Asynchronous Switching Decentralized Controllers for Spacecraft Formations

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Abstract— **A method for the design of decentralized asynchronous controllers for spacecraft formations is presented. The control design problem is specified in terms of the relative positions of all spacecraft in the formation. Each spacecraft has a controller that requires only a subset of the full set of formation relative positions. Each spacecraft controller can also switch asynchronously between many such information subsets, allowing it to replace measured information by information communicated from other spacecraft. A two dimensional simulation example is used to illustrate the approach.**

I. Introduction

Precisely controlled formations of spacecraft can be used to synthesize optical imaging instruments of greater resolving power than could otherwise be achieved with a monolithic instrument on a single spacecraft. One such example, interferometric imaging systems are our primary motivating application, although the work described here applies to a wide range for formation control problems. Several interferometric flight projects, based on formation flying, have been studied including Darwin [1], LISA [2], Terrestrial Planet Finder (TPF) [3] and *Starlight* (formerly ST-3) [4].

Interferometric imaging application is illustrated conceptually in Figure 1. Each spacecraft acts as a collector, reflecting light from the imaging target to a combiner spacecraft. The light from any two collectors is combined at a detector and, if the optical pathlengths are held fixed and equal, an interference pattern can be measured. Each measurement of the amplitude and phase of this pattern amounts to a sample of the spatial Fourier transform of the image. Multiple measurements, using either multiple collectors simultaneously or repositioning fewer collectors, allow reconstruction of the image. The effective aperture depends on the collector separation and future mission objectives call for effective apertures of the order of kilometers, resulting in resolutions that cannot be matched by any monolithic spaceborne telescope. Multiple collectors can also be used to create nulls in the spatial response of the array thereby enabling the imaging of dim objects adjacent to bright ones [5]. This is a promising technology for searching for planetary objects in other solar systems.

This work considers deep space missions, where the formation is in heliocentric orbit rather than planetary orbit. The analysis results we present are quite general and in applying them to our formation flying problem we make some specific assumptions. The most significant of these is that the spacecraft can sense their relative position and not their absolute positions.

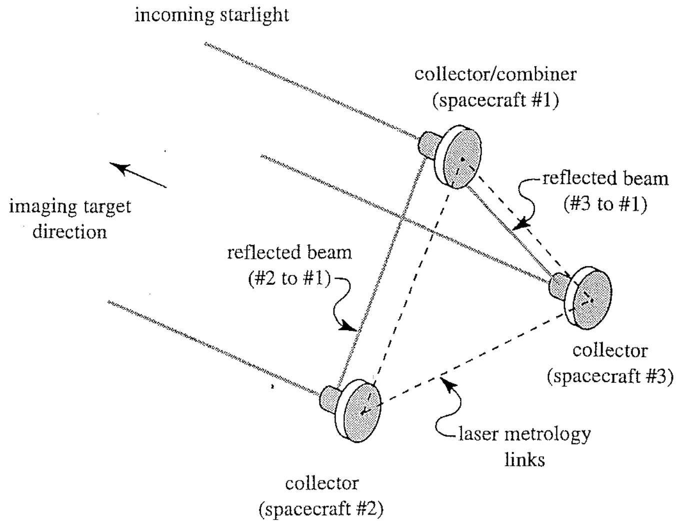

Fig. 1. Interferometric imaging configuration using multiple spacecraft in formation. Spacecraft separations, and the equivalent apertures, are of the order of tens to hundreds of meters.

The spacecraft in such a formation are free flying and their dynamics are coupled only through the application objectives and measurements of relative spacecraft positions and velocities. To maintain the performance of the formation in deep space missions it is necessary only to maintain the relative positions and absolute orientations of the spacecraft.

The stringent optical path length constraint—in the tens of nanometers—is achieved by hierarchical actuation. Depending on the application this may include movable platforms, optical delay lines, and precision piezoelectric actuators on the individual mirrors. The optical path length requirements translate into spacecraft relative positioning requirements in the micrometer to centimeter range [1], [3].

There are many possible topologies for the sensing, control, and communication within the formation. Communication bandwidths, synchronization constraints, and sensor capabilities affect the performance of any chosen topology. These issues have been studied in a formation flying context; see, for example, [6], [7], [8], [9], [10].

