RECONFIGURABLE TOPOLOGIES FOR DECENTRALIZED CONTROL OF SPACECRAFT FORMATIONS IN DEEP SPACE

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Free flying, precision controlled satellite formations can be used to create scientific instruments with resolution capabilities that cannot be matched by monolithic spacecraft. Sensing and communication limitations create challenges in the design of the autonomous coordination and control systems for such formations. Our work focuses on deep space formations where the formation is defined in terms of relative spacecraft positions. This paper discusses control and communication topologies that asynchronously distribute the sensing, communication and computation tasks amongst the spacecraft in the formation. Our approach allows the dynamic reconfiguration of the measurement and communication topologies while maintaining the specified formation performance objectives. An illustrative simulation example is presented.

INTRODUCTION

Precisely controlled formations of spacecraft can be used to synthesize optical and radio instruments of greater utility than could otherwise be achieved with a single monolithic spaceborne instrument. Interferometric imaging systems are one particularly challenging example and will be used here as the motivating application. These systems are of current interest and several interferometric flight projects, based on formation flying, have been studied including Darwin [1], LISA [2], Terrestrial Planet Finder (TPF) [3] and Starlight (formerly ST-3) [4]. Early work on spaceborne interferometers can be found in [5, 6, 7].

We describe the interferometric imaging application (illustrated conceptually in Figure 1) and use this example as a motivating basis for more general formation control problems. Each spacecraft acts as a collector, reflecting light from the imaging target to a combiner spacecraft. The light from

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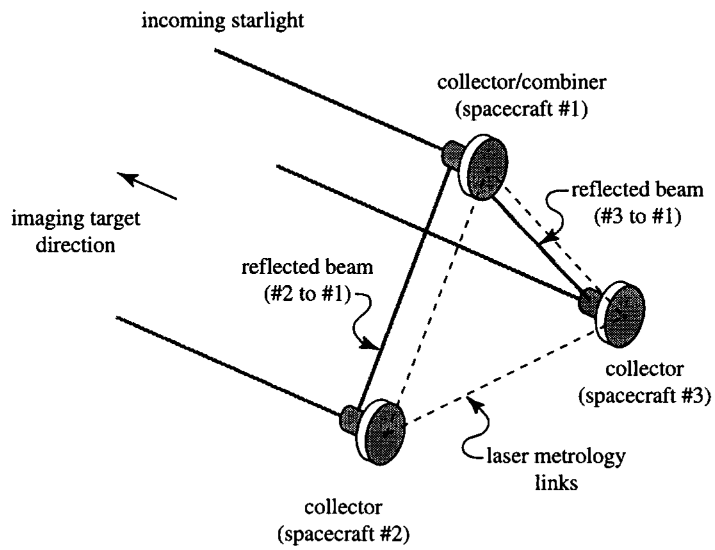

Figure 1: Interferometric imaging configuration using multiple spacecraft in formation. Spacecraft separations, and the equivalent apertures, are of the order of tens to hundreds of meters.

any two collectors is combined at a detector and, if the optical pathlengths are held fixed and equal, an interference pattern can be measured. Each measurement of the amplitude and phase of this pattern amounts to a sample of the spatial Fourier transform of the image. Multiple measurements, using either multiple collectors simultaneously or repositioning fewer collectors, allow reconstruction of the image. The effective imaging aperture is a function of the collector separation. Formation flying can give effective apertures of the order of kilometers, resulting in resolutions that cannot be matched by any monolithic spaceborne telescope. Multiple collectors can also be used to create nulls in the spatial response of the array thereby enabling the imaging of dim objects adjacent to bright ones [8]. This is a promising technology for searching for planetary objects in other solar systems. See [9] for illustrative examples of interferometric imaging.

This work focuses on deep space missions, where the formation is in heliocentric orbit rather than planetary orbit. A significant consequence of this is that the spacecraft can sense their relative position and not their absolute positions. The spacecraft in the formation are free flying and their dynamics are coupled only through the application objectives and measurements of relative spacecraft positions and velocities. To maintain the performance of the formation in deep space missions it is necessary only to maintain the relative positions and absolute orientations of the spacecraft. Actuation for control purposes is performed on the individual spacecraft.

The stringent optical path length constraint—in the tens of nanometers—is achieved by hierarchical actuation. Depending on the application this may include moveable platforms, optical delay lines, and precision piezoelectric actuators on the individual mirrors. The optical path length requirements translate into spacecraft relative positioning requirements in the micrometer to centimeter range [1, 3].

The range of operation, and the bandwidth, of each of these actuation systems varies widely. The actuator allocation design will depend on the specific details of the configuration. In this paper we focus on the formation flying level of the hierarchy and consider only generic actuation. We assume that the control system is able to exert a force suitable for positioning in three dimensions. Our emphasis is on the control topology at the level of communication between the spacecraft, rather than the nested actuation heirarchy within a spacecraft.

There are many possible topologies for the sensing, control, and communication within the formation. Communication bandwidths, synchronization constraints, and sensor capabilities affect the performance of any chosen topology. These issues have been studied; see, for example, [10, 11, 12, 13, 14, 15, 16, 17, 18] for work on leader/follower and other topologies, and [19] on estimation topologies.

