Tunable filters and time delays with coupled whispering gallery mode resonators

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ABSTRACT

We demonstrate tunable optical filters fabricated from metalized lithium niobate disk resonators and discuss further applications of the resonator chains for tunable photonic delay lines.

Keywords: Whispering Gallery Modes, Tunable Optical Filter, High-Order Optical Filter, Tunable Delay Lines

1. INTRODUCTION

Microwave filters with narrow bandwidth and wide tunability are crucial to the realization of advanced communications and radar architectures. The narrow bandwidth naturally allows increased channel capacity in a communications band, while tunability provides spectral diversity and increased efficiency. Unfortunately, microwave filters that also have a bandpass with a flat top and high sidemode rejection cannot simultaneously provide narrow bandwidths (high quality factor, Q) and wide tunability. Typically, the highest Q microwave filters in the x-band have bandwidths in the range of a few MHz. Moreover, those filters have either a fixed center frequency, or are tunable through bandwidths that represent a minor fraction of the center frequency. Tuanbility by 20generally regarded as "wide". Those high-Q filters also introduce several dB of insertion loss in microwave circuits; the typical insertion loss for a multi-pole filter with a few MHz linewidth at 10 GHz center frequency is about 10 dB. It should also be mentioned that the achievable quality factor of high performance microwave filters generally degrades as the center frequency goes beyond a few tens of GHz.

Photonics filters based on optical whispering gallery mode cavities have been devised to address the shortcomings of microwave filters. Multi-pole, high Q filters based on cascaded whispering gallery mode (WGM) microresonators fabricated with silica have been demonstrated allowing compact packages and robust performance at 10 – 100 GHz bandwidths, with corresponding optical Qs on the order of $10^5 - 10^4$,[1–6] and are in fact commercially available. These filters provide passbands with a flat top and sharp skirts, suitable for high performance applications. Since the microwave signals in photonic systems are sidebands of an optical carrier, these filters, in principle, can be used at any microwave frequency, providing the same characteristics throughout the band, from 1 to 100 GHz, and higher.

While delivering high optical Q's and desirable passband spectra, optical WGM filters based on silica ring resonators are limited in their microwave Q, and are not tunable. Mechanical trimming of whispering gallery modes of silica spheres with applied strain[7–9] and temperature[10] tuning have been previously used for the controlled tuning of the resonance frequency in WGM microresonators. An all-optical tunable filter design based on discontinuity-assisted ring resonators has also been proposed theoretically,[11] but, to our knowledge, no experimental implementation of the configuration has been reported.

A recent technique for WGM resonance tuning was demonstrated using microring resonators with a photosensitive coating. In that study, glass microrings were dipped in a polymer coating material and were exposed to UV light. This method produced resonators with relatively small Q (about 800) because of the polymer-induced absorption; but it still allowed large tunability of the optical resonance of the microring, enough for wavelength selective applications.[12]

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Another approach for trimming the frequency of microresonators exploits the photosensitivity of the germanate silica glass. When exposed to UV light, this material undergoes a small permanent change in structure that alters its index of refraction. In the case of a WGM resonator, the spatially uniform change in the index of refraction results in a uniform translation of the resonant frequencies. Such a tunable resonator as well as a second-order optical filter based on two coupled resonators, one of which was tunable, was experimentally realized for optical high-Q (10^8) WGMs.[13–15]

Fabrication of optical WGM resonators with lithium niobate has led to the demonstration of a high-Q microwave filter with a linewidth of about 10 MHz. Such a resonator was used for demonstration of a filter with a tuning range in excess of 10 GHz.[16] Unfortunately, the Lorentzian lineshape of the filter function associated with a single microresonator represents a severe limitation for its application in many systems that , in addition to a narrow bandpass and a large tuning range, also require large sidemode rejection.

Recently, we have adaptated the mechanical optical polishing technique for fabrication of ultrahigh-Q crystalline WGM resonators.[17] The crystals, purchased from several vendors, have repeatable Q-factor values at $\lambda = 1320$ nm at the level of 6×10^9 for CaF_2, 10^9 for Al_2O_3, and 2×10^8 for $LiNbO_3$. We believe that the Q-factors are limited primarily by the material absorption because no Rayleigh scattering on residual surface roughness were detected, i.e. there were no backwardly propagating radiation and no resonance doublets observed.

