Uncertainty of stability variances based on finite differences

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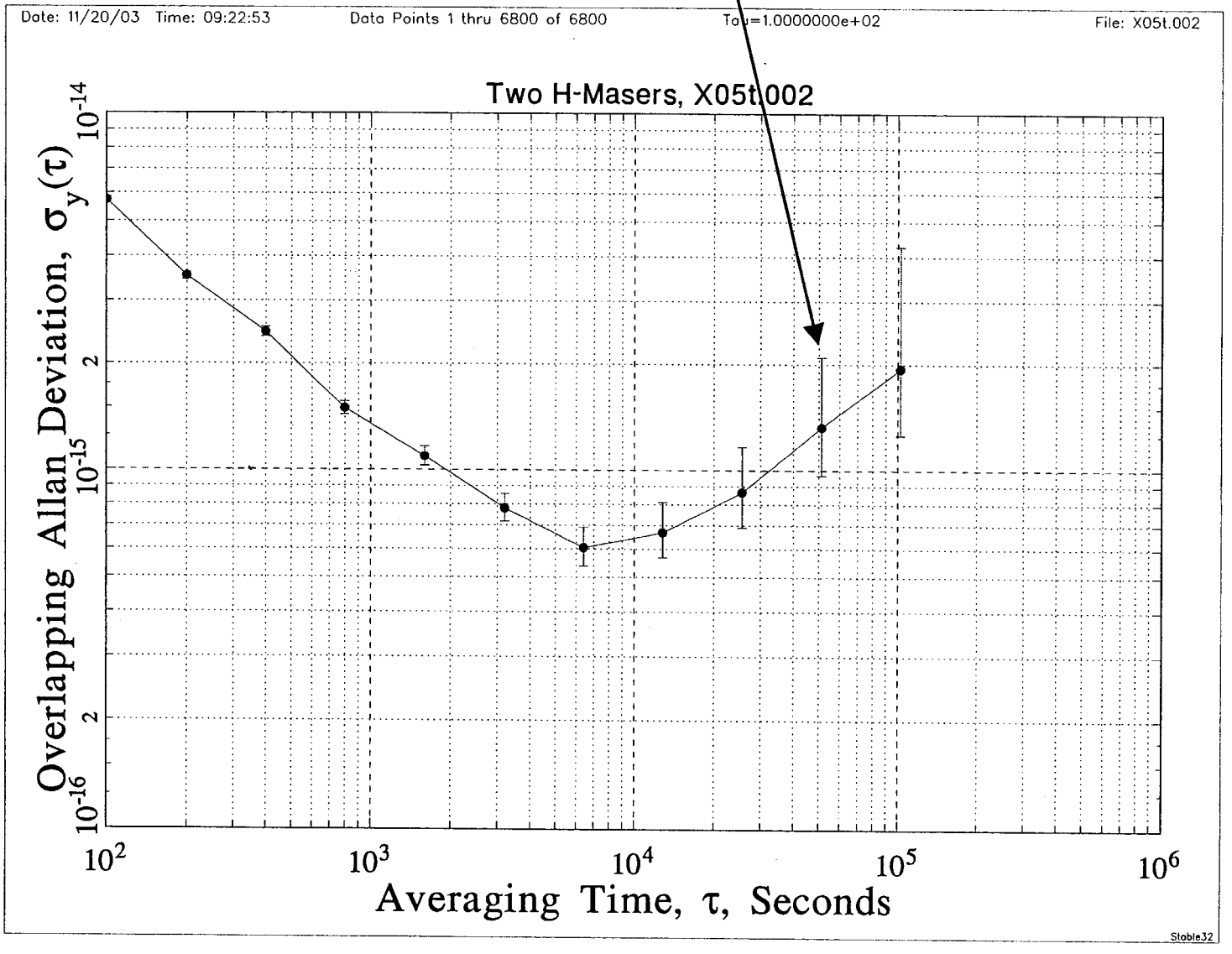
Subject: error bars



C. Greenhall, PTTI, 2003-12-03

History

Lesage-Audoin 1973-1977	Allan, nonoverlapped
Yoshimura 1978	Allan, nonoverlapped
Howe-Allan-Barnes 1981	Allan, overlapped
Yoshimura 1989	Allan, overlapped
Greenhall 1983, 1991	Allan, both
Walter 1994	mod Allan (overlapped)
Greenhall 1997	mod Allan (overlapped)

Goals

Replace the past algorithms, formulas, and approximations by one consistent approach to computing confidence intervals for estimators of stability variances.

Cover the currently-used stability variances and their estimators (and then some).

Generate numbers, not formulas.

Publish an algorithm in pseudocode.

Scope

Variance $\sigma^2(\tau)$		Estimator	Noise	
$d = 1$: 1st difference var	Unmodified	Nonoverlapped	WHPM, $\alpha = 2$	
$d = 2$: Allan	Modified	Overlapped	FLPM	1
$d = 3$: Hadamard			WHFM	0
			...	
			RRFM	–4

$$\mathrm{E}\, z^2(t)$$

Unmodified: $z = \Delta^d x$

Modified: $z = \Delta^d \bar{x}_\tau$

$$\frac{1}{M}\sum_{n=1}^{M} z^2(n\delta)$$

Nonoverlapped: $\delta = \tau$

Overlapped: $\delta = \tau_0$

$$S_x(f) \sim f^{\alpha-2}$$

Restriction: $\alpha + 2d > 1$ so that $\Delta^d x(t)$ is stationary

Exclusions: No trend removal: $\mathrm{E}\,\Delta^d x(t) = 0$

No special long-term stability measures: total variance etc.