Stability maps, Unstable periodic orbits and Transfers in the Jovian System

Benjamin Villac
Jet Propulsion Laboratory

And

Martin Lara
Real Observatorio de la Armada

ABSTRACT

Low cost transfers of a spacecraft in a multi-gravitational orbital environment, which are based on connections between unstable periodic orbits, present several trade-offs, such time-of-flight v.s. transfer fuel cost. It is shown that the dynamical characteristics of the target periodic orbits is related to the maximal (recovery) time allowed for a spacecraft without control. Mildly unstable periodic orbits, which result in good recovery margins, are then shown to structure the boundary of stability regions, resulting in the concept of boundary unstable periodic orbits (b.u.p.o.). The use of a Fast Lyapunov Indicator (FLI) allows us to place the b.u.p.o. in their dynamical context and to exhibit their associated connections, thus offering a quick overview of potential dynamical transfers for spacecraft applications. The concepts are illustrated in the planar, circular, restricted three-body problem, and two different transfer based on the use of stability maps are presented.

ACKNOWLEDGEMENTS

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PLANAR, CIRCULAR, RESTRICTED THREE-BODY PROBLEM

- **Simple model** for the the dynamics of a **spacecraft in a planetary satellite system**.
- Assumes the (point mass) **planet and satellite to be in Keplerian circular orbits** about each other. The spacecraft of negligible mass is only influenced by these two primaries.
- The equations of motion are **autonomous, Hamiltonian**, when expressed in a **moon-centered rotating frame**. They depend only on a single parameter, $\mu \in (0,1)$, after convenient choice of length, time and mass units.
- **Effective potential:**

$$\Omega(x,y)=\frac{1}{2}\left((x+\nu)^2+y^2\right)+\frac{\nu}{r_1}+\frac{\mu}{r_2}$$

$$r_1=\sqrt{(x+1)^2+y^2}\,;\, r_2=\sqrt{x^2+y^2}\,;\, \nu=1-\mu$$

- **Equations of Motion:**

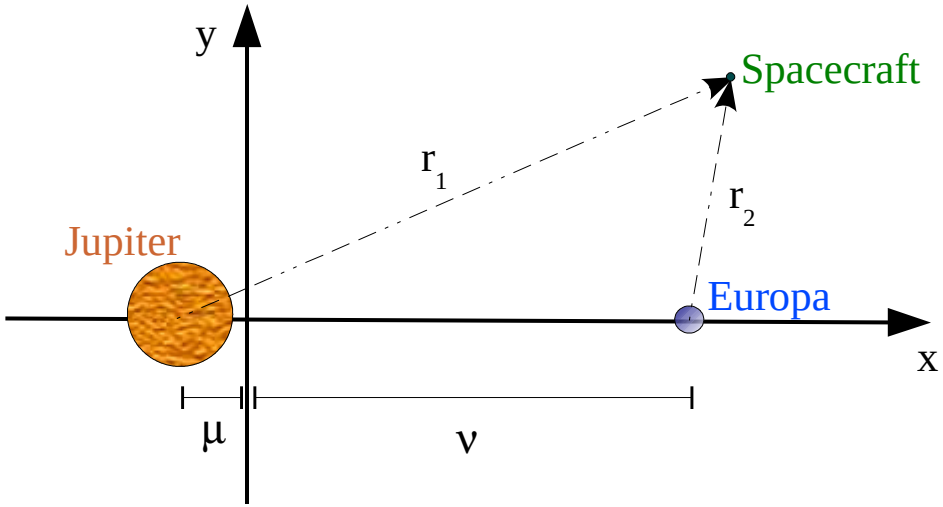
$$\ddot{x}-2\dot{y}=\Omega_x$$
$$\ddot{y}+2\dot{x}=\Omega_y$$

- **Jacobian integral of motion:**

$$C=\Omega(x,y)-v^2\,;\, v^2=\dot{x}^2+\dot{y}^2$$

- A fixed value of C restricts the motion to an **energy manifold** of dimension three.
- **Time-reversal symmetries:**

