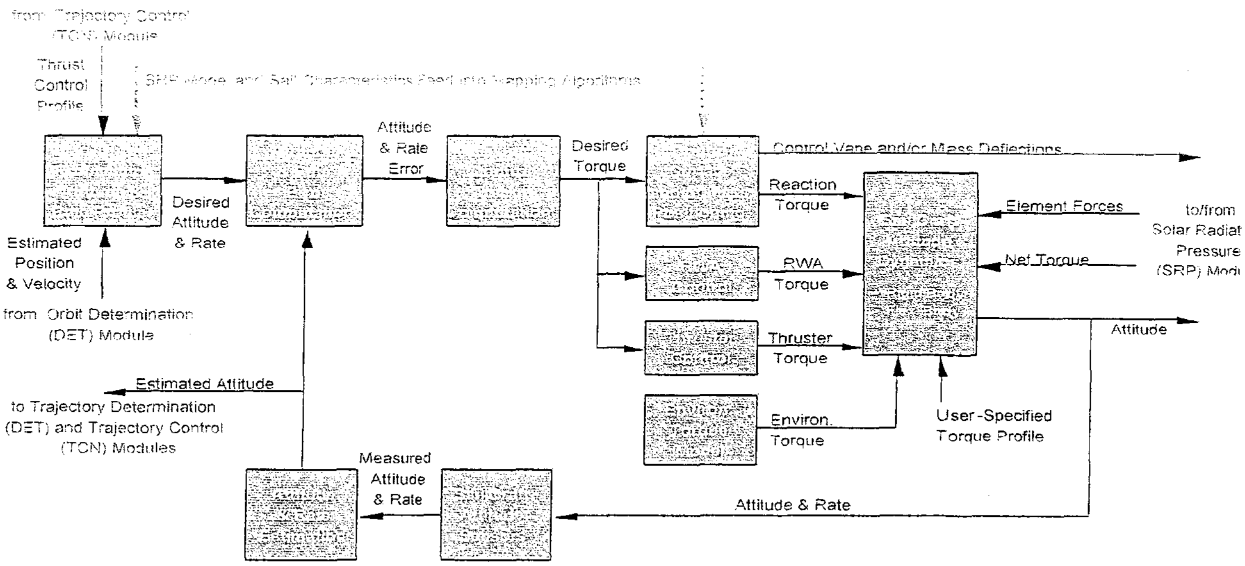

Figure 2. Attitude Dynamics and Control Module Block Diagram for the SSIST

Features of individual functional blocks shown in Figure 2 are discussed in detail below.

The first functional block in the ADC Module maps TCN-derived thrust commands into desired sail attitude. This mapping algorithm employs sail physical dimensions and surface reflective characteristics along with sailcraft ephemeris to calculate the required sail attitude to point the thrust vector in the desired direction. The problem formulation is summarized below.

Figure 3 illustrates an ideal flat sail whose surface normal vector $\hat{\mathbf{n}}$ is oriented at a sun incidence angle α relative to the sun-to-sailcraft position vector $\mathbf{r}$. For a perfectly reflective sail surface, the thrust induced by solar radiation pressure would be aligned with the surface normal vector (because the angle of incident photons equals the angle of reflected photons). On the other end of the spectrum, a sail surface that absorbs all incident photons would produce a thrust vector aligned with the sun-to-sailcraft position vector.
In practice, a sail surface will exhibit both reflection and absorption, and so the sail thrust vector lies somewhere in the plane between the (negative of the) sail surface normal vector (as depicted in the figure) and the sun-to-sailcraft position vector.

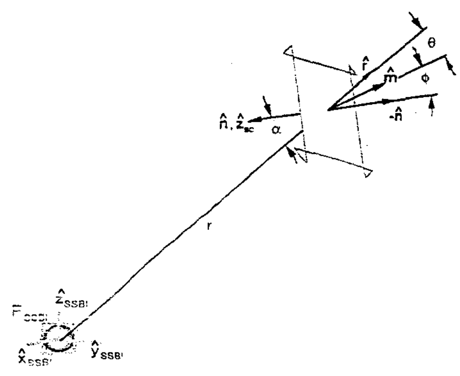

Figure 3. Thrust, Normal, and Position Vector Relationship for Ideal Flat Solar Sail

Cone angle θ is defined as the angle between the thrust direction unit vector $\hat{m}$ and the sun-to-sailcraft position unit vector $\hat{r}$. Center-line angle ϕ is defined to be the angle between the thrust direction unit vector and the (negative of the) sail surface normal vector. Sun incidence angle equals the sum of the center-line angle and the cone angle.