While $LiNbO_3$ resonators are fabricated from commercially available Z-cut wafers by consequent preparation of cylindrical preforms and polishing the rim of the preforms, Al_2O_3 and CaF_2 resonators are produced from commercially available optical windows in the same way as $LiNbO_3$ resonators. Typical resonator has toroidal shape with several millimeter in diameter and several hundred micron in thickness.

In this paper we present information on progress in fabrication of single WGM crystalline resonator filters as well as tunable multi-pole filters based on cascaded lithium niobate WGM resonators.

We realized optical filters with bandwidths of about ten kiloHertz using CaF_2 WGM resonators, and tens of MegaHertz using $LiNbO_3$ and MgO:$LiNbO_3$ resonators. The CaF_2 resonators have very stable ultrahigh Q-factors compared with fused silica resonators, where Q degrades with time. Limited tuning of the CaF_2 filters can be realized with temperature. The insertion loss of the filter was at 5 dB level. Lithium niobate WGM resonators have lower Qs, but they allow convenient tuning with an applied DC voltage. The best tunability for $LiNbO_3$ single resonator filter was ± 20 GHz by applying DC voltage of ± 50 V to an electrode placed over the resonator.

We have also realized a miniature resonant electro-optically tunable third-order Butterworth filter.[18] The filter is based on three WGM disc cavities fabricated from a commercially available lithium niobate wafer. The filter, operating at the 1550 nm wavelength, has approximately 30 MHz linewidth and can be tuned in the range of ± 12 GHz by applying DC voltage of ± 150 V to an electrode. The free spectral range of each resonator is approximately 13.3 GHz, and the filter may be tuned practically at any optical frequency in the transparency range of lithium niobate.

We also have also proposed an application of cascaded resonators for real-time shaping of their modal structures.[19, 20] A key feature of our approach is that it points to a simple tuning of the frequency and the width of the resonator system resonance, allowing the tuning of the group delay of optical signals, a highly desirable feature for signal processing applications.

Large group delays in chains of coupled resonators have been previously studied in the literature.[21, 22] It was shown recently that the Q-factor of the coupling-split modes for a system of N identical coupled resonators is greater than that of a single resonator in the chain by a factor of N, and even more, in the case of optimum coupling.[23] Stopping light all optically with a chain of interacting tunable optical resonators was discussed in.[24]

We analyze the special configuration of two WGM resonators leading to sub-natural (i.e. narrower than loaded) linewidths. We discuss the applications of such devices as delay elements, which can take advantage of recent demonstration of WGMs in crystalline resonators to provide frequency and bandwidth tuning capabilities.

2. TUNABLE LORENZIAN PHOTONIC FILTER

One of obvious applications of optical resonators is fabrication of optical filters. Transmission and reflection of a monochromatic electromagnetic wave of frequency ν by a WGM optical lossless resonator may be characterized by coefficients

$$T = \frac{\gamma}{\gamma + i(\nu - \nu_0)}, \quad R = \frac{i(\nu - \nu_0)}{\gamma + i(\nu - \nu_0)}, \tag{1}$$

where T and R describe the amplitude transmission and reflection respectively, γ and ν_0 are the linewidth and resonance frequency of a mode of the resonator (we assume that $|\nu - \nu_0|$ is much less than the cavity free spectral range). The power transmission $|T|^2$ through the resonator is Lorentzian.