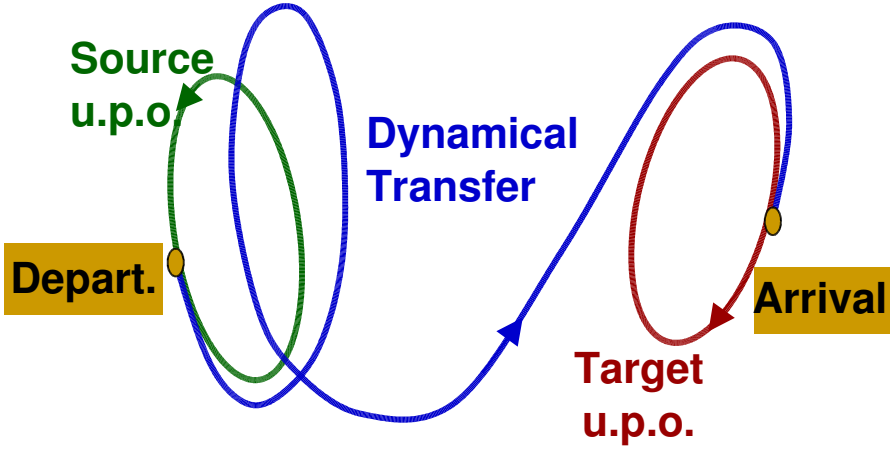
$$(x,y,\dot{x},\dot{y},t)\rightarrow(-x,y,\dot{x},-\dot{y},-t)$$
$$(x,y,\dot{x},\dot{y},t)\rightarrow(x,-y,-\dot{x},\dot{y},-t)$$

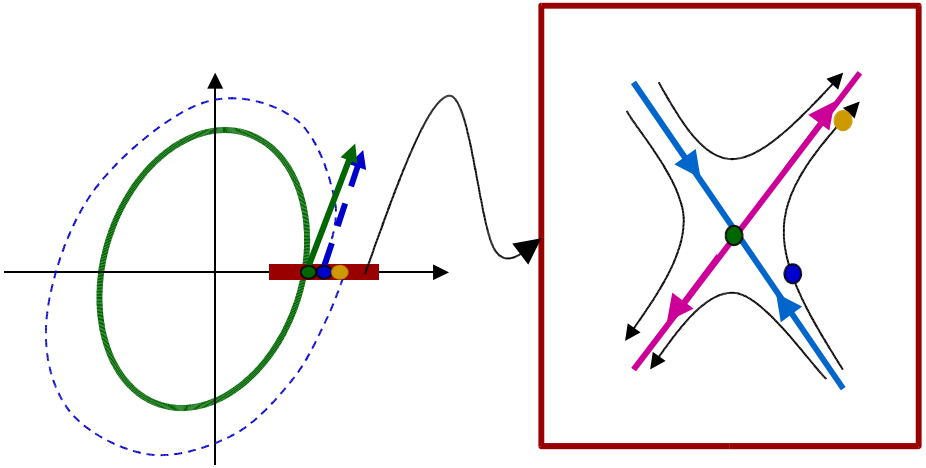


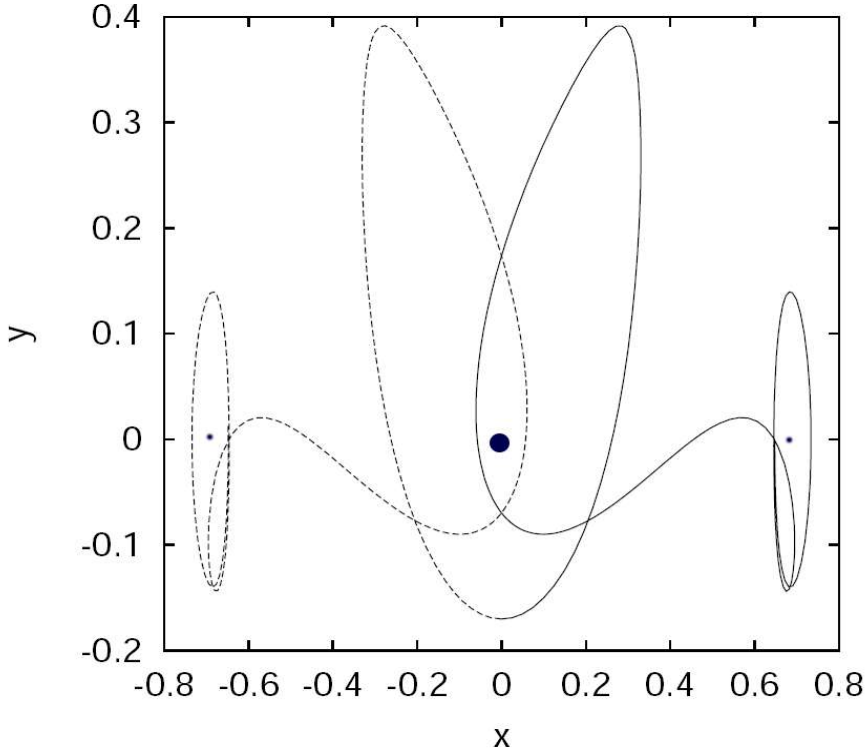
Jupiter-Europa system: $\mu = 2.528 \times 10^{-5}$
Length scale: $L \approx 670900$ km
Time scale: $T \approx 3.5$ days