Centralized or decentralized topologies can be considered for both the control design and the implementation. A centralized—or global—control design topology is one in which the actuation is calculated from information or measurements of all formation variables. A decentralized design implies that the control actuation for each spacecraft depends only on a subset of the formation variables. Decentralization is a matter of degree, and can be used to trade-off between formation performance and controller/communcation complexity. See [20] and the references therein for a discussion of decentralized control and estimation in spacecraft formations.

Centralized or decentralized topologies may also be considered in the implementation. In a centralized implementation all measurements would be sent to a single spacecraft, the required actuation calculated, and then communicated back to the individual formation members. An equivalent controller can also be implemented by having each spacecraft maintain a copy of that part of the controller which generates its actuation. Measurement information is then transmitted through the formation for use by the individual controllers. Communication bandwidth and synchronization constraints make it advantageous to reduce the communication of time critical information required between spacecraft.

A formation-wide optimal control design problem, based on relative position measurements, is posed and solved. This gives a global, centralized control algorithm for formation control. We then exploit the redundancy in relative position information to develop a family of partially decentralized controller implementations of the optimal centralized controller. This also allows individual spacecraft to switch asynchronously between relative measurement options, and this can be exploited when line-of-sight measurements and communication are unavailable during a maneuver. The aerial formation control work described in [21] is similarly motivated but develops decentralized designs which are not suited to reconfiguration.

FORMATION CONTROL TOPOLOGIES

Formation definition and sensing

We begin by considering a typical formation and defining the notation associated with the various local and relative position and absolute attitude variables. Consider a formation of N spacecraft. For simplicity it is sufficient to define on each spacecraft a reference attitude, ϕ_i, $i = 1, \ldots, N$, with respect to an inertially fixed direction, ϕ_{ref}. Figure 2 illustrates these definitions.

We define a local inertial frame within which each spacecraft is located at position $p_i = [x_i, y_i, z_i]^T$ (where ^{T} denotes transpose). The origin of this frame is not critical for the application we consider

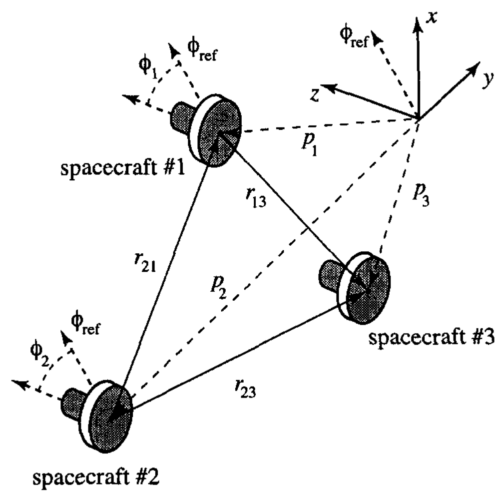

Figure 2: Spacecraft formation: the local and relative position variables are shown.

here. The relative position between each two spacecraft is defined as,

$$r_{ij} = p_j - p_i = \begin{bmatrix} x_j \\ y_j \\ z_j \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}, \quad i,j = 1,\ldots,N, \;\; i \neq j.$$

Naturally, $r_{ij} = -r_{ji}$, and in an N spacecraft formation there are $N(N-1)/2$ relative three dimensional distances that can be defined modulo the opposite direction equivalences.

In deep space an accurate measurement of $(x_i,\ y_i,\ z_i)$ is not available. It may be possible to obtain range and direction information with respect to Earth, but this will be accurate only to the order of kilometers. The r_{ij} variables can be precisely measured using GPS-based approaches or laser metrology.

In constrast to absolute position, spacecraft attitude can be measured to very high accuracy. On-board star trackers are typically used to provide attitude information for each spacecraft, and these have typical accuracies in the range ± 1 milliarcseconds (mas) to ± 200 mas.

We define the formation in terms of the variables that can be accurately measured: the relative spacecraft positions and the attitude of each spacecraft,

$$\begin{aligned} r_{ij} &: \; i,j = 1,\ldots,N, \;\; i \neq j \\ \phi_i &: \; i = 1,\ldots,N. \end{aligned}$$

This definition does not locate the formation in any inertial frame but this is not critical for deep-space applications. Note that there is some redundancy in the above as the $N(N-1)/2$ relative positions are not independent. We will exploit this redundancy in looking for control topologies that do not require all relative positions to be measured.