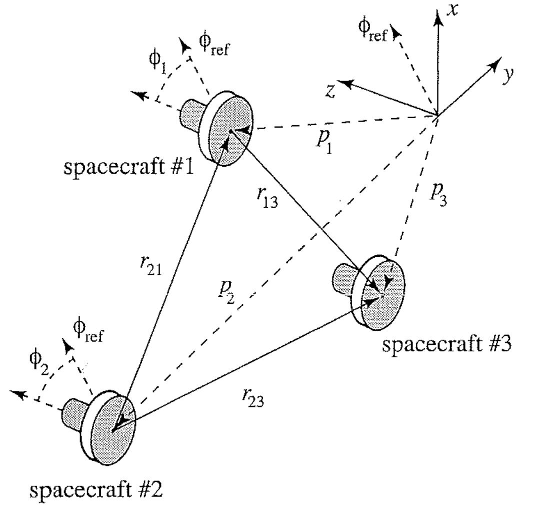

Fig. 2. Spacecraft formation: the local and relative position variables are shown.

The emphasis of this work is on the development of decentralized topologies. The terms "centralized topology" and "decentralized topology" apply to both the control design and implementation. Our objective is to be able to develop controllers which can be implemented in a decentralized fashion. This means that each spacecraft controller requires only a subset of the full formation information in order to be able to implement their component of the formation control. See [11] and the references therein for a discussion of decentralized control and estimation in spacecraft formations.

A formation-wide optimal control design problem based on relative position measurements is posed and solved. This gives a global, centralized control algorithm for formation control. We then exploit the redundancy in relative position information to develop a family of partially decentralized controller implementations of the optimal centralized controller. This also allows individual spacecraft to switch asynchronously between relative measurement options, and this can be exploited when line-of-sight measurements and communication are unavailable during a maneuver. The aerial formation control work described in [12] is similarly motivated, and also is formulated as a relative state control problem.

The focus of this paper is the design of the asynchronously switching network of formation controllers. The theoretical background is described in more detail in [13], [14].

II. Formation Control Problem

A. Formation definition and sensing

We begin by considering a typical formation and defining the notation associated with the various local and relative position and absolute attitude variables. Consider a formation of N spacecraft. For simplicity it is sufficient to define on each spacecraft a reference attitude, ϕ_i, $i = 1, \ldots, N$, with respect to an inertially fixed direction, ϕ_{ref}. Figure 2 illustrates these definitions.

We define a local inertial frame within which each spacecraft is located at position $p_i = [x_i, y_i, z_i]^T$ (where ^{T} denotes transpose). The origin of this frame is not critical for the application we consider here. The relative position between each two spacecraft is defined as,

$$r_{ij} = p_j - p_i = \begin{bmatrix} x_j \\ y_j \\ z_j \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix},$$

$i, j = 1, \ldots, N$, $i \neq j$. Naturally, $r_{ij} = -r_{ji}$, and in an N spacecraft formation there are $N(N-1)/2$ relative three dimensional distances that can be defined modulo the opposite direction equivalences.

In deep space an accurate measurement of $(x_i,\ y_i,\ z_i)$ is not available. It may be possible to obtain range and direction information with respect to Earth, but this will be accurate only to the order of kilometers. On the other hand, the r_{ij} variables can be precisely measured, typically to the order of ± 10 nm.

In contrast to absolute position, spacecraft attitude can be measured to very high accuracy. On-board star trackers are typically used to provide attitude information for each spacecraft, and these have typical accuracies in the range ± 1 milliarcseconds (mas) to ± 200 mas.

We define the formation in terms of the variables that can be accurately measured: the relative spacecraft positions and the attitude of each,

$$\begin{aligned} r_{ij} \ &: \ i, j = 1, \ldots, N, \ i \neq j \\ \phi_i \ &: \ i = 1, \ldots, N. \end{aligned}$$

This definition does not locate the formation in any inertial frame but this is not critical for our applications.

B. Formation control problem

A reference tracking formulation is used to define the control problem, where the relative position and attitude reference signals would be provided by a supervisory system.

The formation specification includes the inertial attitude of each spacecraft. Because the formation dynamics are coupled only through the control objective, attitude errors on each spacecraft are coupled to attitude errors on the other spacecraft only through the control system. In the deep space mission application, each spacecraft is also assumed to have a local measurement of its attitude. This means that correcting attitude errors can be viewed as a strictly local control problem: only local measurements are required to determine the attitude error, and only local actuation is required to attenuate the attitude error. Control of ϕ_i can therefore be treated as decentralized, both in terms of design and implementation. For this reason we drop control of ϕ_i from further consideration and focus on the more difficult problem of the control of r_{ij}.