It is assumed that sailcraft position and velocity are known quantities and that desired thrust is specified. As such, the cone angle is determined from the inner product of $\hat{m}$ and $\hat{r}$, and an expression for the center-line angle is derived using the equation for sail force induced by solar radiation pressure which is a function of sun incidence angle. Assuming that the sail surface is not perfectly reflective, the derivation yields a quartic polynomial for the sine of the center-line angle.

Once the desired sailcraft attitude has been determined, a user-specified limit is applied to maintain the sail within the maximum desired excursion angle between the sail normal vector and the sun direction vector. Similarly, limits on maximum allowed angular velocity are imposed.

The second functional block shown in Figure 2 computes attitude and angular rate errors, taking the difference between desired and estimated states. The error signals drive a proportional-derivative (PD) compensator stage, identified in the figure as the "Desired Control Torque Computation" block, to produce sailcraft control torque signals. The PD compensation stage applies user-specified limits to the control torque magnitude to preclude large attitude and rate errors from producing physically unachievable torque commands.

Desired control torque signals are mapped into control vane and/or boom gimbal deflection angle commands to orient the sailcraft in the desired attitude. This mapping procedure occurs in the functional block "Control Surface and/or Mass Position" of Figure 2. The user may select the sailcraft attitude control architecture, either an articulated control vane implementation or a mass displacement approach (i.e., mass on a gimbaled boom to alter the mass center position relative to the center of pressure).

To derive the formula for control actuator deflection command generation, we begin with a nonlinear expression for the torque τ induced on a solar sail as a function of control actuator state (e.g., control vane deflection angles $\{\theta_i\}$), sail attitude $\mathbf{q}$ relative to the sun direction vector, and solar radiation pressure model parameters as shown in Equation (5).

$$\tau = \mathbf{f}(\{\theta_i\},\mathbf{q}) \tag{5}$$

Equation (5) is linearized about the current vehicle state to obtain an expression for the incremental change in torque resulting from an incremental change in the actuator deflection state as shown in Equation (6).

$$\begin{aligned} \delta\tau &= \left[\frac{\partial}{\partial\{\theta_i\}}(\mathbf{f}(\{\theta_i\},\mathbf{q})) \right] \delta\{\theta_i\} \\ &= \mathbf{M}(\{\theta_i\}_n,\mathbf{q}_n)\delta\{\theta_i\} \end{aligned} \tag{6}$$

A linear relationship for vane deflection angle as a function of torque command is obtained by calculating the pseudo-inverse of the Jacobian matrix $\mathbf{M}(\{\theta_i\}_n,\mathbf{q}_n)$ as shown in Equation (7).

$$\delta\{\theta_i\} = \mathbf{M}^{\dagger}(\{\theta_i\}_n,\mathbf{q}_n)\delta\tau \tag{7}$$

This expression is valid over small variations about the nominal state. Singular conditions arise in the Jacobian matrix for certain sailcraft orientations, and so the matrix is adjusted to mitigate the problem and enable a pseudo-inverse to be calculated.

Changes in commanded torque produce changes in the actuators' desired deflection angles, and so actuator deflection commands are updated by adding the desired angular changes to the current actuator state. User-specified limits on maximum actuator deflection angles are applied. Actuator states and sailcraft attitude are provided to the Solar Radiation Pressure (SRP) Module to compute the resulting thrust and torque applied to the sailcraft.

SRP-generated torque (or forces on constituent sailcraft elements) feed into the functional block "Sailcraft Attitude Dynamics". This module models the vehicle's rigid body and flexible body dynamics. The dynamics module incorporates torque from conventional attitude control actuators (e.g., reaction wheels and thrusters), environmental disturbance sources (e.g., gravity gradient, aerodynamic, and magnetic moment), and a user-defined torque profile (e.g., out-gassing effects after initial sail deployment). Reaction force and torque from articulated actuators are modeled, allowing the user to assess control-structure interaction.