Studying kiloHertz optical filters restricts application of commercially available lasers by those having kiloHertz linewidth. That is why we were unable to use our 1550 nm diode laser, and used 1320 nm laser instead. Light from 1320 nm laser is send into the resonators with a diamond coupling prism. The laser linewidth is less than 5 kHz. The coupling efficiency is better than 50%, though to achieve the maximum Qs we used underloaded resonators. One can gradually turn on and off coupling to WGMs by increasing the gap between the prism and the resonator, unlike to the Fabry-Perot resonators with their fixed coupling to external beams. This permits clear observation of Q-factor up to its unloaded value. Typical filter function (resonance curve) is shown in Fig. 1. It is important to note here that the filter is characterized by absorption resonance in a single

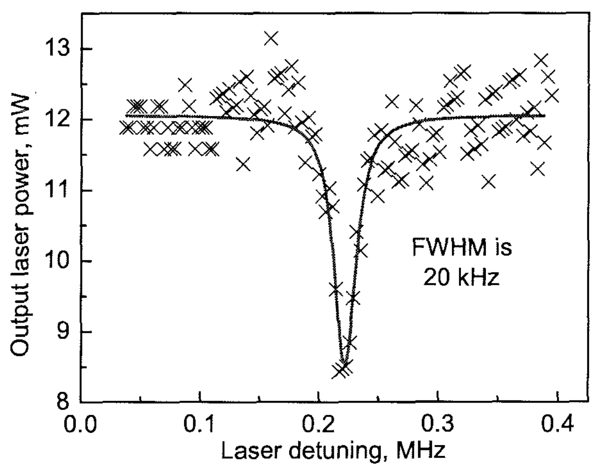

Figure 1. Typical WGM resonance of the calcium fluoride resonator. The experimental data are well fitted with Lorentz curve having 20 kHz full width at the half maximum (FWHM), which corresponds to $Q \approx 10^{10}$.

coupling prism configuration, while in the two prism configuration it is characterized by transmission resonance.

The very-high Q factor of the filter does not survive in a dusty room for more than a half an hour. It relaxes up to $\approx 3 \times 10^9$ and stays unchanged at this value for several days. For a comparison, fused silica resonators have much faster degrading Q-factor unless they are placed in an evacuated box.

To discuss tunability of the filter we note that thermal dependence of the index of refraction for calcium fluoride is $\beta = n_0^{-1} \partial n / \partial T \simeq -10^{-5}/K^o$. It means that the frequency of a WGM mode increases by $10^{-5}\nu$ (several gigaHertz) if the temperature T increases by one degree Kelvin (follows from $2\pi\nu \approx cN/Rn_0(1+\beta)$, where c is the speed of light in the vacuum, $N \gg 1$ is the mode number, R is the radius of the resonator, and n_0 is the index of refraction). This shift is five order of magnitude bigger than the width of the resonance if $Q = 10^{10}$.

Though tuning WGM resonators with temperature is rather efficient, the electro-optical tuning is much more convenient from the practical point of view. Lithium niobate resonators are helpful here. A schematic diagram of the tunable Lorenzian filter configuration based on a disk cavity fabricated from lithium niobate wafer is shown in Fig.2. A Z-cut MgO:$LiNbO_3$ disk resonator has 10 mm in diameter and 30 μm in thickness. The resonator perimeter edge is polished in the toroidal shape. We studied several nearly identical disks. The repeatable value of the quality factor of the main sequence of the resonator modes is $Q = 2 \times 10^7$ at 780 nm wavelength (the observed maximum was $Q = 5 \times 10^7$ at this wavelength), which corresponds to approximately 20 MHz bandwidth of the mode. We should note, that 780 nm laser was used to create the filter/modulator suitable for spectroscopy of rubidium. The filter gives even better performance at 1550 nm. When we used a

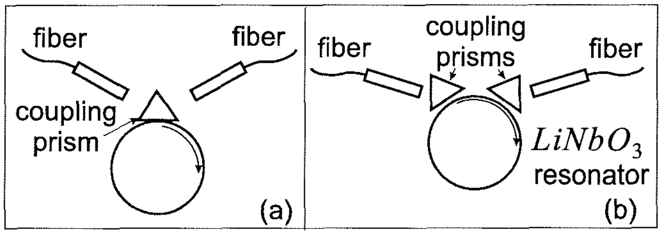

Figure 2: (a) Single prism, and (b) double-prism Lorentzian filter setups.

single coupling prism (absorptive filter, Fig.2a) the typical fiber-to-fiber loss was approximately 5 dB. On the other hand, when we used two prisms (transmittive filter, Fig.2b) and light was sent into, and retrieved out of, the resonator via different coupling diamond prisms, the repeatable value of fiber-to-fiber insertion loss was 12 dB. The minimum/maximum transmission was achieved when light was resonant with the resonator modes in the filter shown in Fig.2a/Fig.2b. Tuning of the filter is achieved by applying voltage to the top and bottom disk surfaces coated with metal. The coating is absent on the central part of the resonator perimeter edge where WGMs are localized.