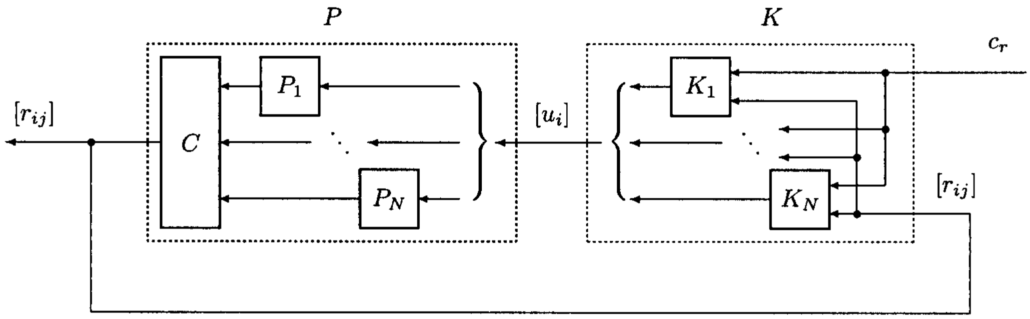

Figure 3: Relative position control design problem for an N spacecraft formation. The reference relative position command vector is denoted by c_r and would be provided by a supervisory system.

Formation control problem

In the deep space mission application, each spacecraft is assumed to have a local measurement of its attitude. This means that correcting attitude errors can be viewed as a strictly local control problem: only local measurements are required to determine the attitude error, and only local actuation is required to attentuate the attitude error. Control of ϕ_i can therefore be treated as decentralized, both in terms of design and implementation. For this reason we drop control of ϕ_i from further consideration and focus on the more difficult problem of the control of r_{ij}. Figure 3 illustrates the relative position tracking problem to be considered.

Using this framework, we pose a relative position formation design problem as follows. Given the collected spacecraft dynamics[3],

$$r_{jk} = P(x, u_i), \qquad j = 1, \ldots, N-1, \quad k = j+1, \ldots, N, \; k \neq j, \quad \text{and } i = 1, \ldots, N,$$

design a stabilzing controller, $u_i = K(c_r, r_{jk})$, to minimize a formation objective cost, $J(r_{jk}, u_i)$. This is a centralized, or "global", control problem in that it is specified in terms of the overall formation objectives, rather than individual spacecraft objectives. Problems such as this are readily handled by existing optimal control theory and supported by analysis and synthesis software. For example, K above may have been designed to meeting an $\mathcal{H}_\infty$ or $\mathcal{H}_2$/LQG objective for the formation.

Switched measurement topologies

The full set of relative position measurements contain redundancies that can be expressed as algebraic constraints. For example,

$$r_{ij} + r_{jk} + r_{ki} = 0, \quad \text{for all } i, j, k \text{ and at every time } t.$$

[3]For notational simplicity we do not explicitly express the time dependence. For example, x denotes the vector valued time signal, $x(t)$.

For the formation to be well defined these constraints must also apply to the relative position commands, c_r.

This constraint can be expressed in the form,

$$r = \begin{bmatrix} r_{12} \\ r_{13} \\ \vdots \\ r_{N-1,N} \end{bmatrix} = C \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix} = \begin{bmatrix} -I & I & 0 & \cdots & 0 \\ -I & 0 & I & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & & -I & I \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix}.$$

The matrix $C \in \mathcal{R}^{3N(N-1)/2 \times 3N}$, and in a state-space representation may actually be a submatrix of the state to output matrix. It has rank $3(N-1)$ which means that it has a $3(N-1)(N-2)/2$ dimensional null space. Therefore there exists a matrix, $M \in \mathcal{R}^{3N(N-1)/2 \times 3(N-1)(N-2)/2}$ satisfying,

$$M^T r = M^T \begin{bmatrix} r_{12} \\ \vdots \\ r_{N-1,N} \end{bmatrix} = 0, \tag{1}$$

or equivalently, $M^T P(x,u) = 0$ for all u. This is a convenient method of expressing the algebraic reduncancies in the relative position measurements and we will use this to define a class of transformation matrices that have the effect of removing specified relative measurements from the controller. This class of transformation matrices is defined by,

$$H = I - XM^T, \tag{2}$$

where $X \in \mathcal{R}^{3N(N-1)/2 \times 3(N-1)(N-2)/2}$ and satisfies,

$$M^T X = I. \tag{3}$$

Tedious algebra shows that H has the effect of expressing some of the relative positions as linear combinations of the others. The H matrix has the effect of removing one or more r_{ij} measurements from the controller input. H can be viewed as a "switching" matrix which selects the particular r_{ij} variables to be used in the controller.

We give a three spacecraft example to illustrate this point. In this case,

$$\begin{bmatrix} r_{12} \\ r_{13} \\ r_{23} \end{bmatrix} = \begin{bmatrix} -I & I & 0 \\ -I & 0 & I \\ 0 & -I & I \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}.$$

Amongst three spacecraft there are only two independent relative positions, and this fact can expressed as,

$$M_1^T \begin{bmatrix} r_{12} \\ r_{13} \\ r_{23} \end{bmatrix} = 0,$$

where $M_1 = \begin{bmatrix} I & -I & I \end{bmatrix}^T$ is one choice. Now select $X_1 = \begin{bmatrix} I & 0 & 0 \end{bmatrix}^T$ and note that $M_1^T X_1 = I$. The transformation matrix, H_1, is given by,

$$H_1 = I - X_1 M_1^T = \begin{bmatrix} 0 & I & -I \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix},$$

and this gives,

$$\begin{bmatrix} r_{13} - r_{23} \\ r_{13} \\ r_{23} \end{bmatrix} = H_1 \begin{bmatrix} r_{12} \\ r_{13} \\ r_{23} \end{bmatrix}.$$

