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LOW-THRUST ORBIT TRANSFER OPTIMIZATION WITH REFINED Q-LAW AND MULTI-OBJECTIVE GENETIC ALGORITHM

Seungwon Lee, Anastassios E. Petropoulos, and Paul von Allmen
Jet propulsion Laboratory
Californian Institute of Technology
4800 Oak Grove Dr. Pasadena, CA

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An optimization method for low-thrust orbit transfers around a central body is developed using the Q-law and a multi-objective genetic algorithm. In the hybrid method, the Q-law generates candidate orbit transfers, and the multi-objective genetic algorithm optimizes the Q-law control parameters in order to simultaneously minimize both the consumed propellant mass and flight time of the orbit transfer. Recently, the thrust control condition of the Q-law has been refined by the introduction of the concept of relative effectivity. The refined thrust control condition is tested in the hybrid optimization method, and its contribution to the Q-law performance is analyzed in comparison with the previous thrust control condition given by absolute effectivity only.

INTRODUCTION

This paper addresses the problem of finding optimal orbit transfers for low-thrust spacecraft. A common goal for the optimization problem is to find the minimum-time, minimum-fuel, or Pareto-optimal trajectory, where the Pareto-optimality means either minimum time for a given fuel or minimum fuel for a given time. In general, these optimization problems are difficult to solve due to the long transfer time and multi-revolutionary transfer.

Various optimization methods have been used to solve the optimization problems. A majority of the work has utilized either direct or indirect techniques [1]. Another quite different approach is to design heuristic control laws [2,3,4]. The advantage of the heuristic control laws is computational speed-up, while the drawback is that the solutions are non-optimal. Recently, the drawback was overcome by combining the heuristic control law with a global optimizer [5,6]. In the hybrid approach, a Lyapunov feedback control law termed Q-law is served as a heuristic control law [2,3,4], and a multi-objective

* Member of Technical Staff at Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Dr. M/S 169-315, Pasadena, CA 91109, USA. Email : Seungwon.Lee@jpl.nasa.gov. Phone : (818)-393-7720. FAX : (818)-393-4802. Web Site : http://hpc.jpl.nasa.gov/PEP/swlee/.

genetic algorithm as a global optimizer for the parameters of Q-law [5,6]. The synergetic, multi-objective optimization process produces an extended Pareto front with significantly less computational effort than conventional single-objective optimization algorithms. The outcome of the hybrid approach is found to be as optimal as those of the direct/indirect approaches within a still reasonably short computational time [4,5]. Moreover, the hybrid approach generates a wide range of Pareto-optimal solutions in a single optimization run instead of one optimal solution per run, which is the case for most of direct/indirect approaches.

Recently, new Q-law parameters have been introduced to improve the performance of coplanar, circle-to-circle transfers and some transfers involving changes in the argument of periapsis [4]. One of the newly introduced parameters in the refined Q-law is the relative thrust effectivity cutoff. The concept of the relative thrust effectivity is introduced to refine the thrust on/off condition, which previously is solely given by the absolute thrust effectivity. It has been demonstrated that the relative effectivity provides a more sensible thrust on/off condition for a coplanar circle-to-circle transfer than the absolute one, when the nominal Q-law parameters are used [4]. In this paper, we further investigate the effect of the thrust condition given by the relative effectivity in comparison with that of the absolute one when the effectivity cutoff parameters as well as all other Q-law parameters are optimized.

Q-LAW THRUST EFFECTIVITY

The Q-law is a Lyapunov feedback control law and determines when and at what angles to thrust based on the proximity quotient termed Q and the change rate of Q due to the thrust [2,3,4]. The function Q judiciously quantifies the proximity of the osculating orbit to the target orbit. The goal of the Q-law is to drive Q to zero, which is equivalent to arrive at the target orbit. The Q-law chooses a thrust angle that can minimize the proximity function Q the most at any given time. The on/off control of thrust is based on the change rate of Q with the optimal thrust angle.

For the thrust on/off control, two thrust effectivity values are defined: 1) absolute effectivity and 2) relative effectivity. The absolute effectivity is given by the ratio of the change rate of Q at the current true anomaly to the best possible change rate (i.e. maximum magnitude) of Q over the current osculating orbit. As suggested by the name, the relative effectivity is given by the relative performance of the change rate of Q at the current true anomaly in comparison with the best and the worst possible change rates of Q over the current osculating orbit, where the relative effectivity of the best change rate is one and the relative effectivity of the worst change rate is zero.

With the two types of the thrust effectivity, the Q-law determines when to thrust or coast. The Q-law turns on thrust if the thrust effectivity is larger than a user-defined cutoff value, and turns off thrust otherwise. In general, a higher cutoff value leads to a more fuel-efficient and longer flight-time trajectory. The user can choose between the

relative and absolute effectivity to control the trust on/off condition. In this paper, we examine the effect of the choice of the effectivity type on the performance of the Q-law.