Figure 3 illustrates the relative position tracking problem to be considered. Using this framework, we pose a relative position formation design problem as follows. Given the collected spacecraft dynamics, $r_{jk} = P(x, u_i)$, where $j = 1, \ldots, N-1$, $k = j+1, \ldots, N$ and $i = 1, \ldots, N$, design

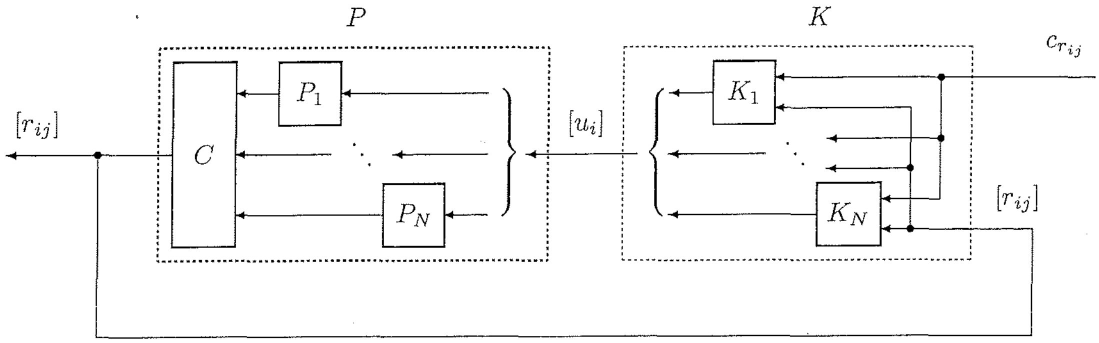

Fig. 3. Relative position control design problem for an N spacecraft formation. Reference relative position commands are denoted by $c_{r_{ij}}$ and would be provided by a supervisory system.

a stabilizing controller, $u_i = K(c_{r_{ij}}, r_{jk})$, to minimize a formation objective cost, $J(r_{jk}, u_i)$. This is a centralized, or "global", control problem in that it is specified in terms of the overall formation objectives, rather than individual spacecraft objectives. Problems such as this are readily handled by existing optimal control theory and supported by analysis and synthesis software. For example, K above may have been designed to meeting an $\mathcal{H}_\infty$ or $\mathcal{H}_2$/LQG objective for the formation.

C. Switched measurement topologies

Note that there is some redundancy in the formation definition as the $N(N-1)/2$ relative positions are not independent. The full set of relative position measurements contain redundancies that can be expressed as algebraic constraints. For example,

$$r_{ij} + r_{jk} + r_{ki} = 0, \quad \text{for all } i, j, k,$$

and at every time t. For the formation to be well defined these constraints must also apply to the relative position commands, $c_{r_{ij}}$.

The approach (detailed in [14]) is based on a subspace characterization of these redundancies. We note that there exists M such that,

$$M^T \begin{bmatrix} r_{12} \\ \vdots \\ r_{N-1,N} \end{bmatrix} = 0,$$

or equivalently, $M^T P(x, u) = 0$ for all u. Define a class of transformation matrices by, $H = I - XM^T$, where X satisfies, $M^T X = I$. The matrix H has the effect of expressing some of the relative positions as linear combinations of the others.

It can be shown that for any stable and stabilizing global formation controller, K, all controllers of the form KH are also stabilizing and achieve the same level of formation control performance. The transformation matrices, H, have the effect of switching the information going into the controller amongst various measured and communicated information sets.

III. Relative Position Based Control Design

Using relative sensing as a basis for control design allows flexibility in the choice of measurement and communication topologies. We now consider the problems that arise as a result of this architectural choice, and provide design methods for developing optimal formation control systems.

We consider a linear, state-space description of the spacecraft dynamics,

$$\dot{x} = Ax + Bu, \qquad r = \begin{bmatrix} C & 0 \end{bmatrix} x.$$

Because the spacecraft are not physically coupled, A and B have a sparse block structure. The output matrix, C, gives the relative position measurements effectively coupling the spacecraft.