UNSTABLE PERIODIC ORBITS and TRANSFERS

- **Connections** between **unstable periodic orbits (u.p.o.)** offer **thrust-free transfers** between (possibly) distant regions of phase space.
- These **connections** are formed by the **intersection of the stable and unstable manifolds** of a source and target u.p.o., respectively.
- The **stable and unstable manifolds** of an u.p.o. are sets of trajectories converging towards the u.p.o. in **forward and backward times,** respectively.





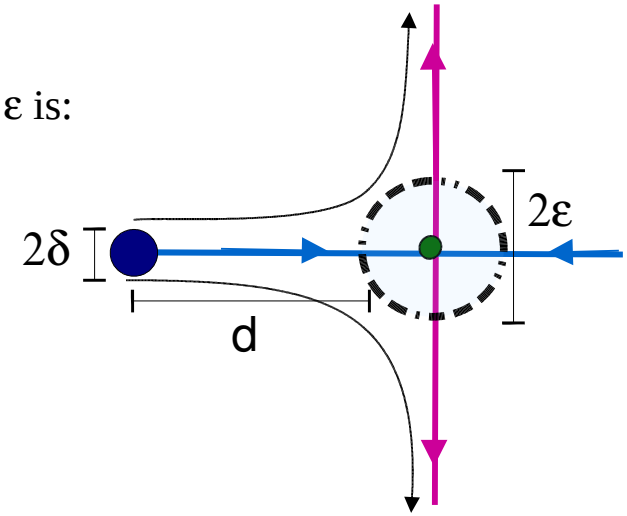


RECOVERY PROPERTIES

- The time taken for the uncertainty in the state of a spacecraft to grow to a maximal tolerance is the **recovery time** allowed by the dynamics.

- **Near an u.p.o.**, too large uncertainty can result in **drastic change in the dynamical behavior** of the trajectory.

- The distance, *d*, from which an u.p.o. can be targeted within an ε is:
  - maximized along the stable manifold
  - depends only on the current and maximal state uncertainties
  - independent of the u.p.o. characteristics.



- The time taken to travel the above distance, d, is dependent on:
  - the instability scale of the u.p.o.
  - the period of the u.p.o.

Mildly unstable u.p.o.s with long periods present better recovery properties

but

longer time-of-flight than highly unstable periodic orbits with short periods

- **Mildly u.p.o.** can be found near **bifurcations of families of periodic orbits**, at the **boundary of stability regions**.

