Filtering for binaries

Andrzej Królak

Jet Propulsion Laboratory
Pasadena
on leave of absence from
Polish Academy of Sciences
Warsaw

National Aeronautics and
Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

Response

$$s(t;A_o,\phi_o,\iota,\psi,f,[f_{dot}],\beta,\lambda) =$$
$$a_1\ h_1(t;f,[f_{dot}],\beta,\lambda) + a_2\ h_2 + a_3\ h_3 + a_4\ h_4$$

$a_i(A_o,\phi_o,\iota,\psi)$ – extrinsic parameters

$f,[f_{dot}],\beta,\lambda$ - intrinsic parameters

$A_o \sim M_c^{5/3}\ f^{2/3}/D$, $f_{dot} \sim M_c^{5/3}\ f^{11/3}$

When f_{dot} can be determined M_c and D can be estimated (Schutz 1986, Seto 2002)

Long wavelength limit

$$h_1(t;f,\beta,\lambda) = u(t;\beta,\lambda)\cos[2\pi\ f\ (t + \text{Doppler}(\beta,\lambda))]$$

Filtering

$$x = n + s$$

Maximum likelihood estimation

$$\ln\Lambda = (x|s) - \tfrac{1}{2}(s|s)$$

ML estimators of the extrinsic parameters are in a close analytic form

$$\mathbf{a} = \mathbf{M}^{-1}\mathbf{N}$$ - ML estimators of a_i

$$F(x;\ f,[f_{dot}],\beta,\lambda) = \tfrac{1}{2}\,\mathbf{N}^T\,\mathbf{M}^{-1}\mathbf{N}$$ - F statistic

$$M_{ij} = (h_i|h_j)\ ,\ N_i = (x|h_i)$$

$$F = (V\,|N_u|^2 + U\,|N_v|^2 - 2\Re\,[W\,N_u\,N_v])/(T_o\Delta)$$

Computation

$$N = \int_0^{T_o} x(t)m(t; f, \beta, \lambda)\exp[\imath 2\pi f(t + Doppler(\beta, \lambda))]$$

Additional difficulty – m depends on frequency

Proposed solution (confirmed by [a limited number of] simulations) - divide data into narrowband sequences

Two step procedure to search for the signals using the F-statistic:

1. Coarse search over a suitable grid in the intrinsic parameter space
2. Fine search around threshold crossing nodes of the grid using a fast maximization algorithm (e.g. Nelder-Mead alg.)

Grid spacing can be perhaps derived form radius of convergence of the maximization algorithm

Number of templates

Quantity to study - $E_1[F]$

(Expectation value of the F-statistic when the signal is present)

Formula (Owen's geometry)

$$N_t = \frac{1}{(1-C_O)^{K/2}} \frac{\Gamma(K/2+1)}{(\pi/2)^{K/2}} \int_V \sqrt{\det G}\, dV$$

G – reduced Fisher matrix

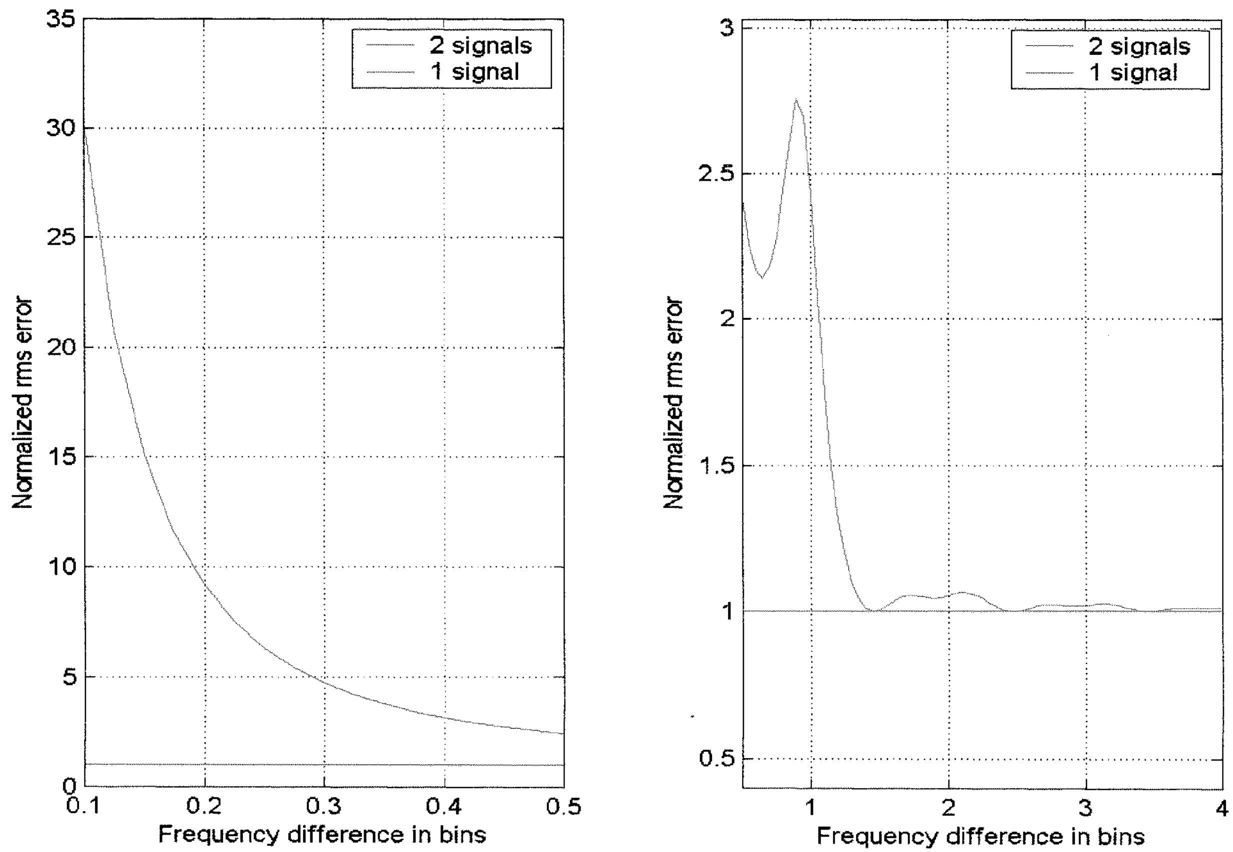
C_o- minimum correlation coefficient

K – no. of intrinsic parameters

We need to set C_o sufficiently large so that we do not miss signals

Signal resolution

Data – time limited function



2 signals
1 signal
Normalized rms error
Frequency difference in bins
35
30
25
20
15
10
5
0
0.1
0.2
0.3
0.4
0.5
2 signals
1 signal
Normalized rms error
Frequency difference in bins
3
2.5
2
1.5
1
0.5
1
2
3
4