Note that r_{12} has explicitly been removed. This effect can be seen more clearly when we consider KH_1 for some controller K designed to use all three relative measurements[4],

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{bmatrix} r_{12} \\ r_{13} \\ r_{23} \end{bmatrix}.$$

Define $\hat{K} = KH_1$ and consider the control action generated,

$$\begin{aligned} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} &= \hat{K} \begin{bmatrix} r_{12} \\ r_{13} \\ r_{23} \end{bmatrix} \\ &= \begin{bmatrix} 0 & K_{11} + K_{12} & -K_{11} + K_{13} \\ 0 & K_{21} + K_{22} & -K_{21} + K_{23} \\ 0 & K_{31} + K_{32} & -K_{31} + K_{33} \end{bmatrix} \begin{bmatrix} r_{12} \\ r_{13} \\ r_{23} \end{bmatrix} \\ &= \begin{bmatrix} K_{11} + K_{12} & -K_{11} + K_{13} \\ K_{21} + K_{22} & -K_{21} + K_{23} \\ K_{31} + K_{32} & -K_{31} + K_{33} \end{bmatrix} \begin{bmatrix} r_{13} \\ r_{23} \end{bmatrix}. \end{aligned}$$

The transformation H_1 has the effect of removing the measurement r_{12} from the controller. There are other obvious choices for X_1 which would remove one of the other relative position measurements. The method is independent of the null space characterization, M_i. A different M_i would simply require a different X_i to achieve the same result.

Equivalent formation control topologies

Before proceeding we will extend our class of transformations. Partition the identity matrix into q block diagonal pieces via,

$$I = \sum_{i=1}^{q} E_i. \tag{4}$$

Now define the transformed controller via,

$$\hat{K} = \sum_{i=1}^{q} E_i K H_i, \tag{5}$$

where the H_i are matrix transformations of the form, $H_i = I - X_i M_i^T$. This has the effect of grouping the controller outputs into q disjoint groups, and applying a different input transformation, H_i, to each.

[4]Both this section and the next consider K to be only the part of the controller in the feedback loop.

We show in [22] that for all controllers $\hat{K}$ defined in this way, $\hat{K}P = KP$. A further consequence of this is that the entire family of controllers defined by (5) achieve the same global optimal formation reference tracking stability and performance. Note that this is true even though $P\hat{K} \neq PK$ and $\hat{K} \neq K$. The family of controllers, $\hat{K}$ differ from each other in the particular r_{ij} measurements that they use to implement the control.

One such $\hat{K}$ represents a particularly interesting topology for formation flying. We define this topology as follows.

Definition 1 *A control topology in which all actuation signals depend only on relative measurements with respect to the actuation location is termed a* local relative control topology.

In our application this topology means that all control calculations can be performed locally, based only on local relative measurements. In other words, the calculation of the actuation, u_i, depends only on r_{ij}, $j = 1, \ldots, N$, $j \neq i$. It does not depend on r_{kj} when $k \neq i$ and $j \neq i$. This topology can be implemented without any communication between the spacecraft. It is interesting to note that one consequence of this is that their always exists a local relative control topology that can achieve the global optimal objective.

The local relative topology has practical implementation advantages. The most significant is that it does not require measurement or state information to be communicated between the spacecraft. This can remove the need to synchronize the spacecraft timing at the control implementation level, and can remove one of the potential bandwidth constraints in the formation control problem. Note that some communication will still be required for supervisory tasks. In some hardware implementations the distinction between relative measurements and communications is not as unambiguous.

It should be noted that this work addresses control topologies, and not estimation topologies. While the tracking response of each topology is equivalent, the noise response may not be. An estimator may derive additional benefit from non-local measurements.

The local relative topology is best suited for implementation with a small number of spacecraft. At some point the cost and complexity of a large number of relative measurements outweighs the disadvantages of communicating measurements and state variables between spacecraft.

Several situations may arise where the control topology must be reconfigured during operation. Examples of this include failure of a sensing system and failure of a communication link. In such cases one or more of the measurements, r_{ij}, is unavailable for control. A transformation matrix, H_j, can be calculated and applied to the global controller to recalculate a new equivalent topology that does not use the unavailable measurement. The new topology will, in general, require some communication between the spacecraft. While H_j is simple to calculate "by hand", the algebraic approach taken here allows for automatic recalculation in the case where multiple measurements are unavailable. Notice also that controller redesign is not required for this eventuality.

Furthermore, since the reconfiguration is algebraic rather than dynamic, there are no transient dynamics associated with changing topologies. For example, given two topologies, calculated by,

$$u_1(t) = KH_1r(t) \quad \text{and} \quad u_2(t) = KH_2r(t),$$

the controller outputs $u_1(t) = u_2(t)$ for all t. As a result we can switch between controllers KH_1 and KH_2 without a transient in $u(t)$.