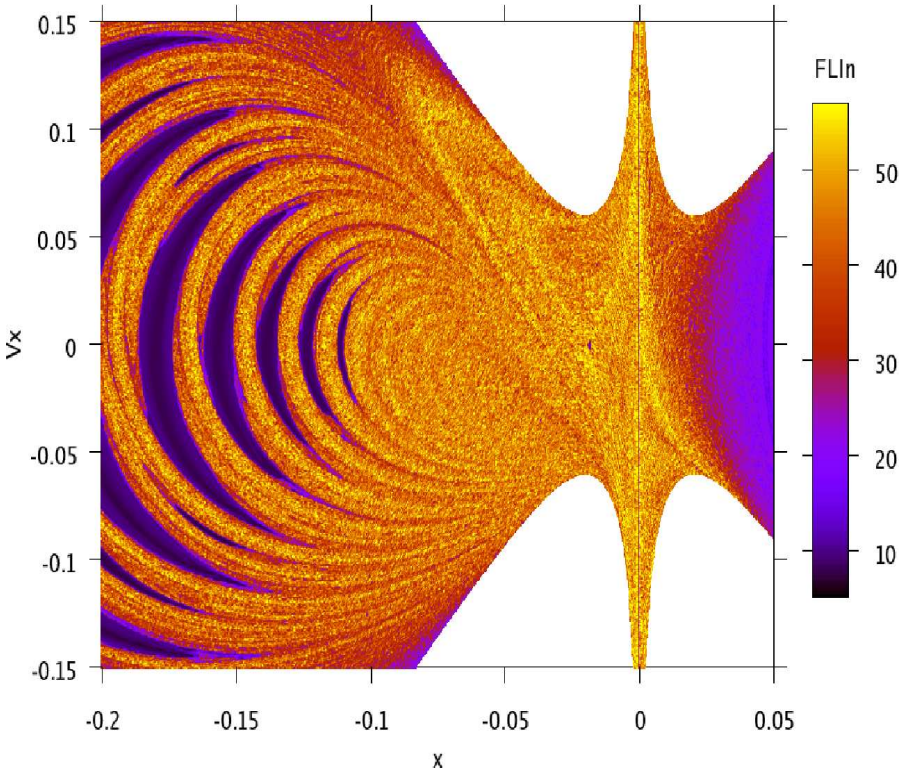
STABILITY MAPS

- **Stability maps** are a **numerical tool** to investigate **phase space structures**.
- The maps associate a scalar index (**indicator**) with each initial conditions in a subset of phase space.
- The indicator considered is based on the expansion properties of the state transition matrix, and is known as the **Fast Lyapunov Indicator (FLI)**:

$$FLI_n(T) = sup_{i,t \le T} \ln \left\| \delta u_i \right\|_n$$

where $\{\delta u_i\}$ represents the evolution of a basis of variational displacement after time t, and $\|.\|_n$ is a scale invariant norm.

- This indicator discriminates (up to a certain time scale) between **regular** and **chaotic motion**:
  - low values of the FLI correspond to regular motion and result in the notion of **stability regions or islands.**
  - Large values of the FLI correspond to chaotic motion



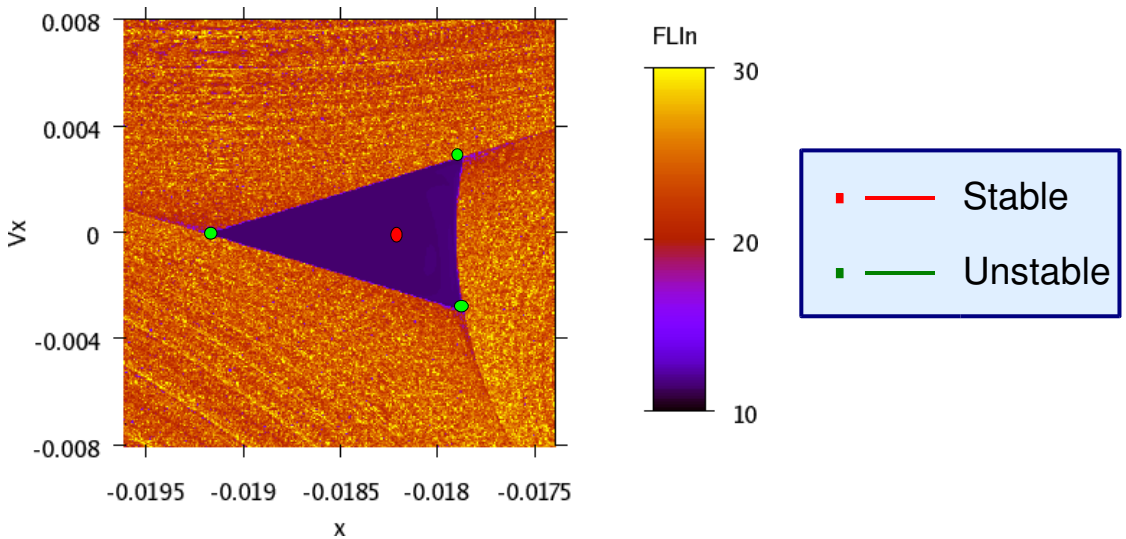
Europa-Jupiter system modeled by the PCR3BP (μ=2.528x10^{-5})