Q-LAW OPTIMIZATION ALGORITHM

The Q-law has about 15 free control parameters, which mission designers can adjust to obtain different trajectories for a given transfer problem. Some of the parameters, such as the absolute and relative effectivity cutoff values, affect the thrust on/off condition. Other parameters define the geometry (gradient, maxima, minima, saddle points, etc) of the proximity quotient Q. Different effectivity cutoff values lead to different lengths or locations of thrust arcs, and different geometries of Q lead to different thrust angles or shift thrust-arc locations. Hence, the mission designer can acquire a different trajectory for a different set of the Q-law control parameters. The desired outcome for the mission designer is knowledge of the trade-off between optimal flight time and propellant mass, and the Pareto-optimal trajectory corresponding to each point on the trade-off curve. Therefore, our optimization problem is to minimize the competing objectives of required flight time and propellant mass by optimizing the Q-law control parameters.

Multi-objective Genetic Algorithm

Our optimization process follows a standard genetic-algorithm process [7,8], while special care is taken for the multi-objective aspect of the orbit transfer [9,10,11]. The Q-law parameters are represented as a real-coded gene. The initial population of the Q-law sets is prepared randomly with a uniform distribution within a reasonable range for each Q-law parameter. Each Q-law set generates a candidate Pareto-optimal orbit transfer. The candidate transfer is evaluated using nondominated sorting [11], where both consumed propellant mass and flight time are evaluated according to the Pareto-dominance concept. Parents are selected by binary tournament in order to avoid a premature loss of diversity in the population. Offspring is generated with biologically inspired operators – simulated binary crossover and polynomial mutation [11]. The new generation undergoes the same evolution procedure, and this process is iterated until a termination condition is met.

Optimization Experiments

Three different optimization experiments are performed in order to assess the contribution of the relative effectivity cutoff parameter η^{rel}_{cut} in comparison with the absolute effectivity cutoff parameter η^{abs}_{cut}. First, both η^{abs}_{cut} and η^{rel}_{cut} are optimized, meaning that both absolute and relative effectivity are monitored to determine the thrust on/off condition. Second, only η^{abs}_{cut} is optimized while η^{rel}_{cut} is set to zero (i.e. the relative effectivity becomes irrelevant to the thrust on/off condition). Third, as opposed to the second experiment only η^{rel}_{cut} is optimized while η^{abs}_{cut} is set to zero (i.e. the absolute effectivity becomes irrelevant to the thrust on/off condition). Note that all other relevant Q-law parameters $\{W_a, W_e, W_i, W_\omega, W_\Omega, m, n, r, \eta^{\phi}_{cut}, \phi_{min}, b, W_P, k, r_{pmin}\}$ are

always optimized in all three optimization experiments. From here on, the three experiments are referred to as EXP-I, EXP-II, and EXP-III, respectively. Table 1 summarizes the setups of the optimization experiments.

Table 1. Effectivity used for thrust/coast condition in three optimization experiments.

Experiment	Effectivity used for thrust/coast condition
EXP-I	Absolute and Relative
EXP-II	Absolute only
EXP-III	Relative only

ORBIT TRANSFER PROBLEMS

Table 2. Initial and final orbit elements of the orbit transfers studied in this paper.

Case	Orbit	a (km)	e	i (degree)	ω (degree)	Ω (degree)
A	Initial	7000	0.01	0.05	0.0	0.0
	Target	42000	0.01	free	Free	free
B	Initial	24505.9	0.725	7.05	0.0	0.0
	Target	42165.0	0.001	0.05	Free	free
C	Initial	9222.7	0.2	0.573	0.0	0.0
	Target	30000.0	0.7	free	Free	free
D	Initial	944.64	0.015	90.06	156.9	-24.60
	Target	401.72	0.012	90.01	Free	-40.73
E	Initial	24505.9	0.725	0.06	180.0	180.0
	Target	26500.0	0.700	116.00	270.0	180.0

The Q-law optimization processes are applied to five different types of orbit transfers, which are different in terms of degree of complexity and the number of orbit elements to change. The five example transfers were first introduced in Ref. [2]. Table 2 and 3 list the initial and final orbit elements, thrust characteristics, spacecraft initial mass, and central bodies associated with the five orbit transfers termed Case A, B, C, D, and E. The orbit transfers range from the simpler, where few elements have target values, to the more complex, where not only do all elements have target values, but also where temporary, large sacrificial changes must be made in some elements to change more effectively other elements, until all elements converge on their target values.