Reconfiguration may be required as a result of the formation pattern itself. If both the communication and relative sensing require line-of-sight contact, the local relative topology may be unfeasible during portions of a formation maneuver. The design example presented in this paper illustrates this case, and shows how asynchronous switching between topologies maintains control of the formation during maneuvers.

RELATIVE POSITION BASED CONTROL DESIGN

Using relative sensing as the basis for control design allows flexibility in the choice of measurement and communication topologies. We now consider the problems that arise as a result of this architectural choice, and provide design methods for developing optimal formation control systems.

We consider a linear, state-space description of the spacecraft dynamics,

$$\dot{x} = A\,x + B\,u, \qquad r = \begin{bmatrix} C & 0 \end{bmatrix} x.$$

Because the spacecraft are not physically coupled, A and B have a sparse block structure. The output matrix, C, gives the relative position measurements, effectively coupling the spacecraft.

State feedback

The first obstacle to design is that the state, x, is not fully observable from the relative position measurements, r. Physically, the unobservability arises from the fact that the position and velocity of the formation centroid cannot be determined by relative position measurements. To obviate this complication we use a similarity transformation of the state, $Tx = \begin{bmatrix} z^T & v^T \end{bmatrix}^T$, to give,

$$\begin{bmatrix} \dot{z} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} A_z & A_{zv} \\ 0 & A_v \end{bmatrix} \begin{bmatrix} z \\ v \end{bmatrix} + \begin{bmatrix} B_z \\ B_v \end{bmatrix} u, \qquad r = \begin{bmatrix} 0 & C_v \end{bmatrix} \begin{bmatrix} z \\ v \end{bmatrix},$$

where (C_v, A_v) is observable. We note that the observable part of the dynamics,

$$\dot{v} = A_v\,v + B_v\,u, \qquad r = C_v\,v,$$

can be used to design a formation controller using relative position measurements. Various control design methods can be applied at this point. We develop one based on Linear Matrix Inequality (LMI) optimization [23] for estimator and state-feedback design.

The state-feedback design problem is formulated in terms of finding a controller that drives all states within an initial ellipsoid, $\mathcal{V}_0 = \{\, v \,|\, v^T V_0 v < 1,\ V_0 = V_0^T > 0 \,\}$, to zero with a bounded cost given by,

$$\|W_v v\|_2^2 + \|W_u u\|_2^2 \le \gamma_v^2.$$

Note that we have chosen to independently penalize both the state error and the control action via the symmetric positive definite weighting matrices W_v and W_u respectively. Finding the minimum γ_v gives the optimal controller for this metric. The controller is given by the following LMI optimization problem.

$$\min_{\gamma_v, Q, Y} \gamma_v \quad \text{subject to:} \quad \gamma_v > 0, \qquad Q = Q^T > 0,$$

$$\begin{bmatrix} \gamma_v^2 V_0^2 & I \\ I & Q \end{bmatrix} > 0,$$

$$\text{and} \qquad \begin{bmatrix} -(QA_v^T + A_v Q + Y^T B_v^T + B_v Y) & Q & Y \\ Q & W_v^{-2} & 0 \\ Y^T & 0 & W_u^{-2} \end{bmatrix} > 0.$$

The required state-feedback controller, $u = Kv$, is given by $K = YQ^{-1}$. Note that the initial state ellipsoid $\mathcal{V}_0$ can be obtained by a transforming an ellipsoid in the original physical variables, and so the weighting matrices are directly associated with physically quantifiable objectives.

The state, v, must be estimated from the relative position measurements, r. Our formulation guarantees the observability of v and we can use a completely analogous dual LMI problem to design an estimator gain matrix, L. For brevity we omit the details of this formulation.

Reference tracking controller design

We now construct a reference tracking controller from the above estimator/state-feedback design of L and K that exploits the redundancy in the relative position reference command.

We begin by using a singular value decomposition (SVD) to determine a (non-unique) matrix M satisfying $M^T r = 0$. The SVD will give a representation for C_v of the form,

$$C_v = \begin{bmatrix} U_v & U_z \end{bmatrix} \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} W^T,$$

and $M = U_z$ is one suitable choice. Given a commanded relative position, c_r, we wish to find a matrix, N_r, that gives a desired stationary state, $v_\infty = N_r c_r$, such that the system holds the commanded relative position vector (i.e. $C_v v_\infty = c_r$). We exploit the fact that c_r must specify a valid formation, i.e. $M^T c_r = 0$.

These requirements can be shown to be equivalent to the conditions, $A_v N_r = 0$ and $C_v N_r = (I - MM^T)$. Any N_r satisfying the equation, $\begin{bmatrix} A_v^T & C_v^T \end{bmatrix} N_r = \begin{bmatrix} 0 & I - MM^T \end{bmatrix}$ meets these requirements. The complete relative position reference tracking controller is now given by the state-space representation,

$$\begin{aligned} \dot{\hat{v}} &= \begin{bmatrix} A_v + B_v K + L C_v \end{bmatrix} \hat{v} + \begin{bmatrix} -B_v K N_r & -L \end{bmatrix} \begin{bmatrix} c_r \\ r \end{bmatrix} \\ u &= K \hat{v} + \begin{bmatrix} -K N_r & 0 \end{bmatrix} \begin{bmatrix} c_r \\ r \end{bmatrix}, \end{aligned}$$

where $\hat{v}$ is the controller state (or equivalently, an estimate of the observable part of the system state, v).