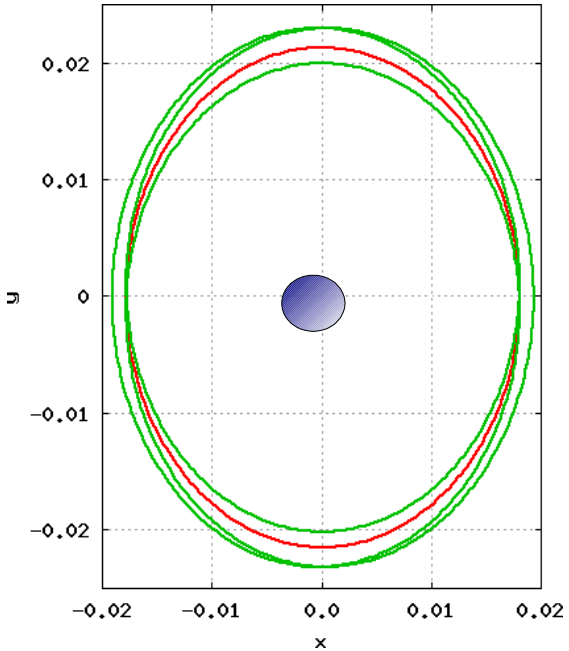
Initial conditions parameterization:

y = 0, C = 3.0; (x, V_x) are varied

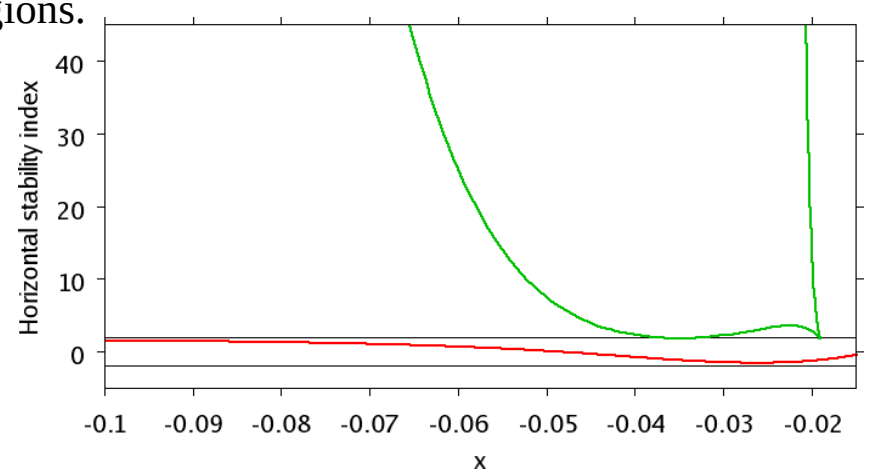
BOUNDARY UNSTABLE PERIODIC ORBITS (1)

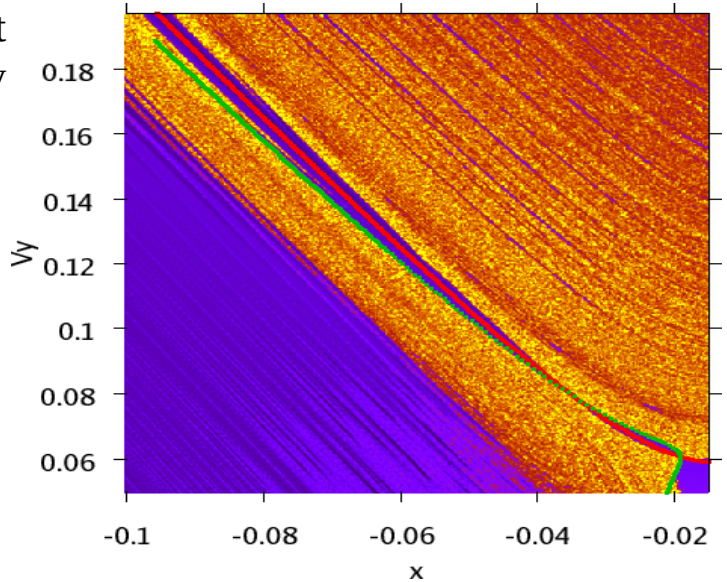
- The boundary of the stability regions is structured by u.p.o., referred to as **boundary unstable periodic orbits (b.u.p.o.).**



