Evaluation of an Energy-Cutoff Algorithm for the Saturn Orbit Insertion Burn of the Cassini-Huygens Mission

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1 Introduction

The Cassini-Huygens spacecraft was launched on October 15, 1997 as a joint NASA/ESA mission to explore Saturn. After a 7-year cruise the spacecraft will enter orbit around Saturn on 1 July 2004 for a 4-year investigation of the Saturnian system. This paper describes the navigation-related aspects and analysis of the burn cut-off algorithm for the orbit-insertion burn. The algorithm is designed to achieve the desired orbital period in spite of possible burn delays and restarts.

The Saturn-Orbit Insertion (SOI) maneuver is arguably the most important in the Cassini-Huygens mission as the opportunity to achieve an elliptical orbit around Saturn passes in a matter of hours. The spacecraft approaches Saturn with a V_∞ of 5.2 km/s. SOI, with a nominal cost of 626 m/s, slows the spacecraft into a Saturn-relative orbit with an orbit period of 116 days (Later, the Periapsis-Raise Maneuver increases that period to 124 days.) SOI primarily alters the semi-major axis of the Saturn-relative orbit; therefore, the burn is best executed nearly parallel to the spacecrafts Saturn-relative velocity.

A second main engine is available to complete the maneuver after some interruption. This capability is greatly enhanced by rotating the spacecraft during SOI. The burn direction during SOI is designed with a constant rotation rate of 0.008°/s (slightly slower than the hour hand of a typical clock).

It has been proposed that the Cassini-Huygens mission use an energy-based cutoff algorithm for SOI. There are a series of approximations that have been made in designing this algorithm from the perspective of navigation. The most significant approximation to investiage is from linearizing the energy integral about the nominal SOI. To make a long story short, an algorithm using this linearization will see about 2 m/s error in reasonable single-delay cases.

Other approximations are being made. Use of the conic approximation has been proposed to enable the use of the on-board propagation software, the Inertial Vector Propagator (IVP). The algorithm ueses a simple Euler

integration and ignores the half-RTI[1] mistmatch between IVP velocity data and Attitude Estimator (ATE) accelerometer data. Intuitively, these errors should be small.

2 Linearization

Orbital energy for a two-body conic is $E(t) = -\mu/r(t) + (1/2)\vec{V}(t) \cdot \vec{V}(t)$, where $r(t)$ is the spacecraft's distance and $\vec{V}(t)$ is the spacecraft's velocity. If an acceleration $\vec{a}_{SOI}(t)$ is applied to the spacecraft, then the change in this energy may be found with

$$\Delta E(t) = \int_0^t \vec{V}(\tau) \cdot \vec{a}_{SOI}(\tau) d\tau \tag{1}$$

Assuming that the influence of $\vec{a}_{SOI}$ is small enough to be linear, then variations in the spacecraft state may be written as

$$\begin{bmatrix} \delta\vec{r}(t) \\ \delta\vec{V}(t) \end{bmatrix} = \Phi(t, t_0) \begin{bmatrix} \delta\vec{r}(t_0) \\ \delta\vec{V}(t_0) \end{bmatrix} + \int_0^t \Phi(t, \tau) \begin{bmatrix} 0 \\ \vec{a}_{SOI}(\tau) \end{bmatrix} d\tau$$

If we partition the state transition matrix

$$\Phi(t, \tau) = \begin{bmatrix} \Phi_{11}(t, \tau) & \Phi_{12}(t, \tau) \\ \Phi_{21}(t, \tau) & \Phi_{22}(t, \tau) \end{bmatrix}$$

then, with $\delta\vec{r}(t_0) = 0$ and $\delta\vec{V}(t_0) = 0$, we can write the spacecraft's velocity as

$$\vec{V}(t) = \vec{V}_{no-SOI}(t) + \int_0^t \Phi_{22}(t, \sigma) \vec{a}_{SOI}(\sigma) d\sigma$$

where $\vec{V}_{no-SOI}(t)$ is the spacecraft's velocity time-history if SOI is not executed. Substituting this into Equation 1, the equation for ΔE, above, gives

$$\Delta E(t) = \int_0^t \vec{V}_{no-SOI}(\tau) \cdot \vec{a}_{SOI}(\tau) d\tau + \underbrace{\int_0^t \int_0^t [\Phi_{22}(\tau, \sigma) \vec{a}_{SOI}(\sigma)] \cdot \vec{a}_{SOI}(\tau) d\sigma d\tau}_{\text{second-order in } \vec{a}\text{; throw away}}$$

[1]Real-Time Interrupt, for Cassini-Huygens it is about 1/8 second.