A. State feedback

The first obstacle to design is that the state, x, is not fully observable from the relative measurements, r. Physically this arises from the fact that the position and velocity of the formation centroid cannot be determined by relative position measurements. To obviate this we use a similarity transformation of the state, $Tx = \begin{bmatrix} z^T & v^T \end{bmatrix}^T$, to give,

$$\begin{aligned} \begin{bmatrix} \dot{z} \\ \dot{v} \end{bmatrix} &= \begin{bmatrix} A_z & A_{zv} \\ 0 & A_v \end{bmatrix} \begin{bmatrix} z \\ v \end{bmatrix} + \begin{bmatrix} B_z \\ B_v \end{bmatrix} u, \\ r &= \begin{bmatrix} 0 & C_v \end{bmatrix} \begin{bmatrix} z \\ v \end{bmatrix}, \end{aligned}$$

where (C_v, A_v) is observable. We note that the observable part of the dynamics,

$$\dot{v} = A_v v + B_v u, \qquad r = C_v v,$$

can be used to design a formation controller using relative position measurements. Various control design methods can be applied at this point. The design example presented in this paper uses an Linear Matrix Inequality (LMI) optimization [15] for estimator and state-feedback design.

The state-feedback design problem is formulated in terms of finding a controller that drives all states within an initial ellipsoid, $\mathcal{V}_0 = \{ v \mid v^T V_0 v < 1,\ V_0 = V_0^T > 0 \}$, to zero with a bounded cost given by,

$$\|W_v v\|_2^2 + \|W_u u\|_2^2 \leq \gamma_v^2.$$

Finding the minimum γ_v clearly gives the optimal controller for solving this problem and this is solved via standard LMI methods. The state, v, must be estimated from the relative position measurements, r. Our formulation guarantees the observability of v and we can use a completely analogous dual LMI problem to design an estimator gain matrix, L.

B. Exploiting Input Redundancies

The linear model of an individual spacecraft's dynamics is essentially a double integrator. Force actuators—typically thrusters—are used for the control inputs, and these may have additional dynamics associated with them. If each spacecraft has zero order or identical first order actuator dynamics, then the input control space contains an additional degree of freedom.

Under these assumptions, B_v has reduced column rank, which means that there is a matrix, $B_\perp$, satisfying $B_v B_\perp = 0$. An SVD can be used to calculate this matrix, and we can define a projection, $(I - B_\perp {B_\perp}^T)$, such that, $B_v(I - B_\perp {B_\perp}^T)\, u = B_v\, u$ and $B_z(I - B_\perp {B_\perp}^T)\, u = 0$. We can calculate control actuation signals of the form,

$$\hat{u} = (I - B_\perp {B_\perp}^T)\, u + B_\perp \nu,$$

which allow us to control the z and v components of the state independently. The input u controls the formation in the manner given in the previous sections and ν can be considered as a control variable for the formation centroid (and other common unobservable states). We now give two relevant uses for this control.

C. Minimizing formation fuel consumption

The input null-space control variable ν can be chosen to minimize the total formation fuel use. At each time instant, given the formation actuation command u, we calculate ν as the solution to the following linear program.

$$\min_{\nu} \|(I - B_\perp {B_\perp}^T)\, u + B_\perp \nu\|_1, \quad \text{where} \quad \|\hat{u}\|_1 := \sum_{i=1}^{N} |\hat{u}_i|.$$

If actuator servo loops have been applied on each spacecraft then the u_i represent commanded thrusts and these are only approximately equivalent to the fuel used on each spacecraft. Note that this approach minimizes the total formation fuel consumption for a given controlled maneuver. It is not necessarily a solution to the problem of finding the minimum fuel maneuver between specified formation configurations. One of the two examples considered in Section IV uses this form of input null-space control.

D. Control of the formation centroid

We now consider the problem of using the variable ν as a means of controlling the formation centroid. The dynamics of the unobservable state can be expressed as,

$$\dot{z} = A_z\, z + A_{zv}\, v + B_z B_\perp\, \nu.$$

We again take the approach of separating this control problem into a estimator and state-feedback design. The lack of observability of z means that the estimator is now open-loop and given by the marginally stable z dynamics above.

Our control of z is implemented via $\nu = -K_z\, \hat{z}$, where $\hat{z}$ is an estimate of z. The objective can again be specified in terms of driving all z in an ellipse, $z \in \mathcal{Z}_0 = \{ z \mid z^T Z_0 z < 1,\ Z_0 = Z_0^T > 0 \}$, to zero with cost bounded by,

$$\|W_z z\|_2^2 + \|W_\nu \nu\|_2^2 \leq \gamma_z^2.$$

This gives an LMI problem identical in form to that used to solve the state feedback problem.