Table 3. Thrust characteristics, spacecraft initial mass, and central bodies associated with the five orbit transfers studied in this paper.

Case	Thrust (N)	Specific Impulse (s)	Initial Mass (kg)	Central Body
A	1	3100	300	Earth
B	0.35	2000	2000	Earth
C	9.3	3100	300	Earth
D	0.045	3045	950	Vesta
E	2	2000	2000	Earth

RESULTS AND DISCUSSIONS

Case A

Figure 1 shows the Pareto fronts obtained with the three optimization experiments. EXP-I and EXP-III generate a slightly better Pareto front than EXP-II for a long-flight-time regime. On the contrary, EXP-I and EXP-II perform slightly better than EXP-III for a short-flight-time regime. Overall, EXP-I outperforms EXP-II and EXP-III for the whole flight time range. This result suggests that the relative effectivity cutoff condition is a sensible choice for a long-flight-time regime while the absolute cutoff condition for a short-flight-time regime. However, it is important to note that the performance difference among the three experiments is within less than one percent in terms of propellant mass.

The temporal variations of the absolute effectivity can give insight to the roles of the absolute and relative effectivity cutoff conditions. Figure 2 (a) shows the temporal variations when the nominal Q-law is used with zero absolute and relative effectivity cutoffs (i.e. continuous thrusting). The absolute effectivity ranges between 0.95 and 1.0 for the most of the flight time and thus the absolute effectivity cutoff does not play a role until it is close to the minimum value around 0.95. When the absolute effectivity cutoff is above the minimum value, the thrust condition becomes very sensitive to the change of the absolute effectivity cutoff. This makes the poor performance of the absolute cutoff condition when the nominal Q-law is used [3].

This problem has been resolved by adjusting other Q-law parameters [5], which in turn changes the temporal variations of the absolute effectivity to make the thrust condition less sensitive to the cutoff value. Figure 2 (b) clearly demonstrates the change of the temporal variations through other Q-law parameter adjustments. The temporal variations are widened and this leads to a less sensitive control of thrust condition with the absolute cutoff value.

Another solution to the poor performance of the absolute cutoff condition can be the use of the relative cutoff condition [4]. Since the relative effectivity always ranges between 0 and 1, the sensitivity of the thrust condition to the relative cutoff value is less

dramatic. A significant improvement of the Q-law performance with the relative cutoff condition has been demonstrated with nominal Q-law [4], where the resulting flight time increases smoothly with the increase of the relative cutoff value instead of having a large gap in the flight times as shown in the absolute cutoff condition case [3].

When optimal Q-law parameters are used, the difference in Q-law performance is not as dramatic as the nominal Q-law. Figure 1 shows that any thrust on/off condition leads to a smooth, widely spread Pareto front, and the difference is only within a few percents of the propellant mass. The relative cutoff condition provides a slightly better performance for the long flight time, thanks to its less sensitivity of the thrust condition to the cutoff value.

The optimal effectivity cutoff values found with the three experiments are plotted in Figure 3. In EXP-1, there is an order switch between the absolute and relative cutoff values at around the flight time of 45 days. Below 45 days, the absolute cutoff value is higher than the relative one, and dominates the thrust condition. Conversely, the relative cutoff value becomes higher after 45 days and controls the thrust condition. In comparison with EXP-II and EXP-III, the optimal absolute values of EXP-I follows closely with that of EXP-II for short flight times while the optimal relative values of EXP-I with that of EXP-III for long flight times. This result is consistent with the performance comparison shown in Figure 1. For a short flight time, the absolute effectivity condition performs better and hence EXP-I chooses high absolute cutoff values. In contrast, the relative condition is more efficient for a long flight time and hence EXP-I chooses high relative cutoff values. In addition, EXP-I chooses moderate absolute cutoff values to truncate the thrust arc if the absolute effectivity is too low even though the relative effectivity is high enough. This gives a slight advantage to the EXP-I thrusting scheme over the EXP-II thrusting scheme in long flight times.

Case B

Figure 4 shows that there is no significant difference in the performances of EXP-I, EXP-II, and EXP-III. Either the absolute effectivity or the relative effectivity provides a sensible thrust on/off condition for this type of orbit transfers. The temporal variations of the absolute effectivity of this orbit transfer shed light on the cause of the uniform performances among the three experiments. As shown in Figure 5, the variation of the absolute effectivity is as wide as almost one in both the nominal Q-law and the optimal Q-law. This wide range makes the absolute thrust on/off condition less sensitive to the choice of the cutoff values. The relative cutoff condition does not have the sensitivity problem of the absolute cutoff condition because in definition the relative effectivity ranges between 0 and 1. However, the shortcoming of the relative cutoff condition is that it becomes too tolerant when the absolute effectivity is very low, and conversely it is too conservative when the absolute effectivity is very high. For example, even a small value of the relative cutoff makes thrust off even though the absolute effectivity is over 0.7 at the

flight time below 20 days in the optimal Q-law case, shown in Figure 5(b). However, the shortcomings of the absolute and relative effectivity can be mitigated by adjusting other Q-law parameters, which changes the temporal variation of the thrust effectivity. It is the interplay between the effectivity cutoff value and other Q-law parameters that leads to the more or less uniform performance among the three experiments.