Experimentally measured electro-optic tuning of the filter spectral response and tuning of center wavelength with applied voltage exhibit linear voltage dependence in ± 100 V tuning range, i.e. the total tuning span significantly exceeds the FSR of the resonator. Changing the tuning voltage from zero to 10 V shifted the spectrum of the filter by 4 GHz for TM polarization, in agreement with theoretical value.

3. TUNABLE THIRD-ORDER BUTTERWORTH FILTER

Coupled optical fiber resonators are widely used as optical and photonic filters.[25,26] Newly developed WGM resonators are also promising for achieving that goal because of their small size, low losses, and integrability into optical networks. We report on the realization of a miniature resonant electro-optically tunable Butterworth third-order filter. Our filter is based on three gold-coated disc WGM cavities fabricated from a commercially available lithium niobate wafer. The filter, operating at the 1550 nm wavelength, is further development of the tunable filter design based on single lithium niobate resonator, described in the previous section.

While tunable single-resonator filters are characterized by their finesse which is equal to the ratio of the filter free spectral range and the filter bandwidth, our three-resonator filter has much more rare spectrum compared with a standing alone WGM resonator, similar to the coupled fiber-ring resonators.[25,26] The tuning speed of the filter is approximately 10 ns, while the real spectrum shifting time is determined by filter's bandwidth and does not exceed 30 ns.

The differences in the size of the cavities is rather important for the device fabrication. Our aim was to produce spectral lines of all the resonators of a similar width to allow the realization of a complex spectral line structure. If resonances of the interacting cavities have differing widths, then as they are made to approach each other, the height of the narrower resonance will simply track the shape of the wider ones, which is of no use for the filter application. The size of a cavity affects the quality of its resonance since cavities of similar size have

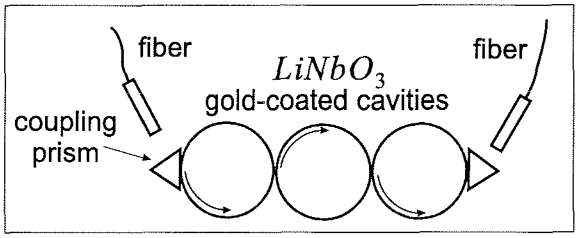

Figure 3: A scheme of a third order tunable optical filter made of gold-coated lithium niobate discs.

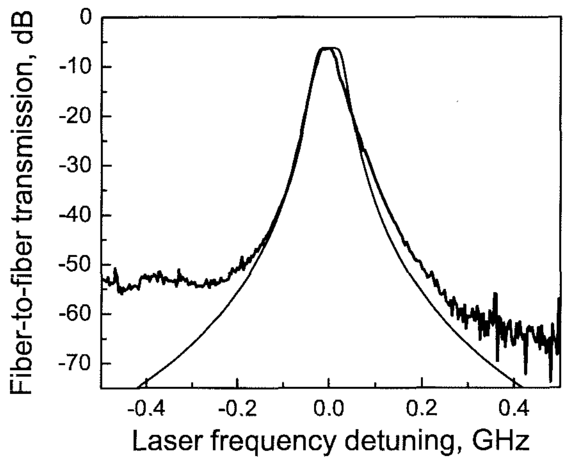

Figure 4. Transmission curve of the filter and its fit with Butterworth profile function $\tilde{\gamma}^6/[(\nu-\tilde{\nu}_0)^6+\tilde{\gamma}^6]$, where $\tilde{\gamma}=29$ MHz, $\tilde{\nu}_0$ determines the center of the filter function and primarily depends on the resonators' geometrical dimensions. Voltages applied to the resonators vary near zero in 10 V range to properly adjust frequencies of each individual resonator and construct the collective filter function as shown.

similar quality factors. Our experiment proves that we are able o fabricate similar resonators approximately with the same parameters.