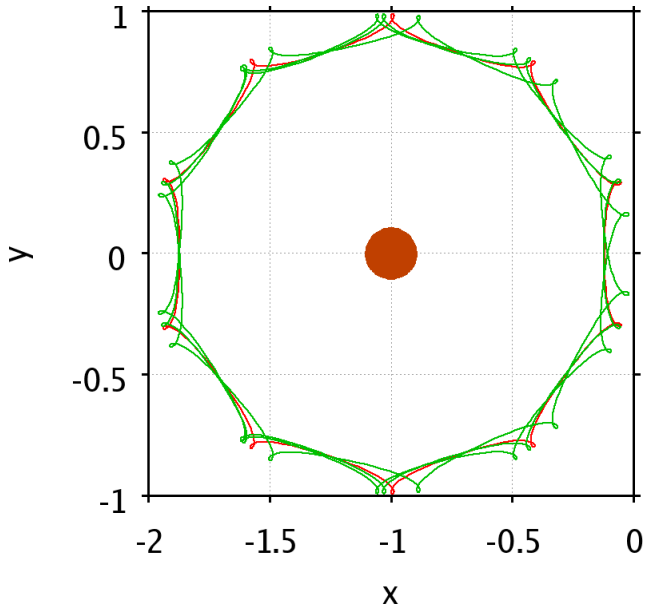


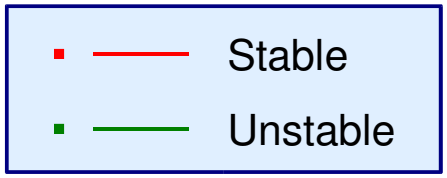
- B.u.p.o. are members of families of periodic orbits that bifurcate from stable families lying inside the stability regions.