The optimal cutoff values found with the three experiments are plotted in Figure 6. The result of EXP-I suggests that the absolute cutoff value dominates the thrust control condition in EXP-I case since the absolute value is mostly higher than the relative value, which fluctuates largely. The absolute cutoff value in EXP-II follows closely with the relative cutoff value found in EXP-III.

Case C

Figure 7 shows that there is no significant difference in the performances of EXP-I, EXP-II, and EXP-III. Either the absolute effectivity or the relative effectivity provides a sensible thrust on/off condition for this type of orbit transfers. Note that for this particular run, EXP-III did not lead to a solution near the shortest flight-time found by EXP-I and EXP-II. However, Another independent run of EXP-III generates the shortest flight time solution. We therefore suspect that the difference in the shortest flight time solutions is within a statistical fluctuation rather than a consistent performance difference.

The temporal variations of the absolute effectivity are plotted in Figure 8. A wide range of the effectivity is shown. The wide range makes the absolute cutoff condition less sensitive to the cutoff value. The optimal Q-law changes the range of the absolute effectivity to be more evenly distributed along the flight time. Figure 9 plots the optimal cutoff values found with the three optimization experiments.

Case D

Figure 10 shows the Pareto fronts found with the three experiments. EXP-II and EXP-III perform slightly better than EXP-I for a long-flight-time regime. This is the opposite result of Case A, where EXP-I performs the best. In principle, EXP-I is the superset of EXP-II and EXP-III and should perform as good as them. However, the optimization process in EXP-I involves a larger search space and thus can be misguided by local minima. This difficulty can be compensated by a larger size of population, which also leads to an undesired consequence – the increase of computational time. If one is interested in the best performance for a given computational time, either EXP-II or EXP-III is a better choice than EXP-I for this type of orbit transfers. This result suggests that both the absolute and relative effectivity measurements provide a suitable guidance on the thrust on/off condition for this orbit transfer. The performance different among the three experiments is within less than two percents in terms of propellant mass.

Figure 11 shows the temporal variations of the absolute effectivity for the nominal

Q-law and an optimal Q-law, which is the shortest flight-time solution from EXP-I. The results show that different Q-law parameters can significantly alter the dynamics of the absolute effectivity. Overall, the absolute effectivity is broadly distributed between 0 and 1 and this makes the choice of the effectivity type less critical in this transfer.

Figure 12 shows the optimal cutoff values found with the three optimization experiments. In EXP-1, there is a switch of the order of the absolute and relative cutoff values at around the flight time of 75 days. Below 75 days, the absolute cutoff value is higher than the relative one, and dominates the thrust condition. Conversely, the relative cutoff value is higher after 75 days and controls the thrust condition. When only one cutoff value is used as in EXP-II and EXP-III, there is a discontinuity of the optimal values around the flight time of 60 days. This indirectly shows that there is a qualitative change in the optimal trajectories around that flight time.

Case E

Figure 13 shows that EXP-II outperforms EXP-III for a long-flight-time regime, and the performance of EXP-I is in between EXP-II and EXP-III. The absolute effectivity is a slightly more efficient measurement for thrust on/off condition than the relative one for this type of orbit transfers. The difference among the resulting propellant mass of the three experiments is smaller than one percent. Figure 14 shows a wide variation of the absolute effectivity along the entire flight time for both the nominal Q-law and an optimal Q-law (the shortest flight-time solution from EXP-I). The optimal cutoff values found with the optimization processes are plotted in Figure 15.

Table 4. Summary of the optimization experiment results.

Case	Best experiment	Best effectivity for thrust/coast condition
A	I	Absolute and Relative
B	Any	Any
C	Any	Any
D	II or III	Absolute only or Relative only
E	II	Absolute only

Table 4 summarizes the results of the optimization experiments for the five orbit transfer problems. None of the five orbit transfer problems shows a significant performance difference among the three experiments. A slight difference is shown in Case A, D, and E, but the performance difference is smaller than a few percents in terms of propellant mass. In general, the relative effectivity becomes important when the orbit transfer involves the narrow variation of the absolute effectivity for most of the flight time as shown in Case A. In other transfers whose absolute effectivity changes broadly between 0 and 1, the absolute effectivity cutoff condition is as efficient as or even better

than the relative one.