The sailcraft attitude dynamics module generates true attitude and angular velocity states. These states are sent to the "Sailcraft Attitude Sensor" module, where the attitude measurement process is modeled. Simulated sensor measurements are passed to the "Attitude and Rate Estimation" module, which produces attitude and angular velocity estimates to complete the attitude control feedback loop.

Data input to the ADC Module are summarized in Table 1. The SSIST Toolkit user provides information defining sailcraft physical characteristics (e.g., sailcraft mass and inertia tensor, sail reflective surface area, size and position of control actuators). The user selects the type of attitude control method to be used (i.e., articulated control vanes versus gimbaled mass) as well as whether to supplement attitude control with conventional spacecraft actuators (e.g., RWA or thrusters). ADC employs user-defined solar radiation pressure model parameters for mapping thrust commands in to attitude commands and for transforming control torque commands into actuator deflection commands. Other user-defined characteristics include limitations on achievable attitude slew rates, actuator deflection angle limits, and maximum angle allowed for pointing the solar array normal away from the sun-to-sail line. The ADC module accepts a user-defined external torque time history, allowing the evaluation of potential disturbance torque sources such as out-gassing after initial deployment.

In addition to user-defined parameters, the ADC Module receives inputs from other SSIST modules. To determine desired sailcraft attitude, the ADC Module requires a commanded thrust from the Trajectory Control Module and current position and velocity from the Orbit Determination Module. The SRP Module provides net torque (or individual element forces) imparted to the sailcraft by solar radiation pressure, given attitude and actuator deflection states.

Table 1. ADC MODULE INPUT DATA

Input Data	Source of Input
Thrust Control Command	Trajectory Control Module
Sailcraft Position and Velocity	Orbit Determination Module
External Solar Radiation Pressure Torque	Solar Radiation Pressure Module
External Solar Radiation Pressure Forces	Solar Radiation Pressure Module
Attitude Control Method (e.g. articulated vanes, mass displacement) and Parameters	User
Solar Radiation Pressure Model Parameters	User
Sailcraft Physical Characteristic Parameters	User
User-Defined External Torque	User
Achievable Maneuver Rates, Actuator Deflection Limits, Maximum Attitude Excursion from Sun Line	User

Table 2 describes the data output from the ADC Module. All of the variables tabulated below (truth model outputs, simulated measurements, and estimated states) are provided to the SSIST Toolkit user for data logging, plotting, and analysis. True attitude and (true) control actuator states (e.g., control vane deflection angles, gimbal angles for mass displacement control) are provided to the SRP Module to compute sailcraft external torque and force due to solar radiation pressure. Attitude measurements from stellar-inertial sensors and estimated attitude are available to the Trajectory Control (TCN) Module and Orbit Determination (DET) Module. TCN may use attitude information in calculating achievable thrust vector commands. DET may use attitude information along with navigation data to calibrate sailcraft thrust as a function of attitude and to estimate sail thrust performance parameters

Table 2. ADC MODULE OUTPUT DATA

Output Data	**Destination of Output**
Sailcraft True Attitude	Solar Radiation Pressure Module, User (for analysis or plotting)
Sailcraft True Angular Velocity	User (for analysis or plotting)
Sailcraft True Angular Acceleration	User (for analysis or plotting)
Sailcraft Measured Attitude	User (for analysis or plotting), Orbit Determination Module, Trajectory Control Module

Sailcraft Rate Gyro Measurements	User (for analysis or plotting), Orbit Determination Module, Trajectory Control Module
Sailcraft Estimated Attitude	Orbit Determination Module, Trajectory Control Module, User (for analysis or plotting)
Sailcraft Estimated Angular Velocity	User (for analysis or plotting)
Control Surface Deflection Angles	Solar Radiation Pressure Module, User (for analysis or plotting)
Translating Control Mass States	Solar Radiation Pressure Module, User (for analysis or plotting)
Gimballed Control Mass States	Solar Radiation Pressure Module, User (for analysis or plotting)

NAVIGATION AND TRAJECTORY DETERMINATION MODULE (DET)

