# High Resolution Shape and Topography of Eros - Preliminary Results From NEAR Imaging Data

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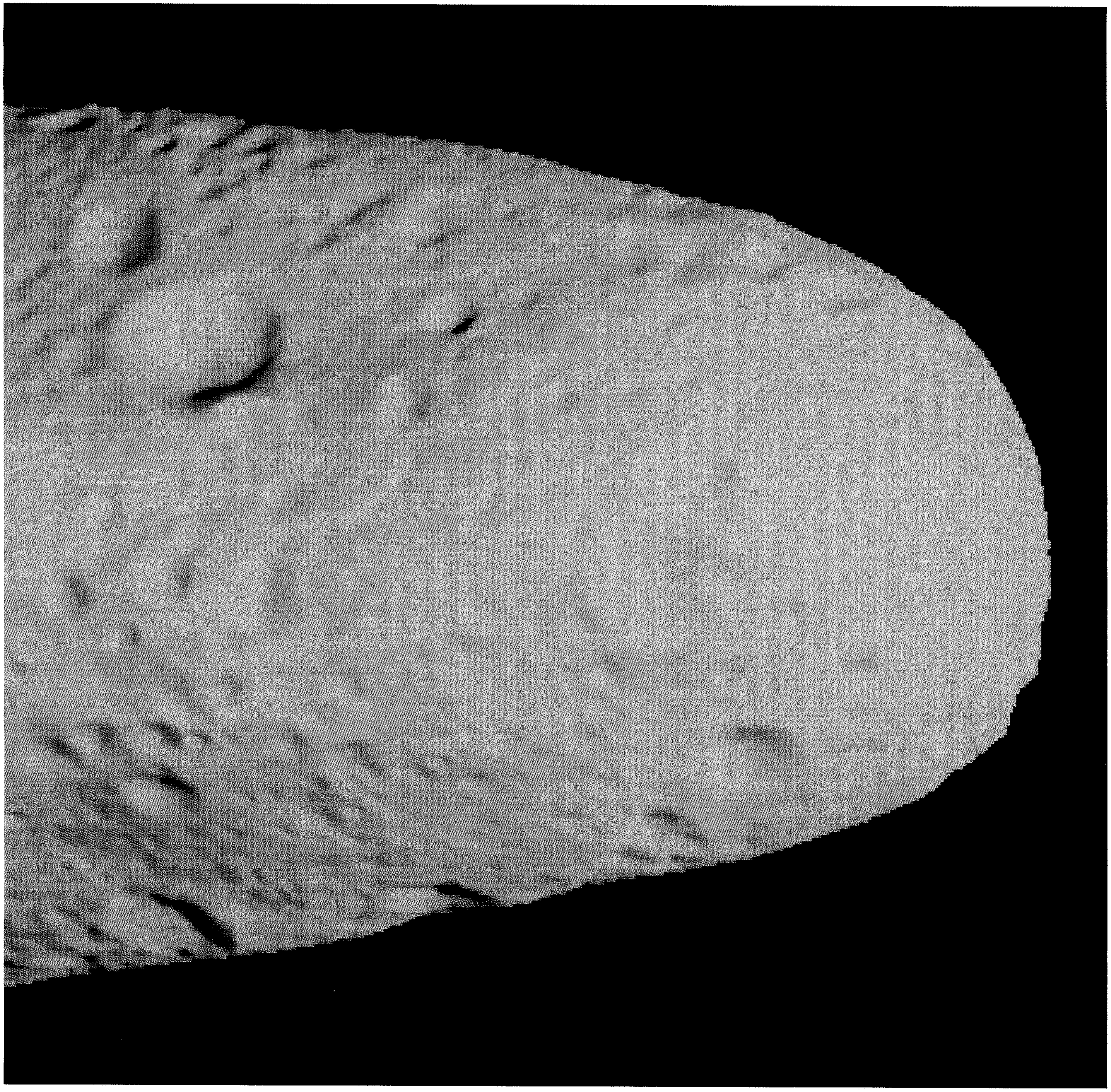
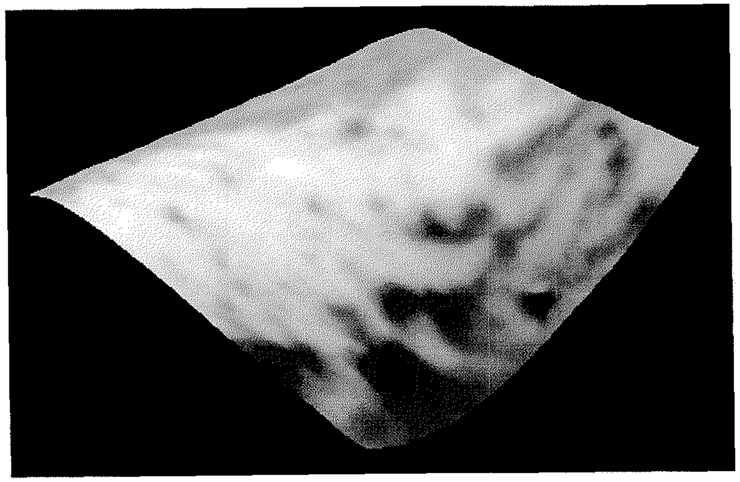


# Abstract

A program has been developed for determining small body shape and topography from imaging data, using multiple image stereography and photoclinometry. A preliminary study using 1600 F4 images of Eros, 90 percent of them between 10 m/pixel and 30 m/pixel, has produced a 1.57 million vector model with an average resolution of about 30 meters. The postfit residual of the 7.2 million vectors which went into the model was less than 8 m/dof. The model predictions correlate better with observed gravity harmonics than does the laser altimetry model, its coverage is more uniform, and it has far less noise. More generally, the method allows the synthesis of all imaging data into a single high-resolution data structure that can be displayed in a variety of ways for geological analysis.