CONCLUSIONS

We have investigated the role of the thrust control condition given by the absolute and/or the relative effectivity in the Q-law performance when the Q-law control parameters are optimized with a multi-objective genetic algorithm. Five different types of orbit transfer problems are studied. For the Q-law performance comparison, we have conducted three optimization experiments: 1) the thrust control condition given by both the absolute and relative effectivity, 2) the thrust control condition given by only the absolute effectivity, and 3) the thrust control condition given by only the relative effectivity. No significant performance difference among the three experiments is found in any of the five orbit transfers studied. A slight difference is shown in some of the transfers, but the performance difference is smaller than a few percents of propellant mass. In general, the absolute effectivity is inefficient when the orbit transfer involves the narrow variation of the absolute effectivity for most of the flight time. Conversely, when the absolute effectivity changes broadly between 0 and 1, the relative effectivity is less efficient than the absolute one. However, the shortcomings of the absolute and relative effectivity are mitigated by adjusting other Q-law parameters, which changes the dynamics of the thrust effectivity. It is the interplay between the effectivity dynamics and other Q-law parameters that leads to the more or less uniform performance among the three experiments.

ACHKNOWLEDGEMENTS

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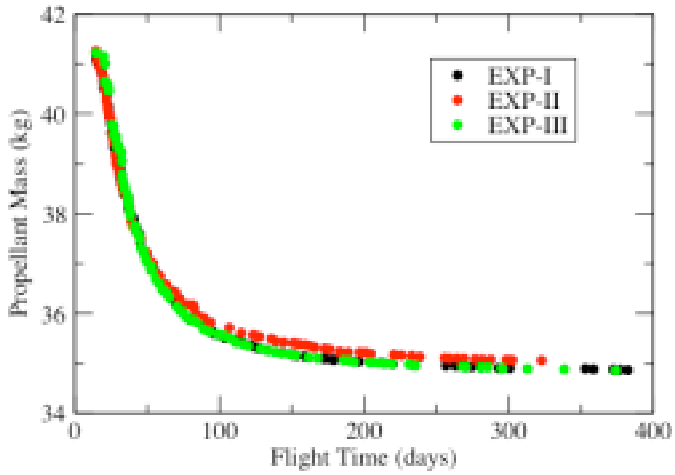

Figure 1. Pareto fronts of Case A obtained from three optimization experiments.

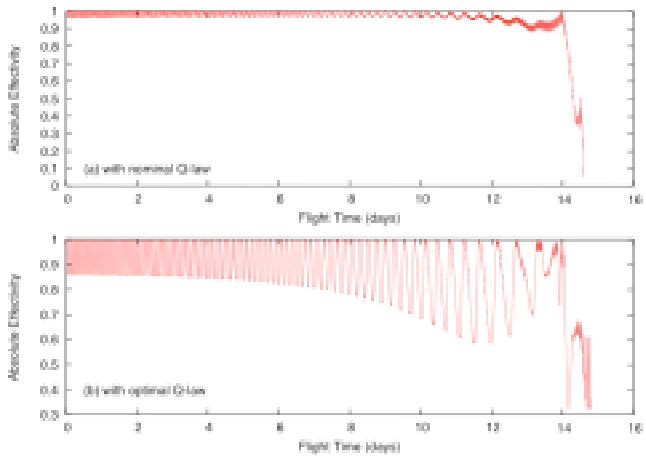

Figure 2. Temporal variations of thrust absolute effectivity for Case A (a) with the nominal Q-law and (b) with an optimal Q-law.

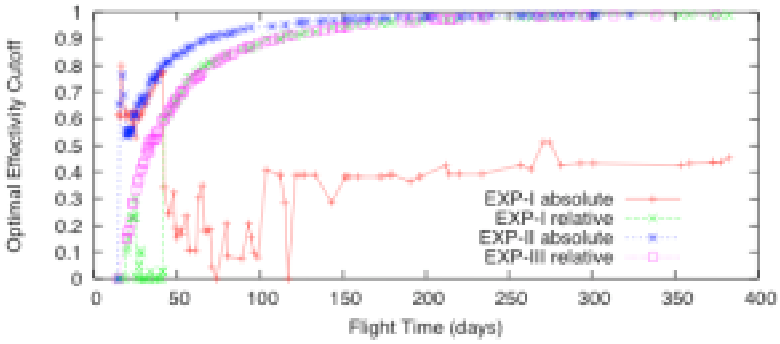

Figure 3. Optimal effectivity cutoff values found for Case A.