Fig.4 shows spectrum obtained in the experiment with three gold-coated lithium niobate resonators. To highlight the filter performance we plotted also the theoretical third-order Butterworth fit of the curve. Obviously, the three-cavity filter has much faster rolloff compared with the Lorentzian line of the same full width at half maximum. On the other hand, the filter function does not look exactly like a third order one because of small differences between the cavity Q-factors and dimensions.

When three cavities are placed in series, the light amplitude transmission coefficient is

$$T=\frac{T_1T_2T_3}{(1-R_1R_2\exp[i\psi_{12}])(1-R_2R_3\exp[i\psi_{23}])-R_1R_3|T_2|^2\exp[i(\psi_{12}+\psi_{23})]}, \tag{2}$$

where T_j and R_j $(j=1,2)$ are the transmission and reflection coefficients of the resonators, and ψ_{jk} is the phase shift introduced by the coupling of resonators j and k.

Let us consider the case of resonators with slightly different resonance frequencies and the values of linewidth, and assume that $\exp(i\psi_{jk})$ are properly adjusted. Under optimum conditions[1] power transmission through the system is

$$|T|^2\simeq\frac{\tilde{\gamma}^6}{\tilde{\gamma}^6+(\nu-\tilde{\nu}_0)^6}, \tag{3}$$

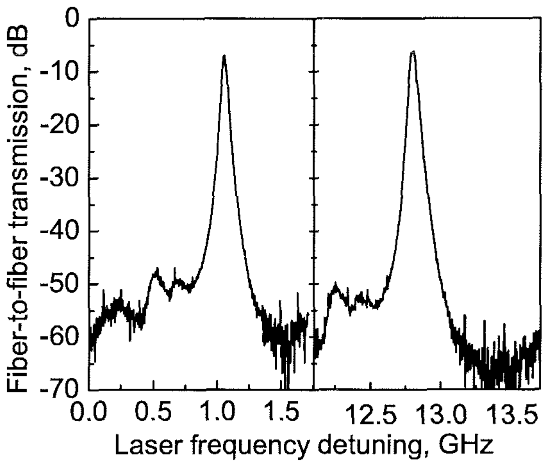

Figure 5. Transmission curve of the filter and the same curve reconstracted at, approximately, 12 GHz frequency shift from the initial transmission curve center ν_0. Optimum voltages are different for all the resonators and are scaled from 100 V to 150 V were applied to the discs.

where $\tilde{\gamma}$ is the bandwidth of the filter, and $\tilde{\nu}_0$ is the central frequency of the filter. The transmission through the resonator system is small for any frequency when the resonant frequencies of the modes are far from each other ($|\nu_j - \tilde{\nu}_0|^2 \gg \tilde{\gamma}^2$). The transmission becomes close to unity when the mode frequencies are close to each other compared with the modes' width γ. Finally, the transmission for the off resonant tuning is inversely proportional to ν^6, not to ν^2, as for a single resonator, Lorentzian, filter. Those are the properties of the third order filters; they much our experimental observations.

Experimentally measured electro-optic tuning of the filter spectral response and tuning of center wavelength with applied voltage is shown in Fig.5. The filter exhibits linear voltage dependence in ± 150 V tuning range, i.e. the total tuning span exceeds the FSR of the resonator. Changing the tuning voltage from zero to 10 V shifted the spectrum of the filter by $1.3 - 0.8$ GHz for TM polarization, in agreement with theoretical value. Though, theoretically, this value does not depend on the resonator properties and is related to the fundamental limitations of optical resonator based high speed electro-optical modulators,[27] the different results for different resonators measured in our experiment occur due to imperfectness of the cavity metal coating as well as due to partial destruction of the coating during polishing procedure.

4. TUNABLE DELAY LINE WITH TWO COUPLED WGM RESONATORS

Tunable filters discussed above allow shifting the spectrum of the resonators, however they do not provide linewidth tuning capabilities. Below we analyze the special configuration of two WGM resonators shown in Fig. 6a (configuration in Fig. 6b has similar responce) leading to sub-natural (i.e. narrower than loaded) linewidths that can be efficiently tuned. This scheme which utilizes linear ring resonators as WGM cavities has been widely discussed in the literature.[1,25,26] The existence of narrow spectral feature has been demonstrated experimentally.[12] In the case of resonators without absorption, the width of the feature may be arbitrarily narrow,[19] however, in reality, the minimum width of the resonance is again determined by the material absorption.