The navigation or orbit determination module DET is the basic component of the Solar System Simulation Tool which evaluates the navigation performance of the sailcraft. DET simulates the trajectory and the observational data required to estimate the spacecraft orbit, evaluates data strategies for satisfying navigation accuracy requirements and analyzes sensitivities of estimates of state and solar pressure parameters to error sources, measurements and filtering strategy. The current state (position and velocity) of the trajectory and uncertain dynamic parameters are estimated based on processing simulated ground based and on board observations. Measurements may consist of radiometric observations (range and Doppler) from the Deep Space Network and on-board optical and accelerometer measurements. DET models the spacecraft dynamics, propagates the trajectory by integrating the equations of motion, simulates the measured observables and processes the observations using a Kalman type filter to estimate the orbit, dynamic modeling parameters and their uncertainties. Solar radiation pressure "thrust" forces used in the trajectory propagation are derived from either the TCN, OPT or ADC modules.

Navigation performance may be evaluated by conducting Monte Carlo simulations or by formal covariance analysis based on a single run. Monte Carlo analysis (as illustrated in Figure 1) is based on generating trajectories and observables for two dynamics models, a ***truth model*** and ***a planned or desired model***. The truth model is the result of propagating the trajectory starting with a perturbed nominal initial state from OPT and using realistic error models and the true controls output by ADC. These are essentially controls which are implemented by the ADC and include effects due to attitude and rate estimation errors, rate constraints and sensor errors. The planned or desired model uses the (unperturbed) initial conditions from the mission design module, and thrust controls which can either be the corrected controls output by TCN or the computed controls implemented by ACS neglecting attitude errors introduced by the feedback loop. Dynamic modeling assumptions and error models for estimated parameters and observations may differ for the two models. The Monte Carlo capability enables users to evaluate rates of convergence, mismodeling errors and effects of nonlinearities. Each Monte Carlo run represents a single realization from a random sampling of the errors that affect the initial conditions and modeling errors.

The DET module can be used in an integrated mode in conjunction with other modules or as a stand alone navigation analysis tool.

1. Integrated Tool Usage: For this mode of operation, simulated trajectories and observations are generated for the true and planned models for each control update span. Estimates of the state and solar pressure parameters are computed by DET and are output to the TCN module. TCN evaluates the solar sail performance based on the current state estimate, updates the control thrust vectors commands required to achieve the optimal flight path and outputs the predicted thrusts to ADC module. ADC simulates the attitude control and dynamics process required to implement the thrust and provides implemented attitude and thrusts (via SRP module) to DET.

2. Stand Alone Usage: In this mode, DET serves primarily for navigation analysis either by Monte Carlo simulations or covariance analysis. A simulated trajectory and observations are used to evaluate observational data requirements and strategies, study sensitivity of estimates to modeling errors and to evaluate the capability to update the solar radiation pressure parameters. The source of thrust commands

can be TCN thrusts which have been corrupted by noise to represent the true thrust or OPT thrusts. Output includes estimates and covariance of the unknown parameters.

Modeling and Filter Assumptions

Dynamic models include gravitational forces for Sun, Earth or any planets; gravitational spherical harmonics, solar radiation pressure acceleration and maneuvers. Variational equations for the estimated dynamic parameters are integrated along with the state equations in a J2000 frame using standard exportable JPL planetary ephemeris. Source of the solar radiation pressure forces are the OPT/TCN updates for the planned case and results of the ADC attitude implementation for the truth model.

Trajectories are propagated for a time interval that corresponds to the time of the next scheduled control update. True observations are computed for the measurements in the control update interval using the true trajectory and the trajectory for the planning model is used for the computed observables and partials. Simulated observations are based on user input measurement schedule. A batch sequential filter processes the difference between the true and computed observations to estimate the current position and velocity and the spacecraft state parameters. The filter includes options to treat certain estimated parameters stochastically by modeling the dynamics as a Gauss-Markov process corrupted by white or correlated process noise. This capability is expected to be useful for stochastic treatment of solar pressure effects. JPL's legacy software which has been used extensively to support deep space and earth orbiter missions is the source of the models for the dynamics and observations.