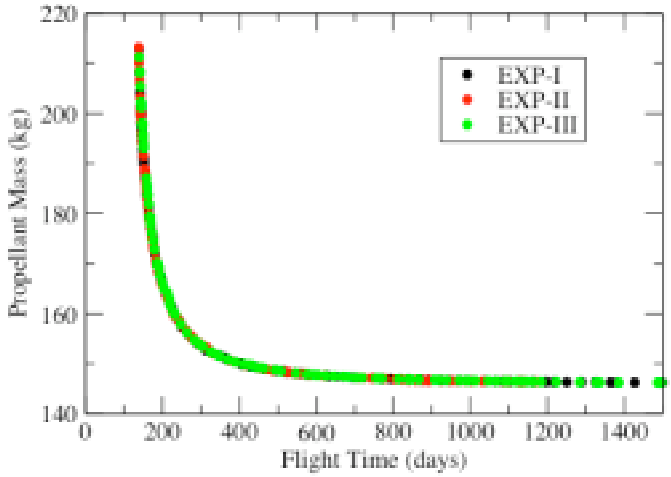

Figure 4. Pareto fronts of Case B obtained from three optimization experiments.

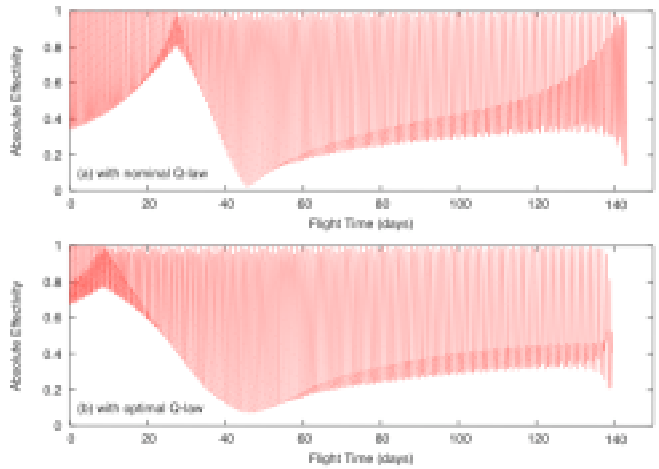

Figure 5. Temporal variations of thrust absolute effectivity for Case B (a) with the nominal Q-law and (b) with an optimal Q-law.

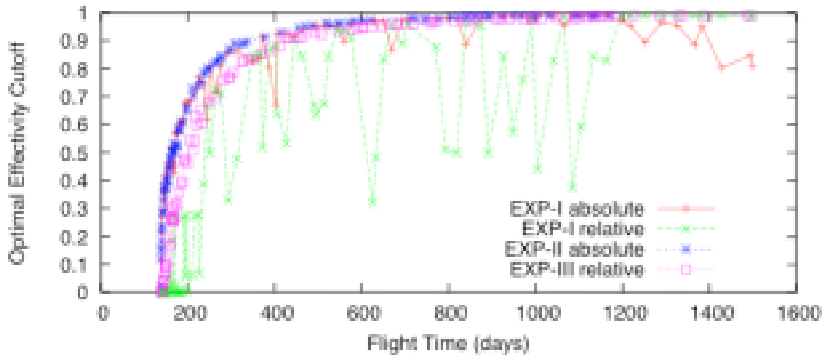

Figure 6. Optimal effectivity cutoff values found for Case B.

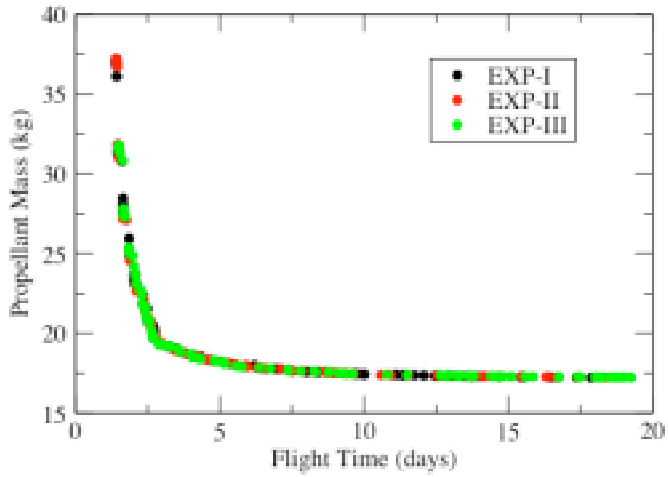

Figure 7. Pareto fronts of Case C obtained from three optimization experiments.