# Basic Element: The Landmark Map

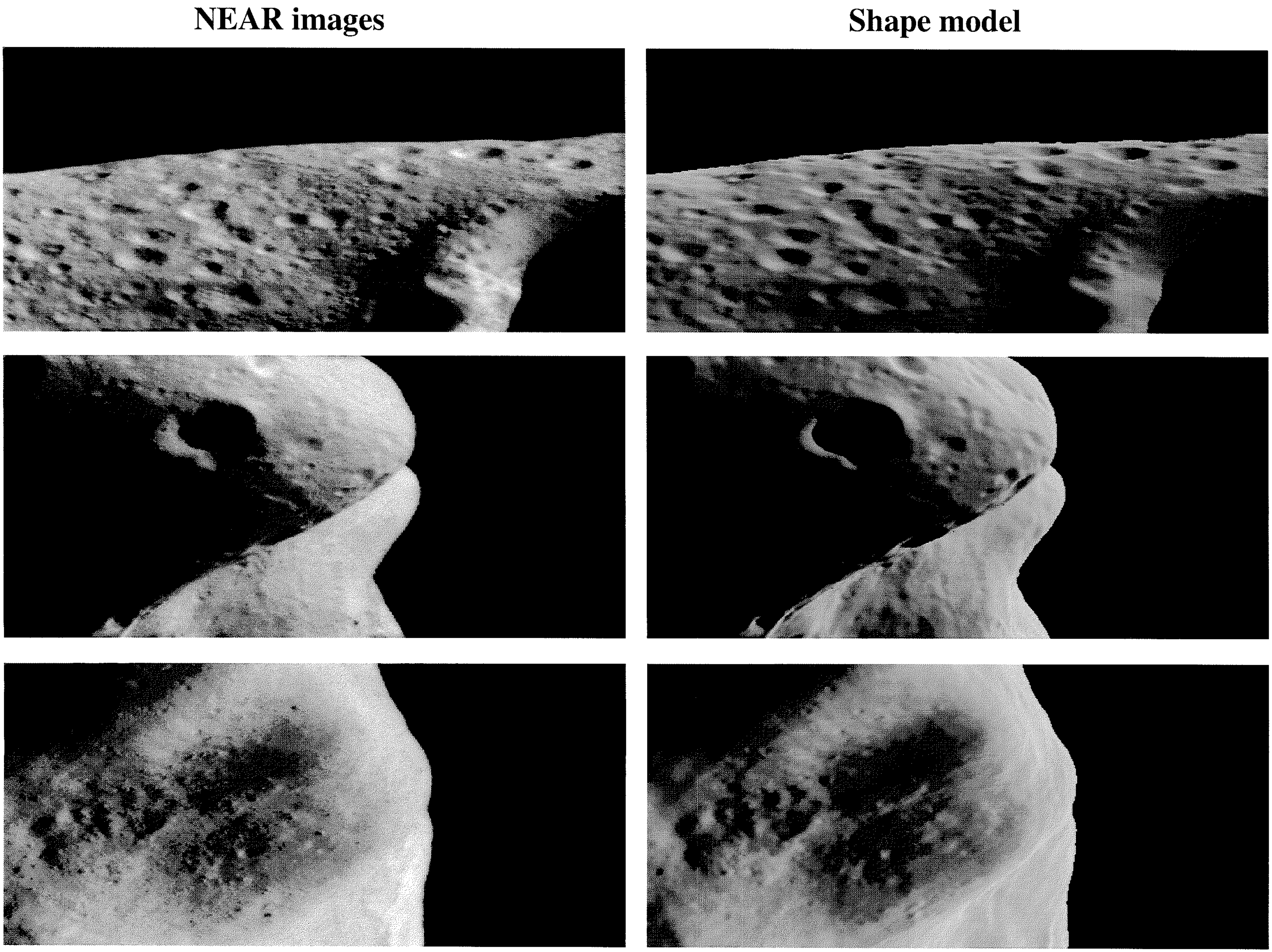


**Specified by:**

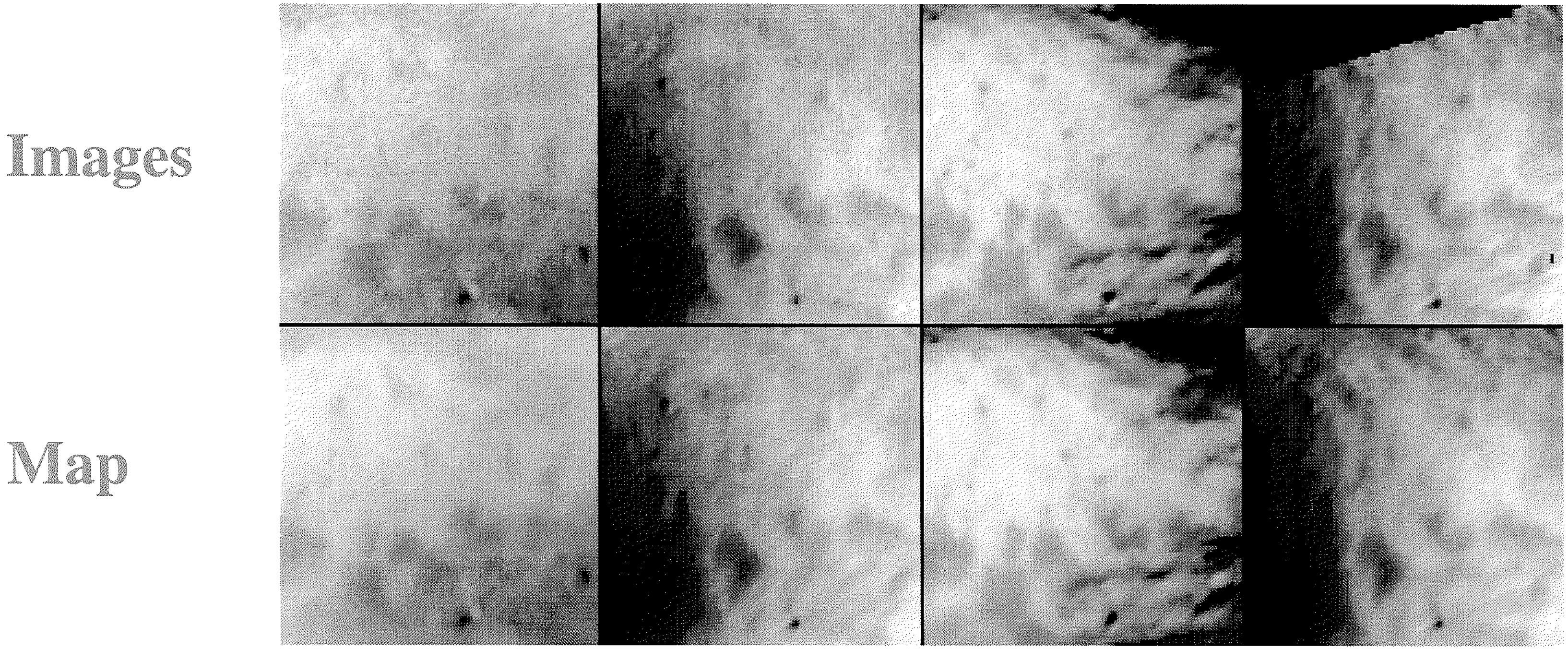
A body-fixed vector V to the center of the map (control point)