Throwing out second-order terms to gives

$$\Delta E(t) = \int_0^t \vec{V}_{no-SOI}(\tau) \cdot \vec{a}_{SOI}(\tau) d\tau \tag{2}$$

This, then, is the change in energy, linearized for "small" accelerations. In order to validate the assumption that SOI is a small acceleration, simulations of the ECB algorithm (with and without this linearization) have been performed.

3 Evaluating the Accuracy of the Linearized Model

The basis of the ECB algorithm is to continue execution of the SOI maneuver until the computed change in energy matches the specified target. The computed change in energy accounts for the measured acceleration and, therefore, any periods during which the maneuver is suspended, viz. while the spacecraft recovers from a fault.

The interest here is how much of a difference it makes when the computed energy is based on Equation 1 versus 2. The singular difference between these two equations is that the spacecraft's velocity in Equation 1 includes the effects of the SOI maneuver but Equation 2 uses $\vec{V}_{no-SOI}(t)$, which does not include any effect due to SOI.

The SOI ΔV results from two simulations, one using the linearized algorithm and the other based soley on Equation 1, were compared for different fault times and delay durations. A contour plot may be made of this difference in SOI ΔV; such a plot is shown in Figure 1. This plot shows the difference in SOI ΔV for an algorithm based on Equation 1 versus an algorithm based on Equation 2.

The left-hand border of the plot represents SOI cases without any fault, the delay durations are zero; therefore, both algorithms give the same result along the left-hand border. The top border represents SOI cases where the fault occurs just at the end of the nominal execution, so no further execution is needed; therefore, both algorithms give the same result along the top border. The bottom border represents cases where the beginning of SOI execution has been delayed.

Reasonable delays may be considered to be up to two hours long. For such reasonable delays, the error due to linearization (use of Equation 2) is

no more than 2 m/s. This is about 0.3 % error in ΔV and clearly shows that SOI's acceleration is within the linear range of this problem.

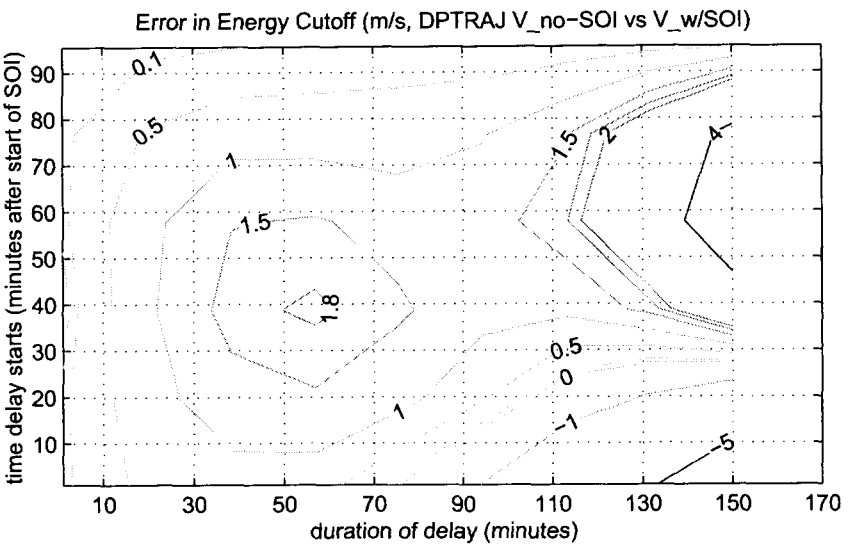

Figure 1: **Contours of Errors due to Linearization.** "time delay starts" is short for "the time at which the delay starts"

4 Further Analysis

Other deterministic errors in the ECB algorithm, due to practical considerations, have been studied. These deal with lower level approximations, such as using a propagated conic to compute velocity, using Cassini's onboard propagator, IVP, for the conic, and smaller errors such as those due to numerical quadrature and timing of accelerometer measurements. None of these other errors are found to be nearly as large as 2 m/s so that the total deterministic error in the ECB algorithm is estimated to be around 2 m/s for reasonable delay durations. This error has been found to be within accuracy requirements for the SOI burn.