Exploiting Input Redundancies

In deep space, the linear model of an individual spacecraft's dynamics is essentially a double integrator. Force actuators—typically thrusters—are used for the control inputs, and these may have additional dynamics associated with them. If each spacecraft has zero order or identical first order actuator dynamics, then the input control space contains an additional degree of freedom. Note that if the actuators are reasonably similar, servo loops can be used to give each spacecraft equivalent actuation dynamics. We now demonstrate how this additional degree of freedom can be exploited to achieve other formation objectives.

Under the above assumptions, B_v has reduced column rank. The key insight is that we need only control $N-1$ of the spacecraft in order to control all of the relative positions defining the formation. This means that there is a matrix, $B_\perp$, satisfying $B_v B_\perp = 0$. An SVD can be used to calculate this matrix, and we can define a projection, $(I - B_\perp {B_\perp}^T)$, such that,

$$B_v (I - B_\perp {B_\perp}^T) u = B_v u \qquad \text{and} \qquad B_z (I - B_\perp {B_\perp}^T) u = 0.$$

Note that the projected control input, $(I - B_\perp {B_\perp}^T)\, u$, drives the observable state, v, in the intended manner, but does not directly drive the unobservable state, z, which contains the dynamics of the centroid of the formation. If $A_{zv} \neq 0$ the unobservable states may still be driven indirectly through the state v. The subsystem, $(A_z, B_z B_\perp)$, may not be completely controllable as some part of the z state may originally have been controllable only via A_{zv} and B_v. Any uncontrollable part of z can be removed via a truncated similarity transform. For simplicity we assume that this has been done and omit the details. Any control signal of the form $u = B_\perp \nu$ directly drives the z part of the state. We can therefore calculate control actuation signals of the form,

$$\hat{u} = (I - B_\perp {B_\perp}^T)\, u + B_\perp \eta,$$

which allow us to control the z and v components of the state independently. The input u controls the formation in the manner given in the previous sections and η can be considered as a control variable for the formation centroid (and other common unobservable states). We now give two practical uses for this control degree of freedom.

Minimizing formation fuel consumption

The input null-space control variable η can be chosen to minimize the total formation fuel use. At each time instant, given the formation actuation command u, we calculate η as the solution to the following linear program.

$$\min_{\eta} \|(I - B_\perp {B_\perp}^T)\, u + B_\perp \eta\|_1, \qquad \text{where} \qquad \|\hat{u}\|_1 := \sum_{i=1}^{N} |\hat{u}_i|.$$

If actuator servo loops have been applied on each spacecraft then the u_i represent commanded thrusts and these are only approximately equivalent to the fuel used on each spacecraft. Note that this approach minimizes the total formation fuel consumption for a given controlled maneuver. It is not necessarily a solution to the problem of finding the minimum fuel maneuver between specified formation configurations. One of the two examples considered in design example uses this form of input null-space control.

Control of the formation centroid

We now consider the problem of using the variable η as a means of controlling the formation centroid. The dynamics of the unobservable state can be expressed as,

$$\dot{z} = A_z\, z + A_{zv}\, v + B_z B_\perp\, \eta.$$

We again take the approach of separating this control problem into a estimator and state-feedback design. The lack of observability of z means that the estimator is now open-loop and given by the marginally stable z dynamics above.

Our control of z is implemented via $\eta = -K_z\, \hat{z}$, where $\hat{z}$ is an estimate of z. The objective can again be specified in terms of driving all z in an ellipse, $z \in \mathcal{Z}_0 = \{\, z \,|\, z^T Z_0 z < 1,\ Z_0 = Z_0^T > 0 \,\}$, to zero with cost bounded by,

$$\|W_z z\|_2^2 + \|W_\eta \eta\|_2^2 \leq \gamma_z^2.$$

This gives an LMI problem identical in form to that used to solve the state feedback problem. This approach is a simple solution to the problem; we may wish to obtain higher performance by taking advantage of the estimate $\hat{v}$ in accounting for the disturbance to z (i.e. in the case where $A_{vz} \neq 0$).

This input null-space controller can now be integrated into the previous reference tracking controller. The final controller for the i^{th} spacecraft is expressed as,

$$
\begin{aligned}
\begin{bmatrix} \dot{\hat{z}} \\ \dot{\hat{v}} \end{bmatrix} &= \begin{bmatrix} A_z - B_z B_\perp K_z & A_{zv} + B_z(I - B_\perp {B_\perp}^T)K) \\ 0 & A_v + LC_v + B_v K \end{bmatrix} \begin{bmatrix} \hat{z} \\ \hat{v} \end{bmatrix} \\
&\quad + \begin{bmatrix} -B_z(I - B_\perp {B_\perp}^T)KN_r & 0 \\ -B_v K N_r & -L \end{bmatrix} \begin{bmatrix} H_0 & 0 \\ 0 & H_j \end{bmatrix} \begin{bmatrix} c_r \\ r \end{bmatrix}, \\
u_i &= E_i \begin{bmatrix} -B_\perp K_z & (I - B_\perp {B_\perp}^T)K \end{bmatrix} \begin{bmatrix} \hat{z} \\ \hat{v} \end{bmatrix} \\
&\quad + E_i \begin{bmatrix} -(I - B_\perp {B_\perp}^T)KN_r & 0 \end{bmatrix} \begin{bmatrix} H_0 & 0 \\ 0 & H_j \end{bmatrix} \begin{bmatrix} c_r \\ r \end{bmatrix},
\end{aligned}
$$