Unit vectors $u_x$, $u_y$, $u_z$ defining the local coordinate system

Local heights and relative albedos at each pixel of the map


NEAR images
Shape model

# Basic Procedure: Image/Map Alignment




**Image/Map correlation locates control point in images**

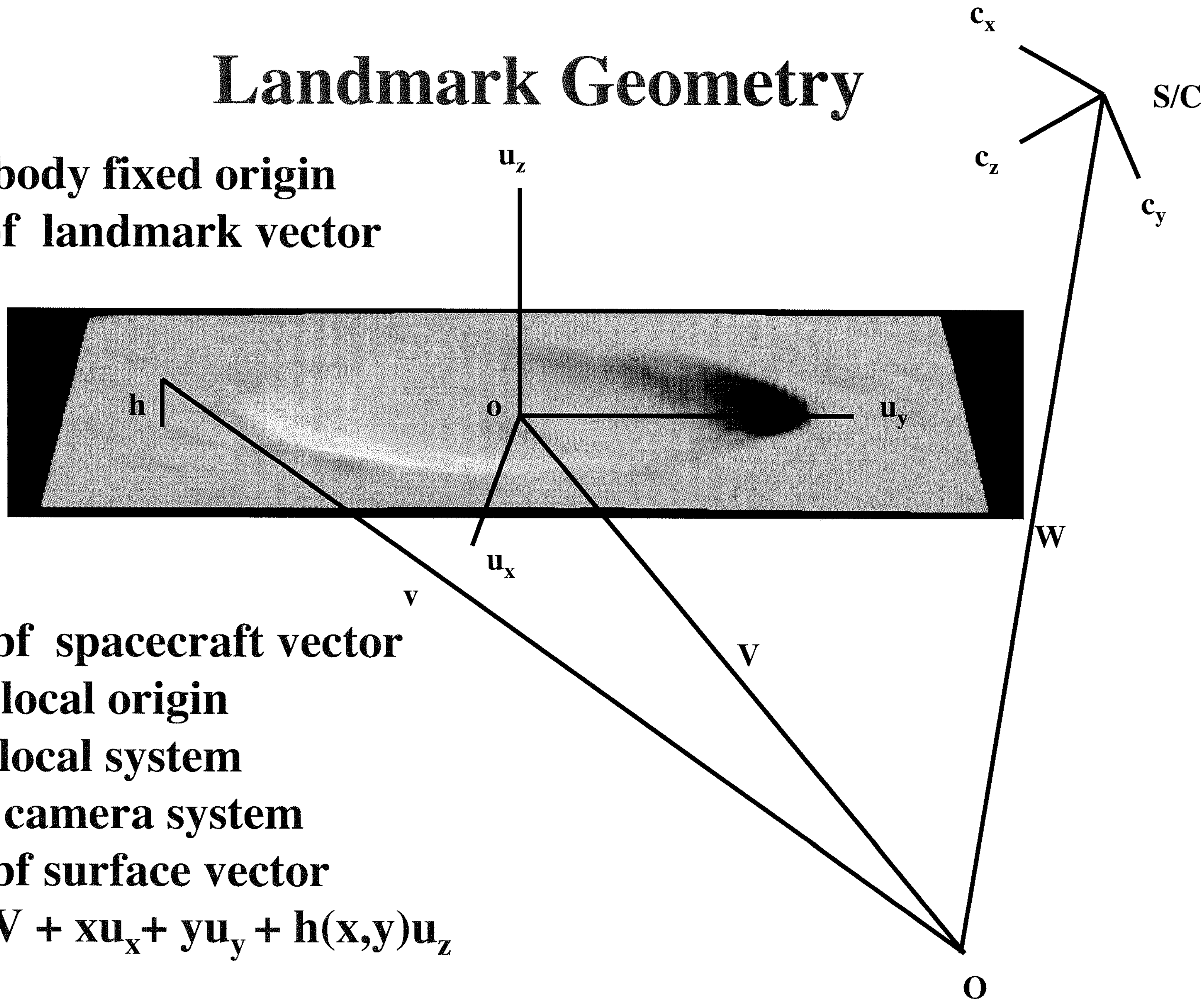
**Brightness of images at each map pixel gives map slope & albedo**

**Slope distribution in map is integrated to give local heights**

# Landmark Geometry

**O = body fixed origin**

**V = bf landmark vector**




**W = bf spacecraft vector**

**o = local origin**

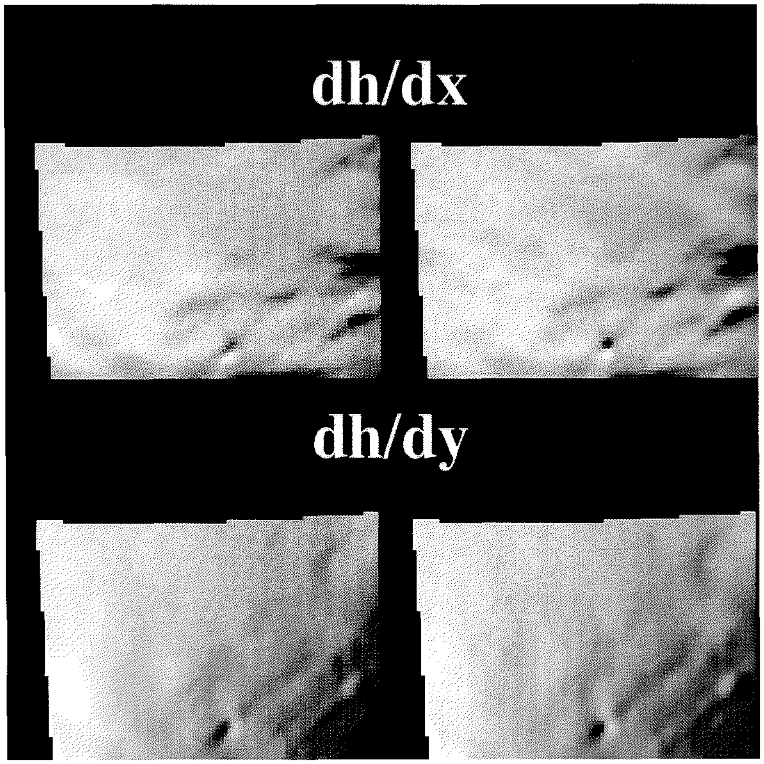
**$u_k$ = local system**

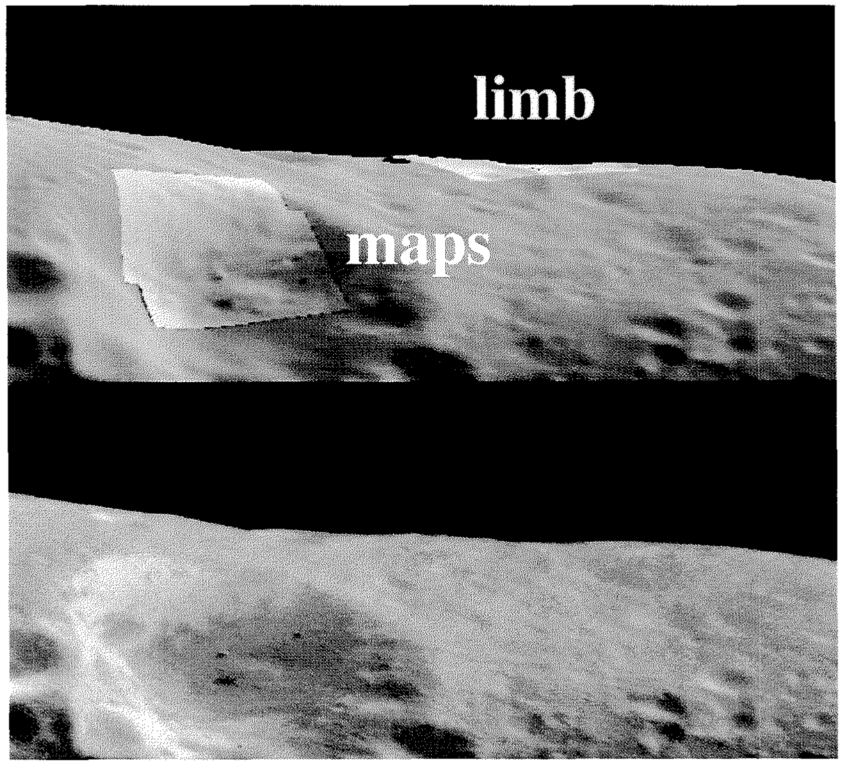
**$c_k$ = camera system**

**v = bf surface vector**

$$= V + xu_x + yu_y + h(x,y)u_z$$

# Data Types for Geometry




**Offset ΔV of two map centers found by correlating slopes and heights in the overlap region**

**Image location (X,Y) of map centers found by map/image correlation or by map/limb projection**

# Geometry Solution

**Landmark point (x,y,h) maps to focal plane location (X,Y) with**

$$X = f((V-W)\bullet c_1+M_{11}x+M_{12}y+M_{13}h)/((V-W)\bullet c_3+M_{31}x+M_{32}y+M_{33}h)$$
$$Y = f((V-W)\bullet c_2+M_{21}x+M_{22}y+M_{23}h)/((V-W)\bullet c_3+M_{31}x+M_{32}y+M_{33}h)$$