If **X(t)** is the spacecraft state at time t, which includes the position and velocity of the spacecraft and additional estimated constant and stochastic parameters- then the corrections to the state $\delta\hat{x}$ based on processing data over the control interval **[t-h, t]** are given by

$$\delta\hat{x}(t) = \delta\tilde{x}(t) + K[\delta z - H\delta\tilde{x}(t)]$$

where

$\delta\tilde{x}$ is the predicted state and $\delta\hat{x}$ the corrected state

δz is the difference between the true and computed observations

H is the partials of the observations $z(t_k)$ with respect to the estimated state $\delta z(t_k)/\partial x(t-h)$

and **K** is the gain matrix which is computed as follows

$$K = \tilde{P}H^T[H\tilde{P}H^T + R]^{-1}$$

where **R** is the measurement noise covariance

$\tilde{P}(t)$ is the predicted state covariance at time t given by

$$\tilde{P}(t) = \Phi(t-h,t)\hat{P}(t-h)\Phi^T(t-h,t) + Q(t)$$

Where $\Phi(t-h,t)$ is the transition matrix $[\partial x(t)/\partial x(t-h)]$ which is computed by integrating the variational equations

Q(t) is the noise on the stochastic parameters

and $\hat{P}$ is the a posterior covariance of the state given by

$$\hat{P} = \tilde{P} - \tilde{P}H^T(H\tilde{P}H^T + R)^{-1}H\tilde{P}$$

The updated state output to the TCN module is given by

$$\hat{X}(t) = \tilde{X}(t) + \delta\hat{x}(t)$$

and the uncertainty of the estimate is given by $\hat{P}(t)$. The actual implementation of the above filter will be based on the batch sequential square root formulation given by Bierman (Ref.7).

Mandatory user input for DET consists of dynamics modeling parameters, observational data types, measurement data schedule, selection of parameters to be estimated, filter strategy, a priori uncertainty for estimated parameters, observational data noise characteristics and characteristics of the process noise. For the stand alone mode of operation, the user may have to input an initial state vector and a solar pressure thrust profile. The module outputs the estimated states, updated solar radiation pressure parameters and covariances for the estimated parameters. The statistics of the difference between the true and planned estimates are retained for navigation performance.

TRAJECTORY CORRECTION MODULE (TCN)

The purpose of the trajectory control module (TCN) is to produce thrust demands T_d for the ADC module to implement via sail pointing. These desired T_d vectors result from predictions of sailcraft state (position and velocity) estimates $\hat{X}$ obtained from the DET module, in comparison with the nominal trajectory X^N computed by the OPT module. When $\hat{X}$ is equal to X^N, the nominal thrust T^N is used. Trajectory errors $X^N - \hat{X}$ are reduced by the addition of thrust changes to T^N resulting in two types of trajectory control laws.

The first type is termed a nominal control law, since it uses state feedback gains arising from the OPT trajectory optimization. This optimization may take into account some perturbations in sailcraft dynamics or environmental disturbances, but is unlikely to accurately represent actual conditions on a particular sailcraft. Indeed, the purpose of the simulation is to examine the effects of these parameter and disturbance variations on a particular sailcraft/mission design. The nominal control law produces the nominal thrust when $\hat{X}$ is equal to X^N:

$$T^N = G^N(X^N).$$

When the estimated state is used instead of x^N, the uncorrected thrust T results

$$T = G^N(\hat{X}).$$

Note that the control law G^N provides a feedback loop which reacts to deviations from the nominal trajectory, resulting in a change in thrust to reduce the trajectory error $X^N - \hat{X}$. The feedback gains in this control law are determined indirectly by the optimization module's performance index, whose partial derivatives are used to produce a convergent iteration for the optimal (nominal) trajectory. Therefore, these feedback gains are obvious candidates for steering a perturbed trajectory back to nominal, and can be expected to produce close tracking of the nominal trajectory when disturbances are small or infrequent.

The second type of control law provides alternative feedback gains, resulting in a corrected thrust T_c. This enables more aggressive trajectory error correction, for example, by employing various methods for designing feedback gains directly. This can be thought of as a correction control law G_c added to the nominal control

$$T_c = G^N(X^N) + G_c(\hat{X} - X^N)$$

which provides a local correction, i.e. to track the original nominal trajectory. In cases of extreme perturbations, it may be advantageous to re-optimize the trajectory beginning at the current state, resulting in an entirely new control law

$$T_c = G_g(\hat{X})$$

which produces a more global optimization of trajectory and smaller target state errors.