The H_j matrices define the input switching of the relative position measurements. These can be precalculated, $H_0 = I$, $H_1 = X_1 M_1, \ldots, H_j = I - X_j M_j^T$, and applied to switch between locally measured or communicated relative position measurements. The switching between the H_j measurement matrices can occur independently and asynchronously between the various spacecraft controllers. That is, the spacecraft need not use identical topologies.

This controller implements both control objectives (precise control of relative positions via state feedback on $\hat{v}$, and open-loop control of the formation centroid via feedback on $\hat{z}$) in a manner which ensures that the objectives do not interact. This input decoupling approach could equally well be used to implement lower bandwidth and/or lower resolution feedback control of the formation centroid if a lower precision measurement of absolute position was available.

A DESIGN EXAMPLE

We illustrate the application of measurement switching, optimal relative state control design, and input null space control on a four spacecraft, two-dimensional, example. Each spacecraft is modeled in each dimension as a double integrator with first order actuation dynamics. To illustrate the most general application of the input null space control we consider the case where each spacecraft has identical actuation dynamics (1.0 second time constant). The spacecraft masses are not identical and are specified as 300, 310, 280 and 280 kg.

The control will maneuver the spacecraft from an initial position—in an arbitrary (x,y) frame—of a square 200 meters on a side, rotating clockwise at 0.003 rad/second, to a non-rotating line in the y direction with 200 meters separation. The initial and final formation specification, and the measurement vector, consists of 12 relative positions (6 in each dimension). Each vehicle begins the maneuver using the local relative control topology. The maneuver is such that line-of-sight contact is lost between various spacecraft at seven instances (including the final position), and at these times the affected spacecraft employ relative measurement switching to accomplish control of the maneuver.

We consider two controllers with differing objectives for the input null space control. The first maintains the estimated formation centroid at the same position throughout the maneuver. The

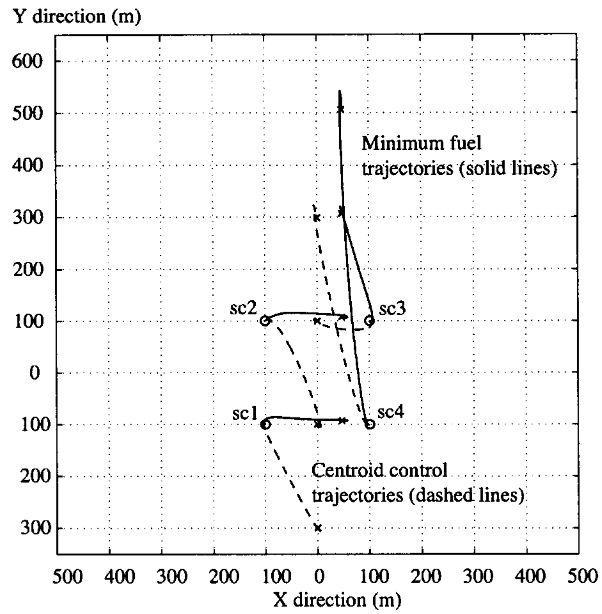

Figure 4: Absolute frame x-y plot of the trajectories for the centroid and minimum fuel controllers.

second allows the centroid to move to perform the maneuver with minimum fuel consumption. In each case, the relative positions track identical trajectories.

Figure 4 illustrates the x-y plane motion of each spacecraft for each of the two input null space control options. In absolute coordinates there are significant differences in the trajectories, and in the final positions. The minimum fuel controller uses 3.1% less fuel illustrating that maintaining the centroid invariant is reasonably efficient, but not optimal. The instances where measurement vector switching occurs, and the measurement topology that each spacecraft uses, are given in Table 1.

Figure 5 illustrates the six relative distances during the maneuver. Each can be associated with a measurement/communication link, and the times at which these links are occluded is also illustrated. Note that the relative positions (shown in Figure 5), and the information switching instances (given in Table 1) are identical for both the centroid control and minimum fuel control cases.

Each spacecraft has sufficient information to reconstruct the relative paths of all of the other spacecraft, predict when specific measurement and communication links will be occluded, and determine a switching strategy using the remaining available links. The control design is optimal with respect to the chosen formation-wide criteria, and in this instance there is sufficient freedom in the input null space to allow additional control objectives. The maneuver demonstrated required relative information to be communicated between spacecraft. This is not always the case; simpler maneuvers may be accomplished without interspacecraft communication or topological reconfiguration.