**Where f=focal length and $M_{ij}=c_i\bullet u_j$**

**For a given picture, W and $c_i$ are found by minimizing the weighted mean square residual (difference) between predicted and observed locations, averaged over all landmarks in the picture.**

**For a given landmark, V is found by minimizing the same quantity, but now averaged over all pictures containing the landmark. Also included in the average are residuals between predicted and observed offsets ΔV between overlapping maps.**

# Map Illumination

A Landmark map is illuminated in the local frame according to

$$I(x,y) = I_0(1+t_3(x,y))F(\text{cosi},\text{cose}) + \Phi$$

$\text{cosi} = (s_1t_1+s_2t_2+s_3)/\sqrt{(1+t_1^2+t_2^2)}$, $\text{cose} = (e_1t_1+e_2t_2+e_3)/\sqrt{(1+t_1^2+t_2^2)}$

$t_1 = -\partial h/\partial x$, $t_2 = -\partial h/\partial y$, $1+t_3$ = relative albedo,

$\Phi$= background, $I_0$ =normalization,

$s_k$= local sun vector, $e_k$= local camera vector

The function F(cosi,cose)=cosi+2cosi/(cosi+cose) does a good job of reproducing imaging data.

# Stereophotoclinometry

**Many images with many sun and viewing angles are used to construct the landmark templates. Slopes $-t_1$ and $-t_2$ and relative albedos $1+t_3$ are determined from the minimization procedure:**

**At each pixel (x,y) of the map, minimize**

$$\sum_k ( E_k(x,y) - I_k(x,y,t) - \delta t \bullet \nabla_t I_k(x,y,t))^2$$

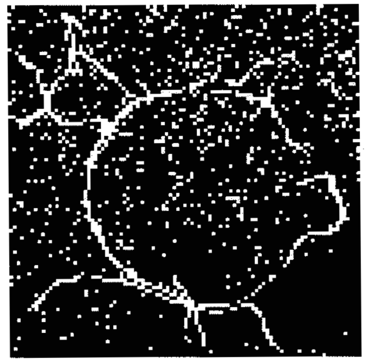
**$E_k$ = Data from kth image at (x,y)**
**$I_k$ = Prediction for kth image at (x,y)**

**Only relative photometry is used. $I_0$ and $\Phi$ are found from the large scale topographic variations known from stereo, shape, limb, or overlapping map data. In effect, this provides an interpolation algorithm for topography down to the pixel scale.**

# Height Integration

The height at each location (x,y) is determined from the neighboring heights, and a possible constraining height $h_c$ from shape model, differential stereography, limb or overlapping map data, from:

$$
\begin{aligned}
h(x,y) = [&w_c h_c(x,y) \\
&+h(x+s,y)+s(t_1(x,y)+t_1(x+s,y))/2 \\
&+h(x-s,y)-s(t_1(x,y)+t_1(x-s,y))/2 \\
&+h(x,y+s)+s(t_2(x,y)+t_2(x,y+s))/2 \\
&+h(x,y-s)-s(t_2(x,y)+t_2(x,y-s))/2]/(w_c+4)
\end{aligned}
$$



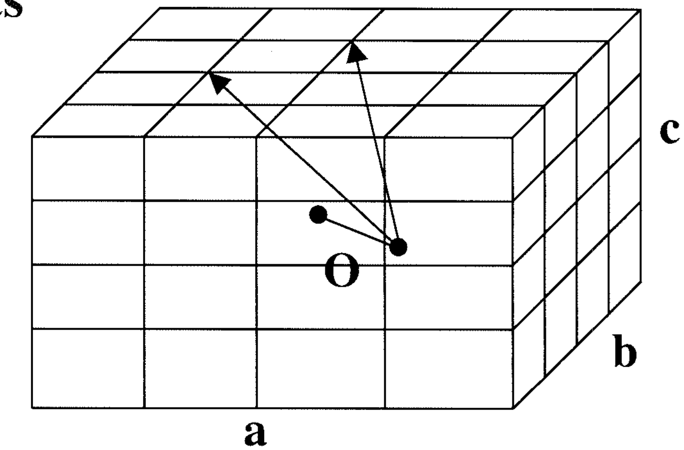
Limb and map overlap points used in a slope-to-height integration.

where s is the map pixel spacing and $w_c$ is a small constraining weight.

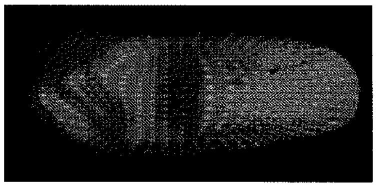
This equation is applied repeatedly to map points chosen at random until a converged solution is reached. If any height does not exist, its term is not included in the average.

# Geoid Geometry

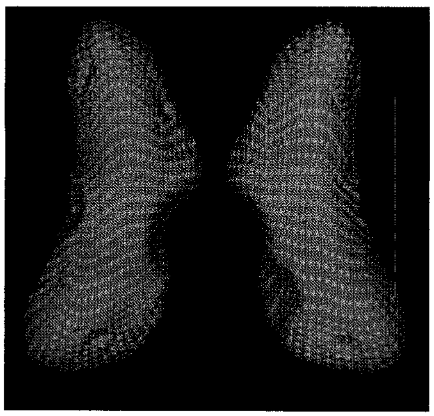
A set of direction vectors is defined by grid points on the faces of a rectangular solid. This also defines the set's connectivity.



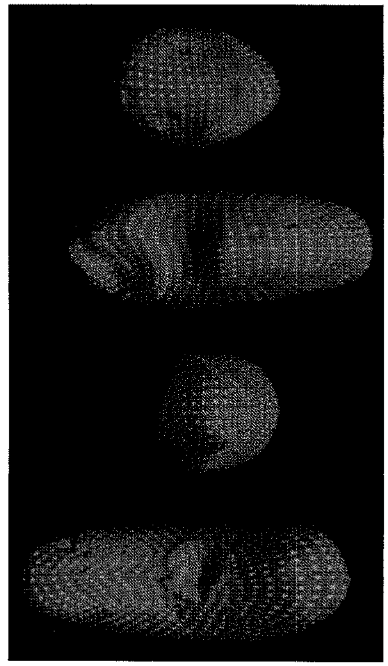

A rough 25k vector shape model (geoid) is made by projecting the maps onto this set.



a, b, c and offset vector O are found so that the projected surface areas of the six faces are equal.



Direction vectors along the edges are held fixed. The remaining vectors in each face are adjusted to most nearly equalize the projected surface area of each cell in the face.