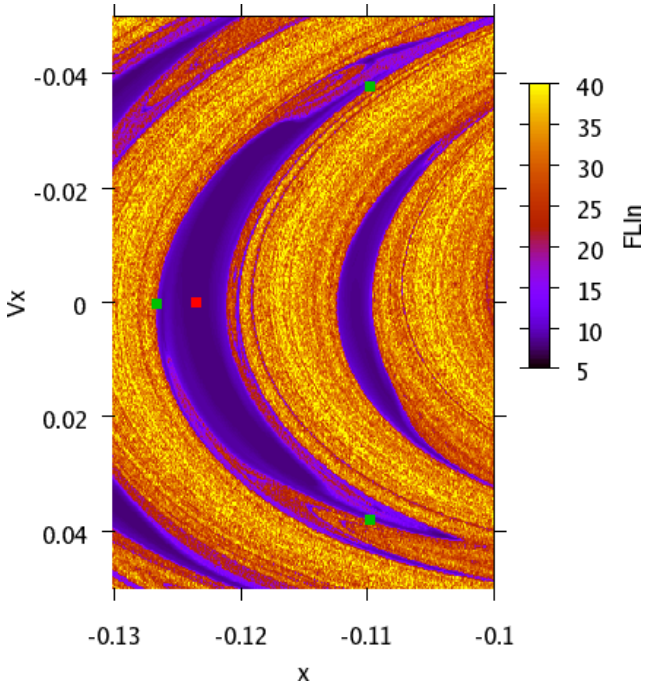


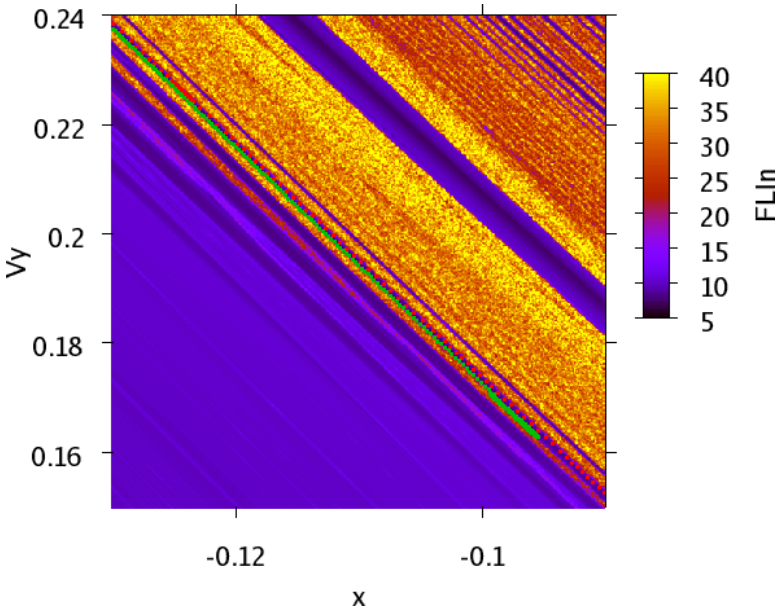
CASE OF THE 10:9 RESONANT ISLAND

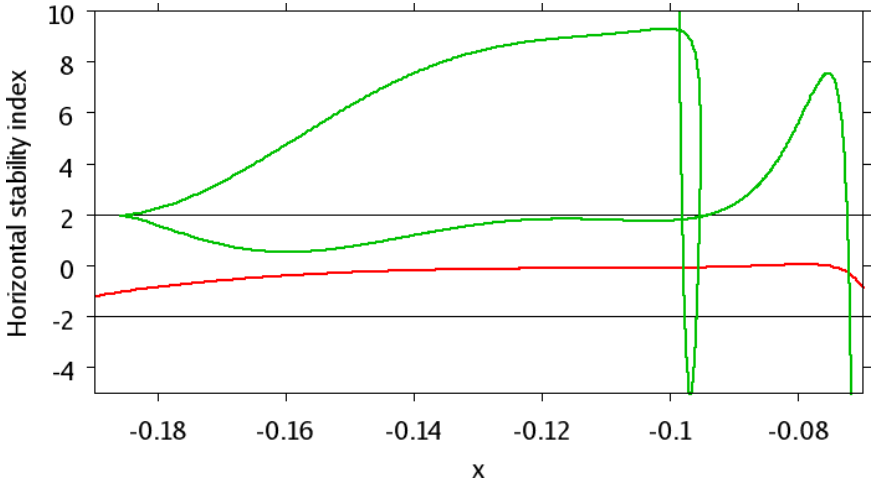




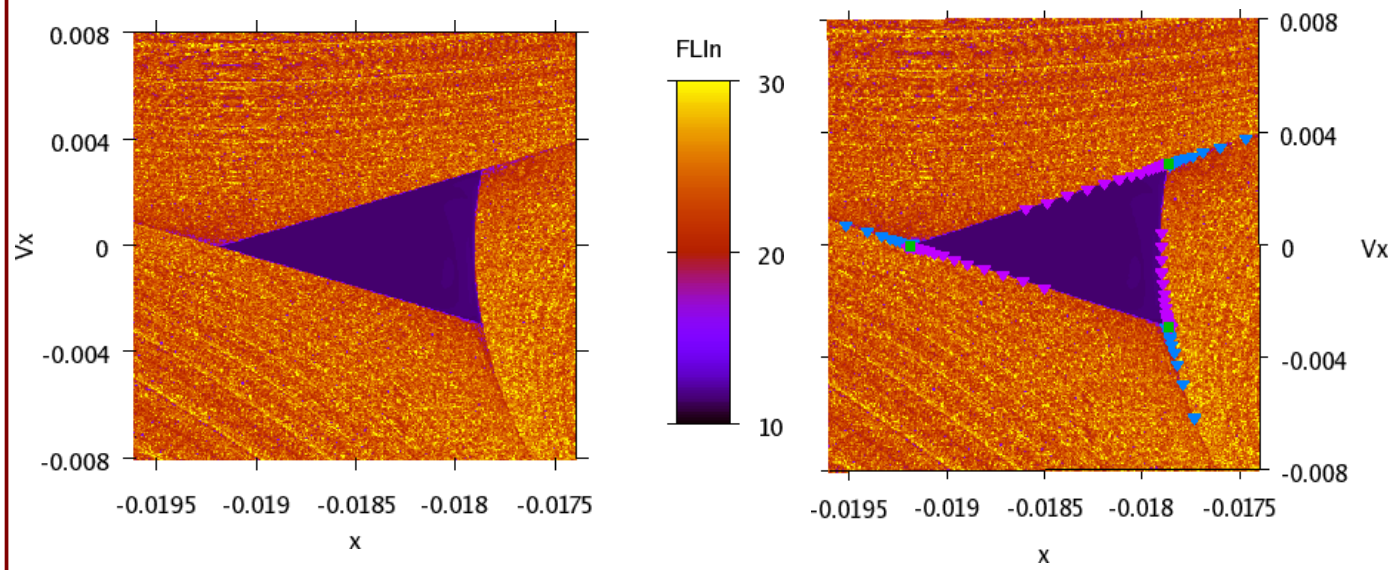