This controller implements both control objectives (precise control of relative positions via state feedback on $\hat{v}$, and open-loop control of the formation centroid via feedback on $\hat{z}$) in a manner which ensures that the objectives do not interact. This input decoupling approach could equally well be used to implement lower bandwidth and/or lower resolution feedback control of the formation centroid if a lower precision measurement of absolute position was available.

IV. A Design Example

We illustrate the application of measurement switching, optimal relative state control design, and input null space control on a four spacecraft, two-dimensional, example. Each spacecraft is modeled in each dimension as a double integrator with first order actuation dynamics. To illustrate the most general application of the input null space control we consider the case where each spacecraft has identical actuation dynamics (1.0 second time constant). The spacecraft masses are not identical and are specified as 300, 310, 280 and 280 kg.

The control will maneuver the spacecraft from an initial position—in an arbitrary (x,y) frame—of a square of 200 meters on a side, rotating clockwise at 0.003 rad/second, to a non-rotating line in the y direction with 200 meters separation. The initial and final formation specification, and the measurement vector, consists of 12 relative positions (6 in each dimension). Each vehicle begins the maneuver using the measured relative positions of each of the other three spacecraft. The maneuver is such that line-of-sight contact is lost between various spacecraft at seven instances (including the final position), and at these times the affected spacecraft employ relative measurement switching, via a series of switching matrices H, to accomplish control of the maneuver.

We consider two controllers with differing objectives for the input null space control. The first maintains the estimated formation centroid at the same position throughout the maneuver. The second allows the centroid to move to perform the

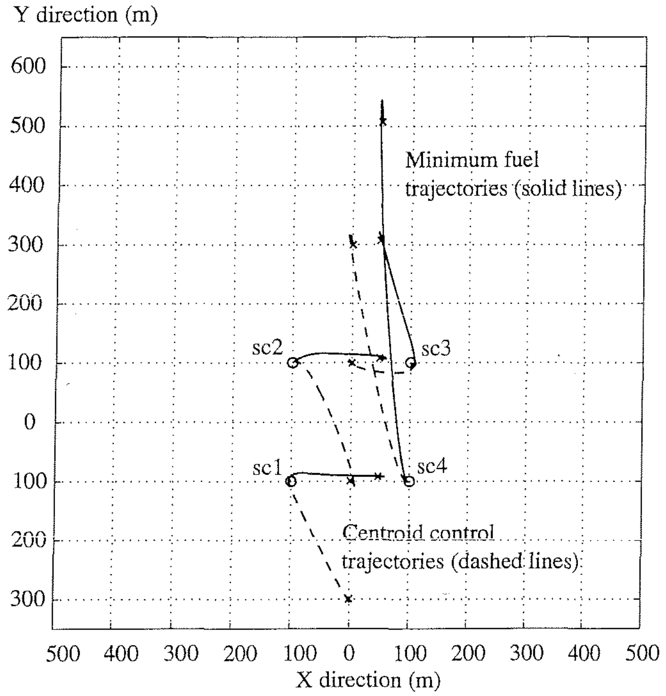

Fig. 4. Absolute frame x-y plot of the trajectories for the centroid and minimum fuel controllers. The spacecraft initial positions are denoted by circles, and the final positions in each maneuver are denoted by crosses.

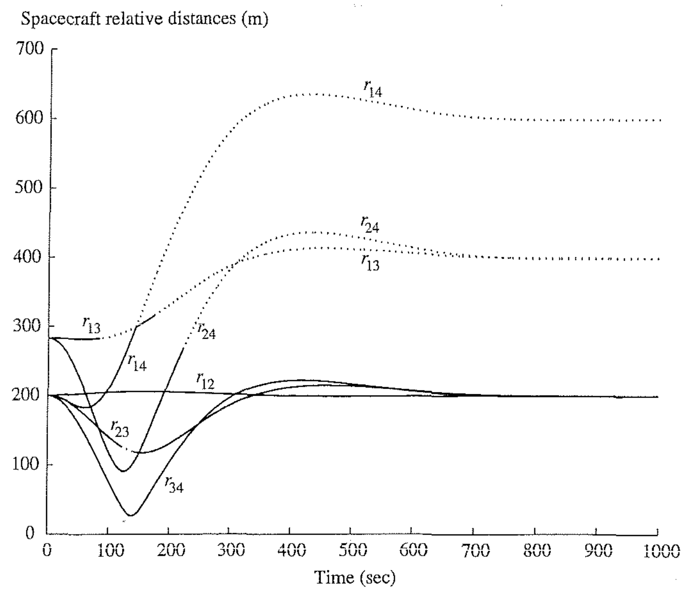

Fig. 5. Spacecraft relative distance vector, r_{ij}, for centroid and minimum fuel controllers. Dotted lines indicate relative links that are occluded by intervening spacecraft.

maneuver with minimum fuel consumption. In each case, the relative positions track identical trajectories.