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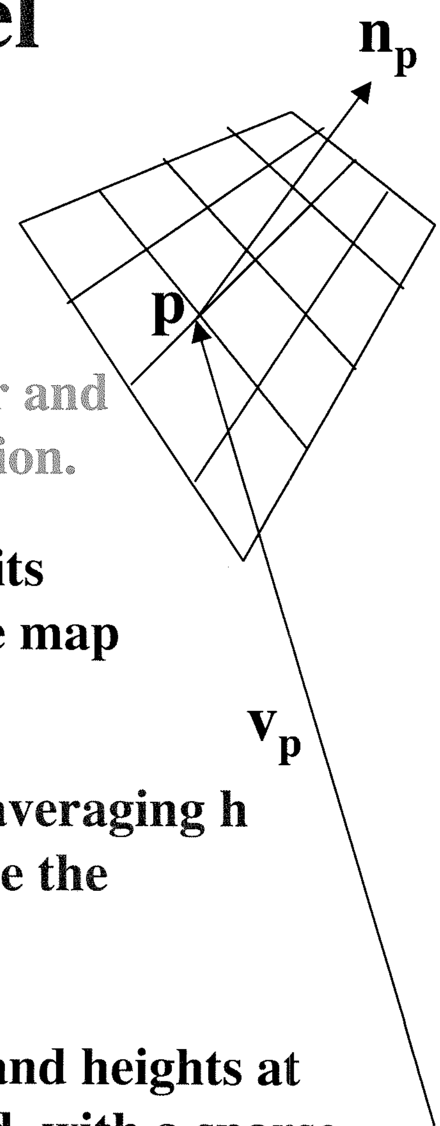



**By construction, the geoid's radius is a single valued function of direction. The surface of Eros is not. However, it can be described by normal deviations from the geoid.**

**Each cell of the low resolution geoid is subdivided. The vector and normal of each intersection p are found by bilinear interpolation.**

**If $n_p$ pierces a map, the intersection is associated with p, and its height h and map normal n are determined. The vector to the map point is $v = v_p + n_p h$.**

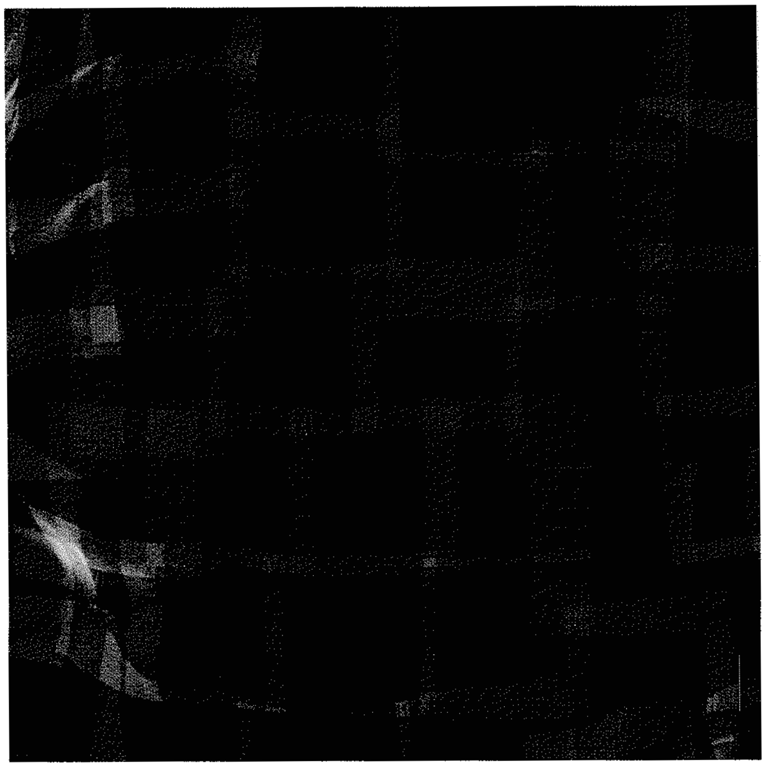
**The surface vector associated with p *could* be determined by averaging h over all maps: $v = v_p + n_p\langle h\rangle$. It seems to be *cleaner* to average the normals $\langle n\rangle$ and then integrate to get the shape.**

**The height at p is found from $\langle n\rangle$ and the average normals and heights at the nearest neighbor points. A Monte Carlo procedure is used, with a sparse set of height-averaged points included to initiate and control the integration**

# High Resolution Topography Maps

**A similar averaged-slope-to-height integration is used to construct high-resolution topography maps of selected regions**

Highest resolution maps used in construction. 12 m/pixel.



3D image of high-resolution map

# Summary of the Study

**The study was done as a proof of concept for DAWN navigation. 1593 randomly selected F4 images, less than 3% of available data, were used. 1417 of there images were between 10 and 30 m/pixel.**

**The surface was tiled with 1000 overlapping landmark maps, most with resolutions of 25 m/pixel or 12 m/pixel, and each containing about 10000 vectors. The RMS postfit residuals were less than 11 m, about 7 m/dof.**

| Eros Results: | Optical Only | | | Laser Altimeter | | |
|---|---|---|---|---|---|---|
| Volume ($km^3$): | 2512.86 | | | 2511.21 | | |
| Area ($km^2$): | 1138.54 | | | 1143.80 | | |
| | 16.71 | 9.36 | -.02 | 16.64 | 9.22 | -.03 |
| I/M ($km^2$): | 9.36 | 71.63 | .01 | 9.22 | 71.66 | .01 |
| | -.02 | .01 | 74.43 | -.03 | .01 | 74.43 |