We find the transmission coefficient for such a configuration

$$T = \frac{[\gamma + i(\omega - \omega_1)][\gamma + i(\omega - \omega_2)]}{[2\gamma_c + \gamma + i(\omega - \omega_1)][2\gamma_c + \gamma + i(\omega - \omega_2)] - 4e^{i\psi}\gamma_c^2}, \tag{4}$$

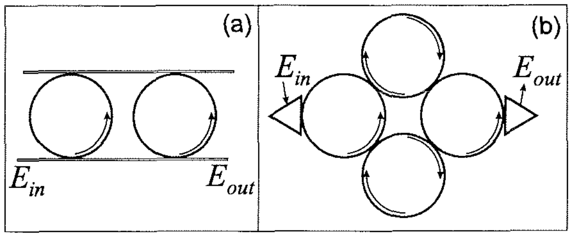

Figure 6. (a) A system of two identical WGM resonator side-coupled to two identical waveguides; (b) Four coupled WGM resonators could have the same response as scheme (a).

where γ, γ_c, ω_1, and ω_2 are the linewidth originated from intrinsic cavity losses, linewidth due to coupling to a waveguide, and resonance frequencies of modes of the resonators, respectively; ω is the carrier frequency (we assume that $|\omega - \omega_1|$ and $|\omega - \omega_2|$ are much less than the cavity free spectral range); ψ stands for the coupling phase, which may be adjusted by changing the distance between the cavities. Choosing $\exp i\psi = 1$ and assuming strong coupling regime $\gamma_c \gg |\omega_1 - \omega_2| \gg \gamma$ we see, that the power transmission $|T|^2$ has two minima

$$|T|^2_{min} \simeq \frac{\gamma^2}{4\gamma_c^2}$$

for $\omega = \omega_1$ and $\omega = \omega_2$, and a local maximum

$$|T|^2_{max} \simeq \frac{(\omega_1 - \omega_2)^4}{[16\gamma\gamma_c + (\omega_1 - \omega_2)^2]^2}$$

for $\omega = \omega_0 = (\omega_1 + \omega_2)/2$. It is important to note that for $\gamma = 0$, the width of the transparency feature may be arbitrarily narrow. However, in reality, the resonance width is limited from below by a nonzero γ.

The origin of this "subnatural" structure in the transmission spectrum of the cavities is in the interference of the cavities' decay radiation. In fact, in the overcoupled regime considered here, the cavities decay primarily into the waveguides, and not into free space. Thus there are several possible pathways for photons to be transmitted through the cavities, and the photons may interfere because they are localized in the same spatial configurations determined by the waveguides. The transmission is nearly cancelled when the light is resonant with one of the resonators' modes. However, in between the modes the interference results in a narrow transmission resonance.

The group time delay originated from the narrow transparency resonance is approximately

$$\tau_g \simeq \frac{16\gamma_c(\omega_1 - \omega_2)^2}{[16\gamma\gamma_c + (\omega_1 - \omega_2)^2]^2} \gg \gamma_c^{-1}. \tag{5}$$

Changing $\omega_1 - \omega_2$ we are able to change the width of the narrow resonance and tune the group delay time. Therefore, the system could serve as an efficient tunable delay line. This delay exceeds the minimum group delay available from a single resonator.

5. CONCLUSION

We have demonstrated experimentally a single metalized $LiNbO_3$ resonator optical filter as well as a coupled system of three metalized $LiNbO_3$ resonators, which is basically a tunable third order filter with a sharp roll-off. The technique may also be used to produce other tunable complex filter functions with any desired line shapes. We have also discussed extremely narrow-band filters with calcium fluoride resonators with limited tenability, and theoretically analyzed narrow resonances achievable with coupled resonators. These types of filter may be utilized for many high-density telecommunication networks, and in RF photonics applications.

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