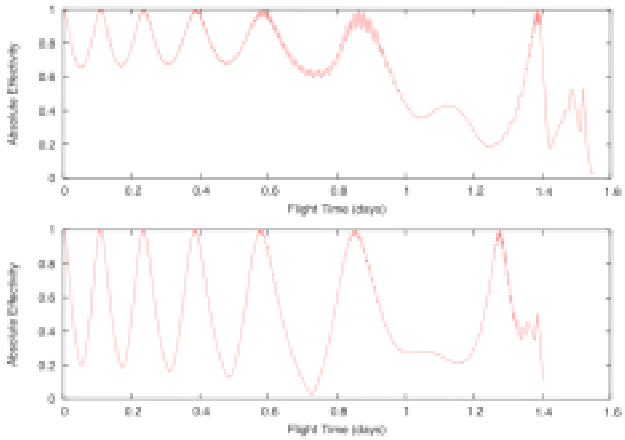

Figure 8. Temporal variations of thrust absolute effectivity for Case C (a) with the nominal Q-law and (b) with an optimal Q-law.

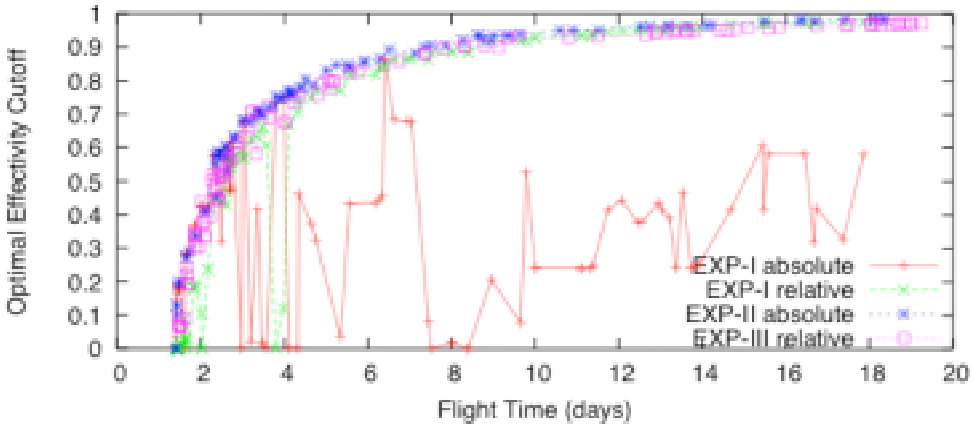

Figure 9. Optimal effectivity cutoff values found for Case C.

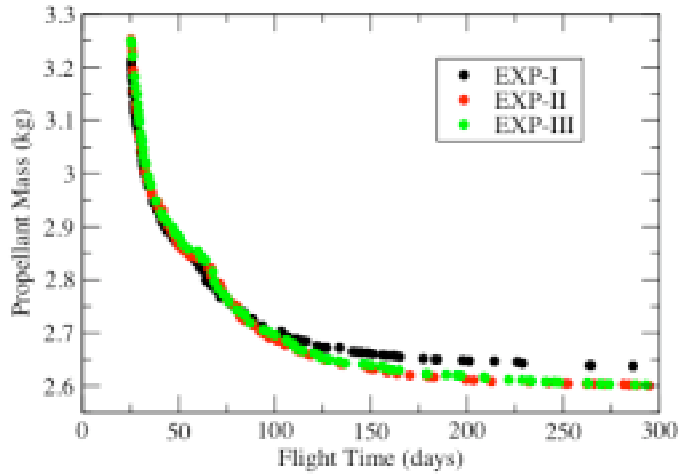

Figure 10. Pareto fronts of Case D obtained from three optimization experiments.

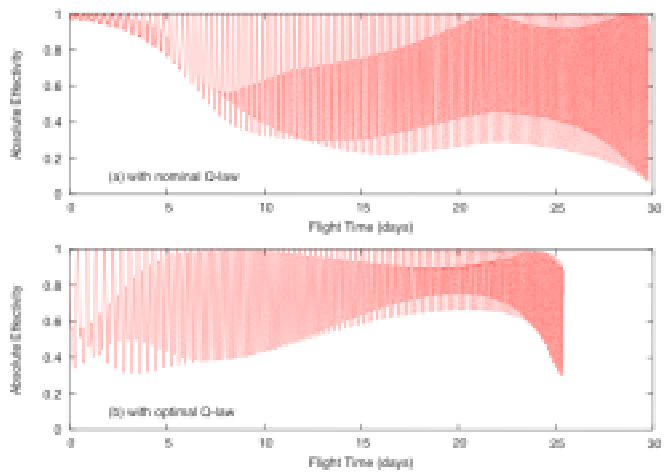

Figure 11. Temporal variations of thrust absolute effectivity for Case D (a) with the nominal Q-law and (b) with an optimal Q-law.