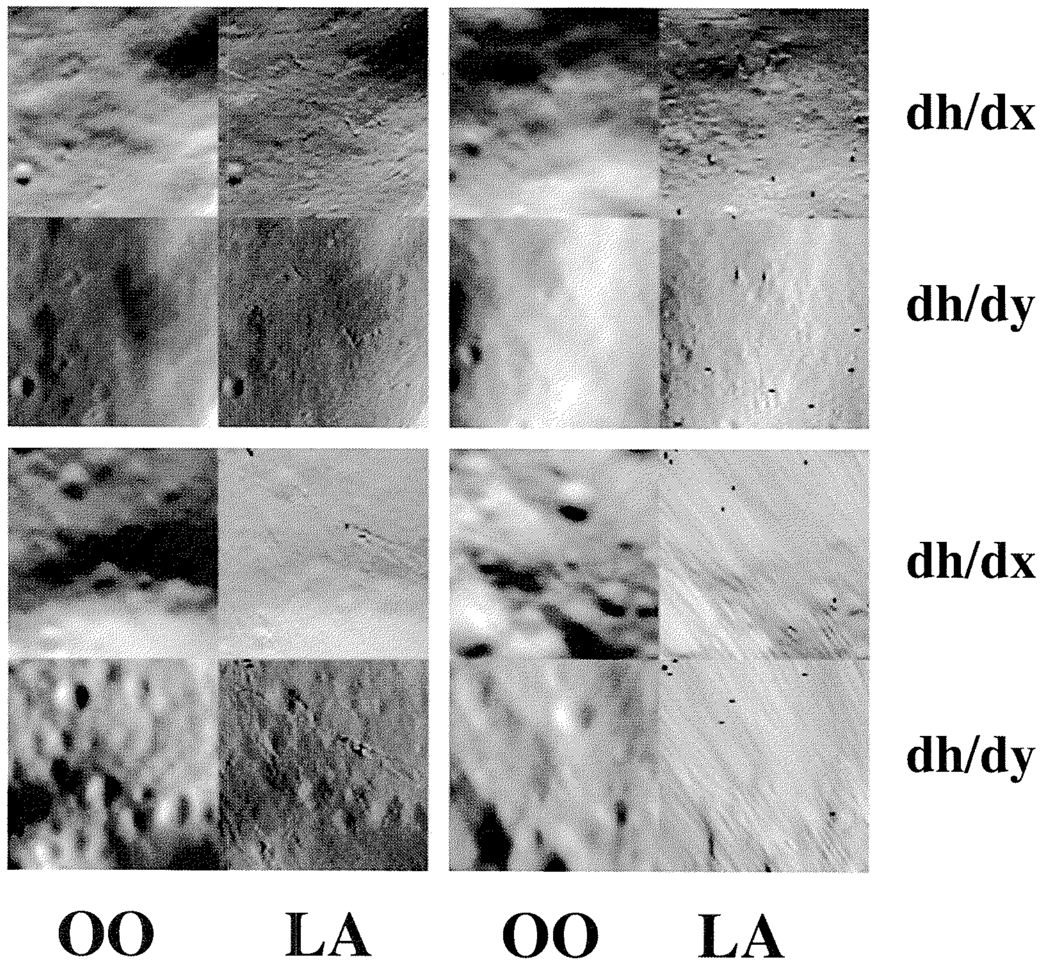
# Optical Only and Laser Altimetry

**Models compared by correlation over 360 overlapping 35 m/pixel patches. Low frequency residuals measure differences in central vectors. High frequency residuals measure rms height differences.**

**22 meter rms random residuals overall between LA and OO.**

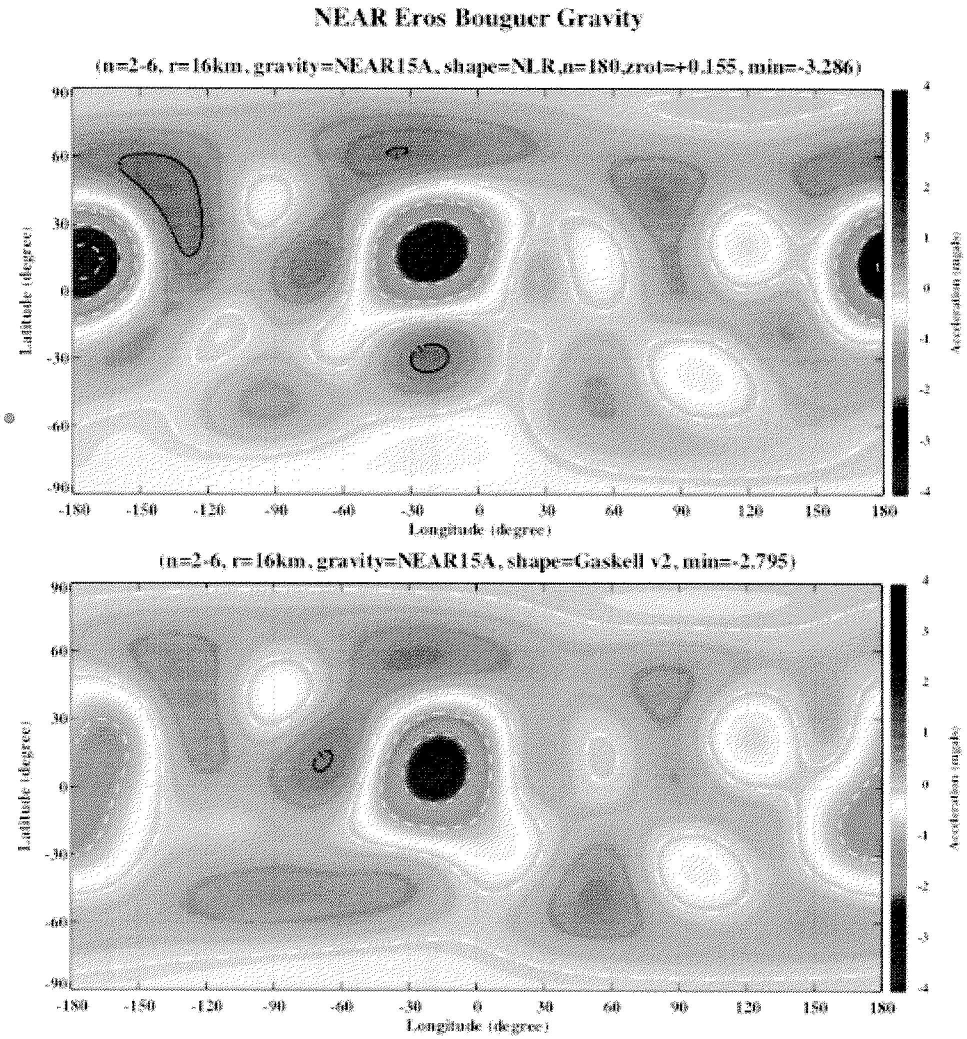
**10 meter rms low frequency residuals.**

**18 meter rms high frequency residuals.**



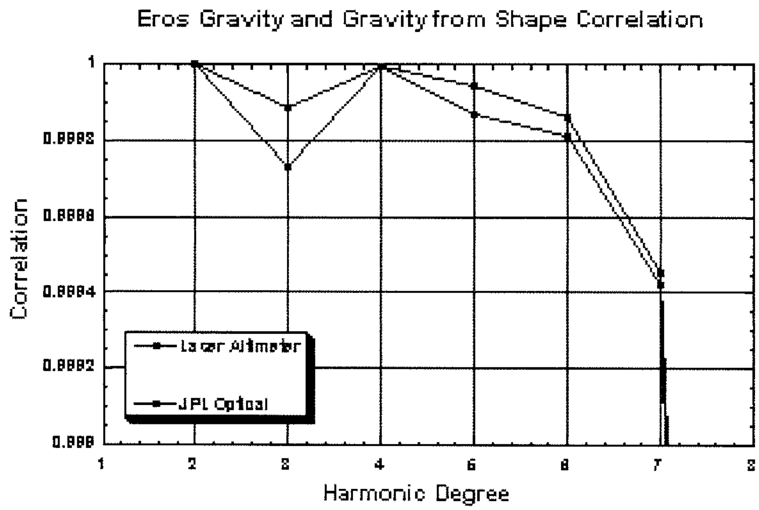
# Eros Gravity

**Gravity harmonics were found for each model, with a homogenous mass distribution, and compared with the measured gravity.**