Figure 4 illustrates the x-y plane motion of each spacecraft for each of the two input null space control options. In absolute coordinates there are significant differences in the trajectories, and in the final positions. The minimum fuel controller uses 3.1% less fuel illustrating that maintaining the centroid invariant is reasonably efficient, but not optimal for this maneuver. The instances where measurement vector switching occurs, and the measurement topology that each spacecraft uses, are given in Table I.

Figure 5 illustrates the six relative distances during the maneuver. Each can be associated with a measurement/communication link, and the times at which these links are occluded is also illustrated. Note that the relative positions (shown in Figure 5), and the information switching instances (given in Table I) are identical for both the centroid control and minimum fuel control cases. The measurement/communication link is considered to be occluded if the edge of a closer 2 meter radius spacecraft comes within 10 degrees of the line-of-sight. In Table I, superscripts * and † denote information communicated via one and two links respectively.

Each spacecraft has sufficient information to reconstruct the relative paths of all of the other spacecraft, predict when specific measurement and communication links will be occluded, and determine a switching strategy using the remaining available links. The control design is optimal with respect to the chosen formation-wide criteria, and in this instance there is sufficient freedom in the input null space to allow additional control objectives. This particular maneuver required relative information to be communicated between spacecraft. This is not always the case; simpler maneuvers may be accomplished without inter-spacecraft communication.

V. Conclusions and Future Research

Relative position based specifications are a suitable choice for deep space formations where absolute position measurements are inaccurate or unavailable. The redundancy in a relative position based design allows the development of a family of equivalent formation controllers, where each spacecraft may use differing relative measurement vector components. Some of these may be communicated from other spacecraft and this allows the optimal formation control to be maintained under reconfiguration when certain measurement and communication links are no longer available. An input null space control approach has been outlined and allows the formation to simultaneously achieve other objectives in a decoupled manner. The minimum fuel controller is one such example.

If data communication latency is significant, the switching between measured and communicated information may require a more detailed stability analysis. This is an area of future research, along with formation control methods that are robust with respect to potential communication latencies.

Our approach essentially employs a transformed state estimator in each spacecraft, and this allows each spacecraft to reconstruct the controls and trajectory of all other spacecraft. This can be done with only $N-1$ relative spacecraft measurements. However, it may be possible to improve the accuracy of the internal estimators by transmitting additional information around the network and this is also an area of research interest.

Time (sec.)	Link Occlusions	Measurement Topology			
		s/c1	s/c2	s/c3	s/c4
0.0		(r_{12}, r_{13}, r_{14})	(r_{12}, r_{23}, r_{24})	(r_{13}, r_{23}, r_{34})	(r_{14}, r_{24}, r_{34})
89.5	r_{13} by s/c4	$(r_{12}, r_{34}{}^*, r_{14})$		$(r_{14}{}^*, r_{23}, r_{34})$	
121.9	r_{13} and r_{23} by s/c4		$(r_{12}, r_{34}{}^*, r_{24})$	$(r_{14}{}^*, r_{24}{}^*, r_{34})$	
141.5	r_{13} by s/c4		(r_{12}, r_{23}, r_{24})		
143.8	r_{14} by s/c3	$(r_{12}, r_{13}, r_{34}{}^*)$		(r_{13}, r_{23}, r_{34})	$(r_{12}{}^*, r_{24}, r_{34})$
146.7	r_{14} by s/c2 and s/c3				
174.4	r_{14} by s/c2 and s/c3 r_{13} by s/c2	$(r_{12}, r_{23}{}^*, r_{24}{}^*)$		$(r_{12}{}^*, r_{23}, r_{34})$	
220.7	r_{14} by s/c2 and s/c3 r_{13} by s/c2 r_{24} by s/c3	$(r_{12}, r_{23}{}^*, r_{34}{}^\dagger)$	$(r_{12}, r_{23}, r_{34}^*)$		$(r_{12}{}^\dagger, r_{23}{}^*, r_{34})$

TABLE I

RELATIVE POSITION VECTOR SWITCHING INSTANCES

VI. ACKNOWLEDGMENTS

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