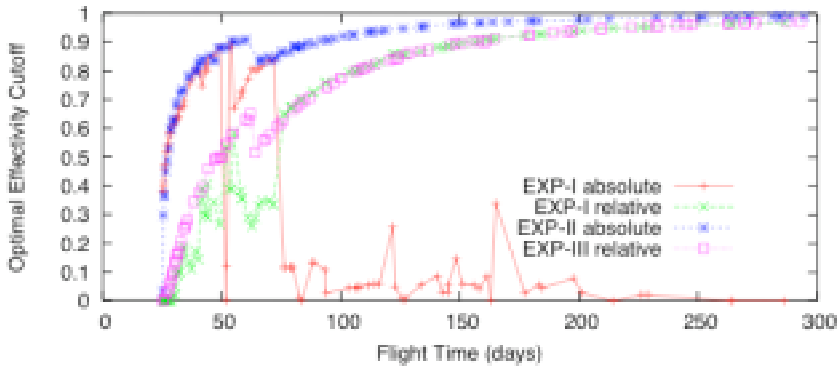

Figure 12. Optimal effectivity cutoff values found for Case D.

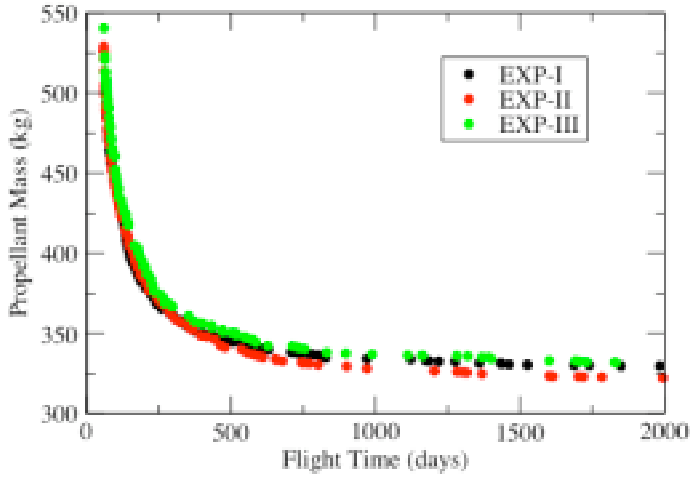

Figure 13. Pareto fronts of Case E obtained from three optimization experiments.

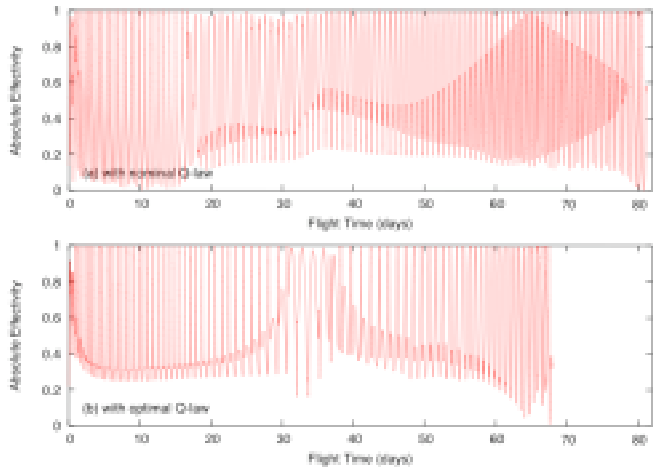

Figure 14. Temporal variations of thrust absolute effectivity for Case E (a) with the nominal Q-law and (b) with an optimal Q-law.

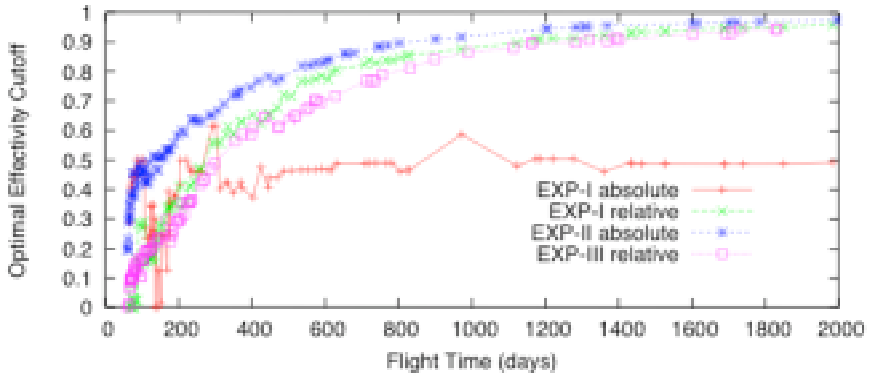

Figure 15. Optimal effectivity cutoff values found for Case E.