**Right: Comparison on a 16 km radius sphere.**

**Below: Correlations of harmonics as a function of degree.**



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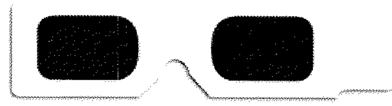
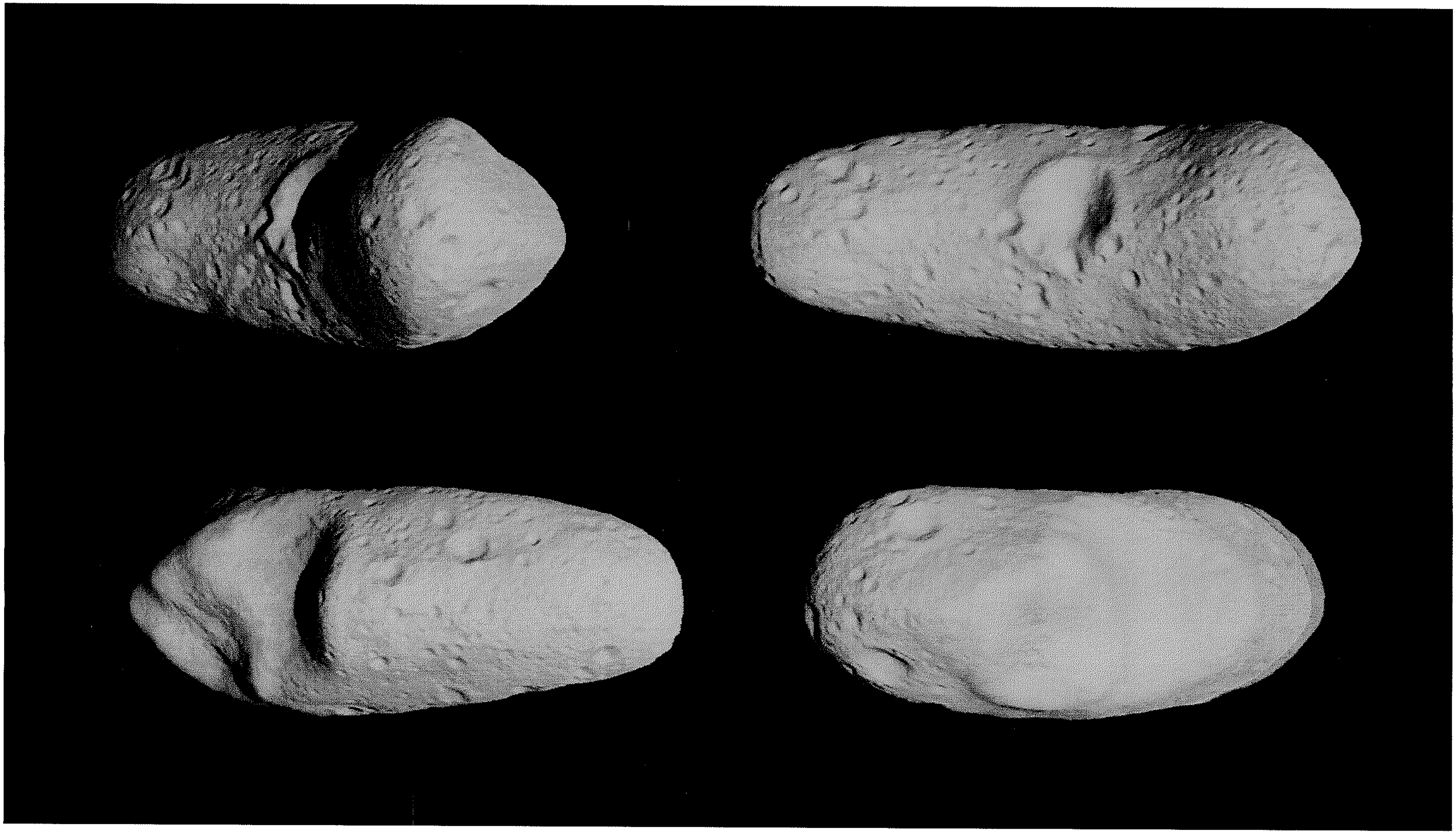
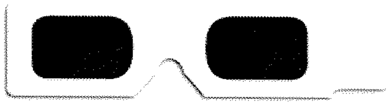
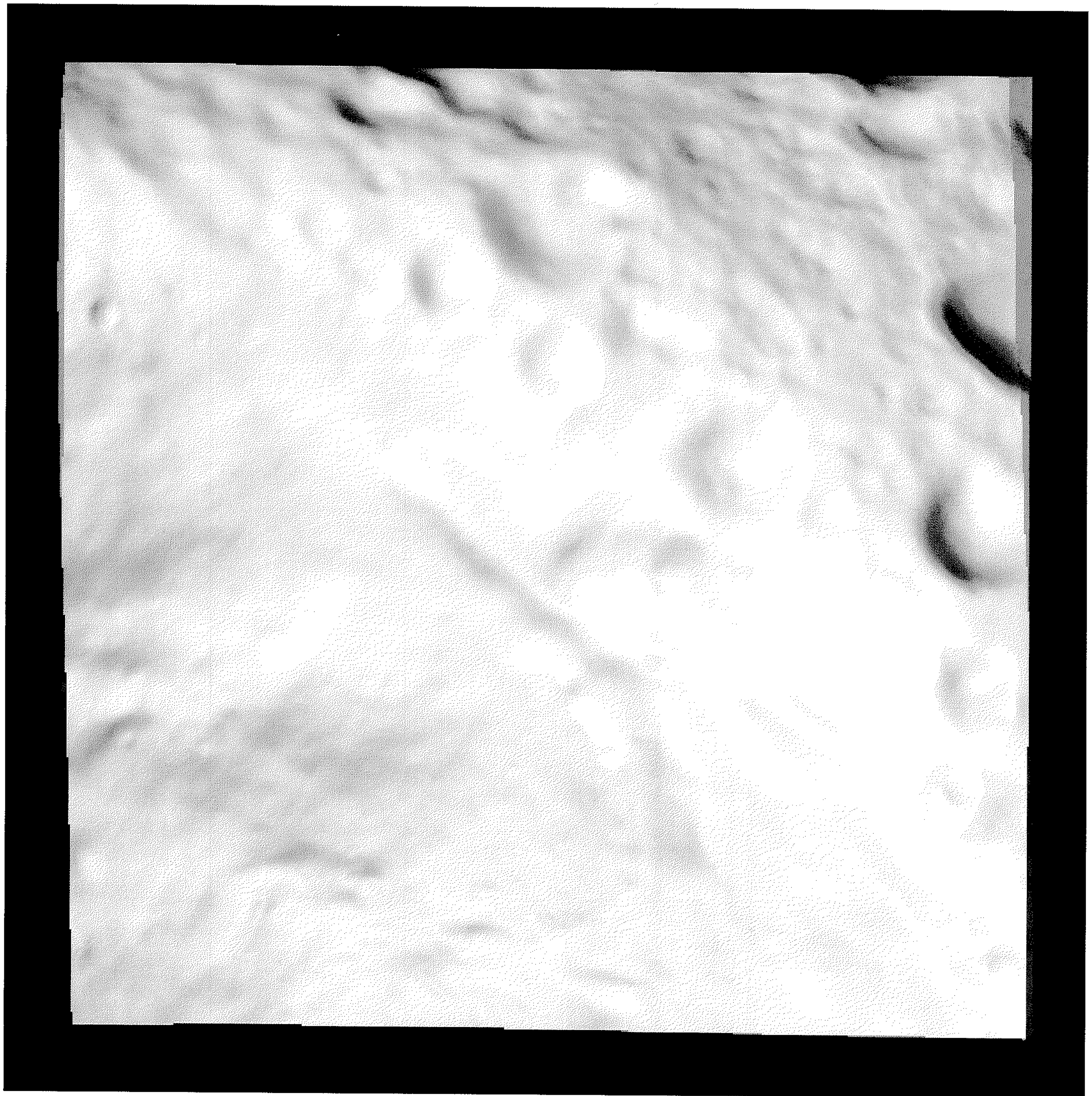