In some segments of the mission, a trajectory may be simply a desired equilibrium state, for example in sub-L1 station keeping missions such as GEOSTORM. Here, direct control over feedback gains to regulate excursions from the equilibrium may be desirable .

Figure 4 shows a top-level block diagram of the TCN module architecture illustrating nominal and corrected control laws, user interfaces and design aids, and connections with DET, OPT, and ADC modules..

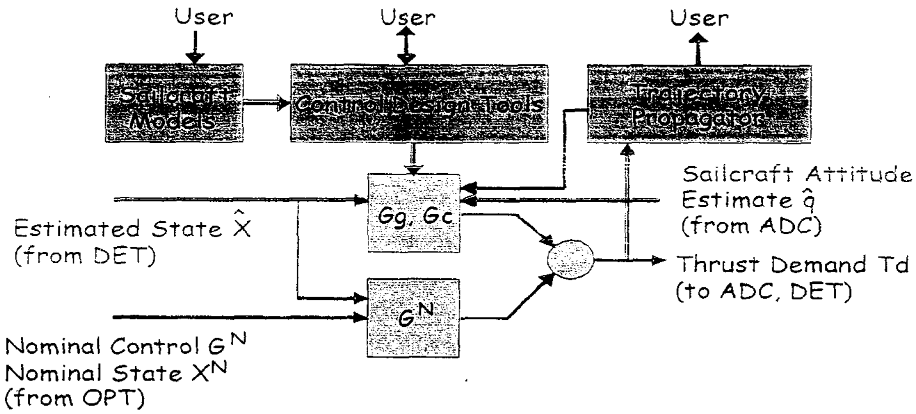

Figure 4. TCN Architecture

TCN Development Issues

Although sailcraft have advantages in propulsion over other spacecraft, trajectory control is made difficult by the restriction of the thrust vector to a two dimensional manifold. The spacecraft dynamics are essentially decoupled, having only very weak coupling from gravitational interaction with other bodies. This produces a two-dimensional subspace in the state space of spacecraft position and velocity which is poorly controllable. Success in correcting perturbations via trajectory control is therefore highly dependent on the alignment of the poorly controllable subspace relative to the spacecraft natural dynamics and disturbances. This alignment, in turn, is affected by choices of nominal trajectories and operating modes of the sailcraft. Tools are to be developed for the TCN module that allow the user to assess the degree of controllability and thrust vector correction motions required for particular mission designs and particular disturbances. These can be used to trade various mission designs, leading to likely trajectory candidates for detailed control design and simulation using the entire toolkit suite

Control design will be supported by providing several basic design and analysis tools. Initially, local control will be supported by non-linear dynamics linearization, pole placement and linear quadratic design tools, together with non-linear simulation of controlled behavior. Particular reference missions (e.g. GEOSTORM) will be used to drive requirements for these tools, and to provide examples of their use. Tools for more global re-optimization may be developed if needed for these reference missions.

SUMMARY

This paper has described an integrated set of software tools being developed for use by the solar sail community to design and analyze the guidance, navigation and control of solar sail missions. The software toolset is being implemented to run on Windows based systems that have access to MATLAB. An intermediate delivery of the software is scheduled for delivery in June 2004 for beta testing with the final version being delivered in June 2005. This intermediate delivery will not include the full range of high fidelity capabilities however it will include sufficient capabilities to be used for mission design and GNC simulation. It is expected that the experience and feedback gained from users of this delivery will be incorporated in the final delivery-especially user's comments on the capabilities, ease of use of the program and the display and output provisions. SSIST testing and validation is expected to rely on using existing JPL and Ball Aerospace legacy software to independently generate test cases for comparing and validating results for the SSIST modules. Test cases will be developed using an existing GEOSTORM optimal trajectory and control profile for the transfer and the sub L1station keeping phases that has been extensively studied. The final version of the software delivery will be demonstrated by applying the mission design and simulation capabilities using realistic data on sail craft characteristics derived from the Part 4A of the NRA program.

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