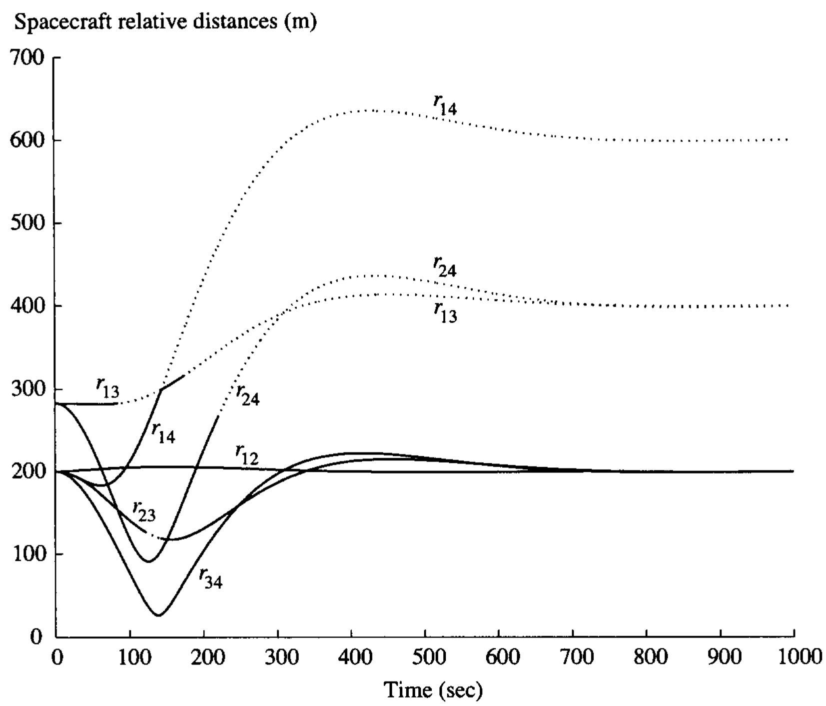

Figure 5: Spacecraft relative distance vector, r_{ij}, for centroid and minimum fuel controllers. Dotted lines indicate relative links that are occluded by intervening spacecraft.

Time (sec.)	Link Occlusions	Measurement Topology s/c1	s/c2	s/c3	s/c4
0.0		(r_{12}, r_{13}, r_{14})	(r_{12}, r_{23}, r_{24})	(r_{13}, r_{23}, r_{34})	(r_{14}, r_{24}, r_{34})
89.5	r_{13} by s/c4	$(r_{12}, r_{34}{}^*, r_{14})$		$(r_{14}{}^*, r_{23}, r_{34})$	
121.9	r_{13} and r_{23} by s/c4		$(r_{12}, r_{34}{}^*, r_{24})$	$(r_{14}{}^*, r_{24}{}^*, r_{34})$	
141.5	r_{13} by s/c4		(r_{12}, r_{23}, r_{24})		
143.8	r_{14} by s/c3	$(r_{12}, r_{13}, r_{34}{}^*)$		(r_{13}, r_{23}, r_{34})	$(r_{12}{}^*, r_{24}, r_{34})$
146.7	r_{14} by s/c2 and s/c3				
174.4	r_{14} by s/c2 and s/c3 r_{13} by s/c2	$(r_{12}, r_{23}{}^*, r_{24}{}^*)$		$(r_{12}{}^*, r_{23}, r_{34})$	
220.7	r_{14} by s/c2 and s/c3 r_{13} by s/c2 r_{24} by s/c3	$(r_{12}, r_{23}{}^*, r_{34}{}^\dagger)$	$(r_{12}, r_{23}, r^*_{34})$		$(r_{12}{}^\dagger, r_{23}{}^*, r_{34})$

Table 1: Relative position vector switching instances. The measurement/communication link is considered to be occluded if the edge of an intervening 2 meter radius spacecraft comes within 10 degrees of the line-of-sight. Superscripts * and † denote information communicated via one and two links respectively.

CONCLUSIONS AND FUTURE RESEARCH

Relative position based specifications are a suitable choice for deep space formations where absolute position measurements are inaccurate or unavailable. The redundancy in a relative postion based design allows the development of a family of equivalent formation controllers, where each spacecraft may use different relative measurement vectors. Some of these vector components may be communicated from other spacecraft. The freedom in selecting amongst multiple topologies allows the optimal formation control to be maintained under reconfiguration when certain measurement and communication links are no longer available. An input null space control approach has been outlined, and it allows the formation to simultaneously achieve other objectives in a decoupled manner. The minimum fuel controller is one such example.

If data communication latency is significant, the switching between measured and communicated information may require a more detailed stability analysis. This is an area of future research, including formation control methods that are robust to potential communication latencies.

Our approach essentially employs a transformed state estimator in each spacecraft, and this allows each spacecraft to reconstruct the controls and trajectory of all other spacecraft. This can be done with only $N-1$ relative spacecraft measurements. However, it may be possible to improve the accuracy of the internal estimators by transmitting additional information around the network, and this is also an area of research interest.

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