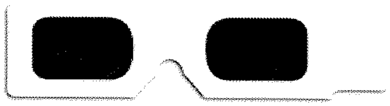




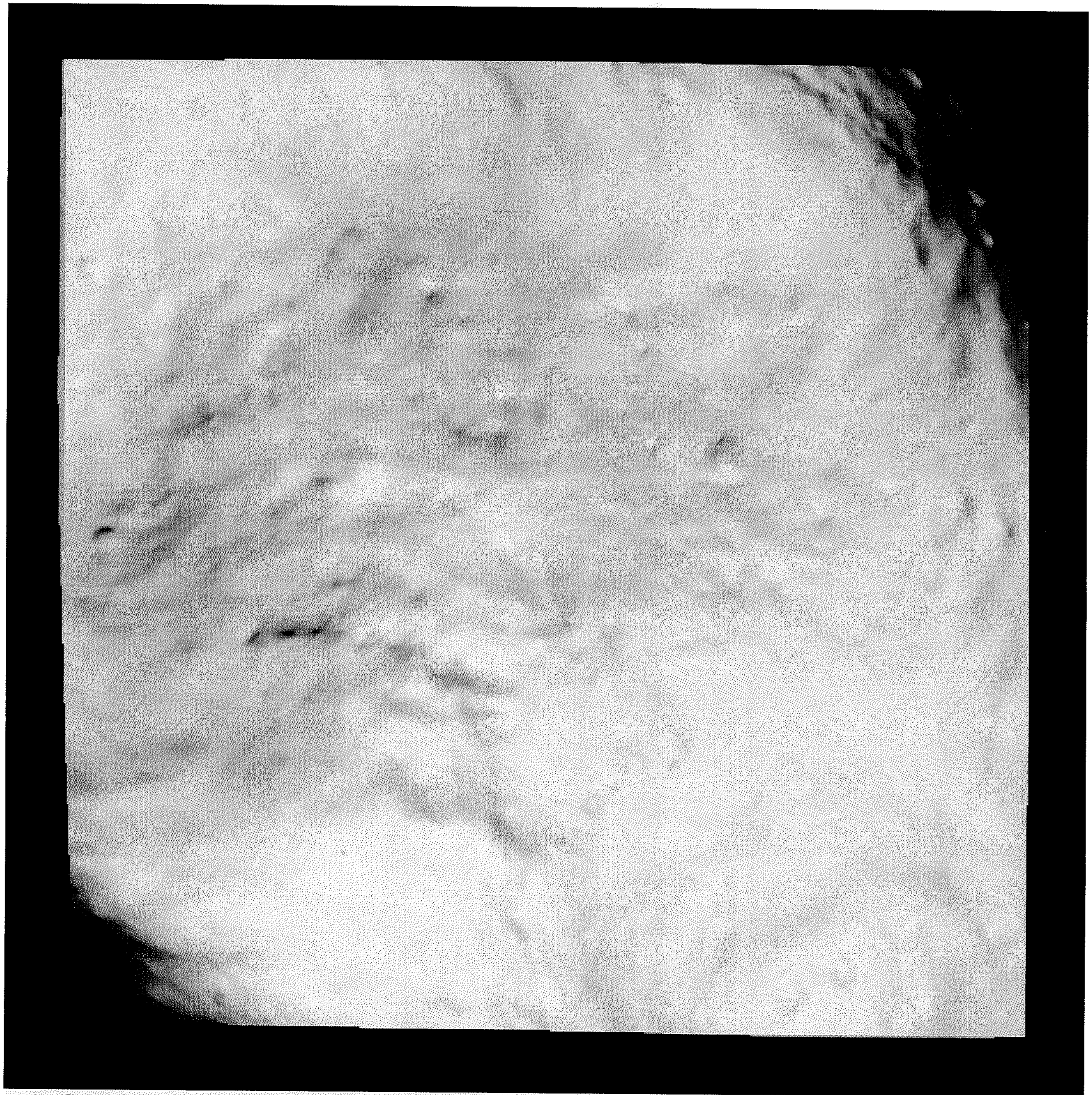
**One end of a prominent system of ridges on Eros, on the edge of the "saddle" Himeros**



X00003 Lt=41.00N Ln=271.00W Rd= 5.910 Sz=5.125 km



**Shoemaker Regio, the rockiest place on Eros**



SR0003 Lt=10.00S Ln=335.00W Rd= 8.340 Sz=7.175 km

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## Abstract

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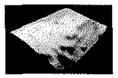

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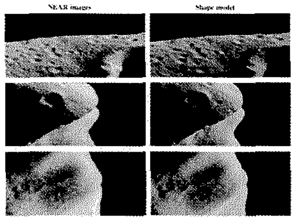
Local heights and relative albedos at each pixel of the map

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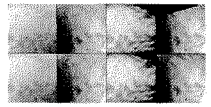


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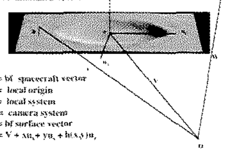

W = bf spacecraft vector
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$= V + xu_x + yu_y + h(x,y)u_z$

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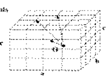

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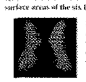

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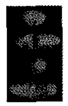

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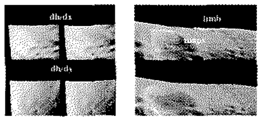


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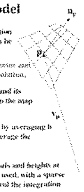


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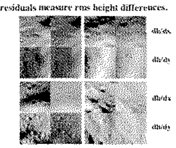


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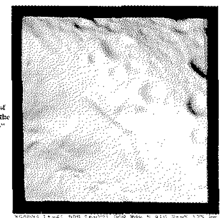

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$$Y = f((V-W)\cdot c_2 + M_{21}x + M_{22}y + M_{23}h)/((V-W)\cdot c_3 + M_{31}x + M_{32}y + M_{33}h)$$

Where f=focal length and $M_{ij} = c_i \cdot u_j$

For a given picture, W and $c_i$ are found by minimizing the weighted mean square residual (difference) between predicted and observed locations, averaged over all landmarks in the picture.

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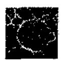
## Height Integration

The height at each location (x,y) is determined from the neighboring heights, and a possible constraining height $h_c$ from shape model, differential stereography, limb or overlapping map data, from:

$$\begin{aligned} h(x,y) = \{ & w_c h_c(x,y) \\ & + [h(x+s,y) + s(t_1(x,y)+t_1(x+s,y))/2 \\ & + h(x-s,y) - s(t_1(x,y)+t_1(x-s,y))/2 \\ & + h(x,y+s) + s(t_2(x,y)+t_2(x,y+s))/2 \\ & + h(x,y-s) - s(t_2(x,y)+t_2(x,y-s))/2]\}/(w_c+4) \end{aligned}$$



Limb and map overlap points used in a slope-to-height integration.

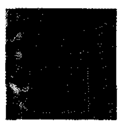
where s is the map pixel spacing and $w_c$ is a small constraining weight.

This equation is applied repeatedly to map points chosen at random until a converged solution is reached. If any height does not exist, its term is not included in the average.

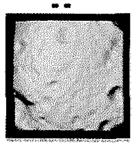
## High Resolution Topography Maps

A similar averaged-slope-to-height integration is used to construct high-resolution topography maps of selected regions

Highest resolution maps used in construction. 12 m/pixel.



3D image of high-resolution map

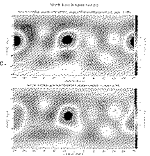


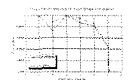
## Eros Gravity

Gravity harmonics were found for each model, with a homogeneous mass distribution, and compared with the measured gravity.

Right: Comparison on a 16 km radius sphere.

Below: Correlations of harmonics as a function of degree.





Shoemaker Regio, the rockiest place on Eros