The **b.u.p.o.** are generally not simple (**longer periods**) and present low values of the characteristic multipliers (**mild instability**).



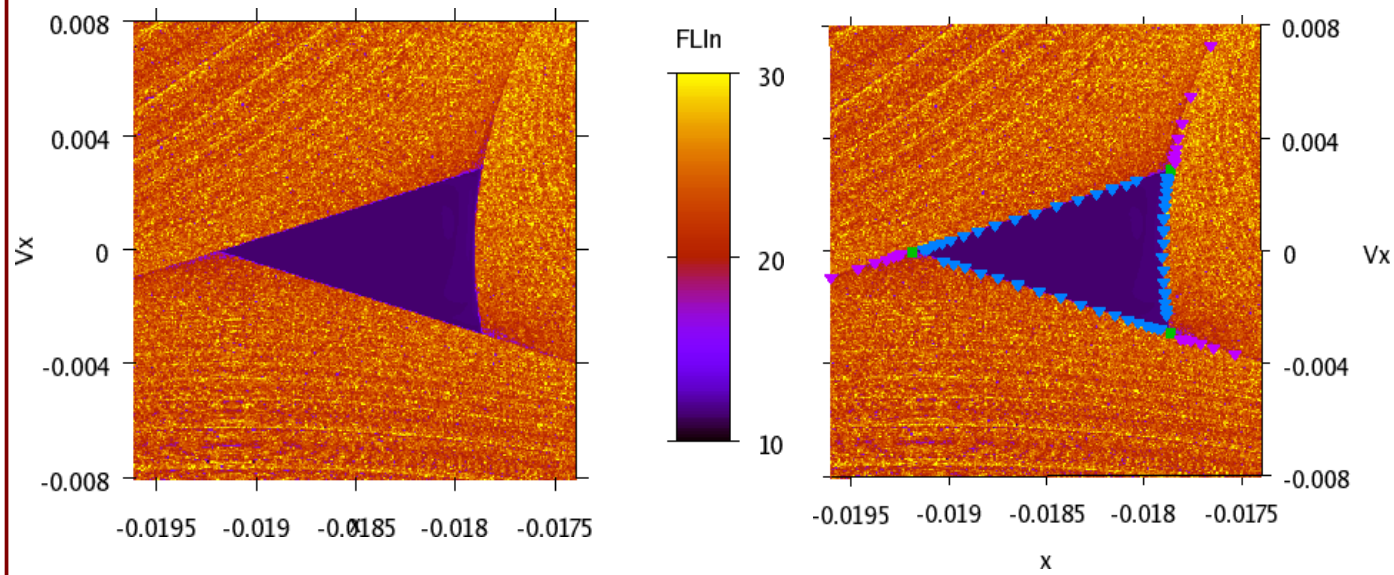




STABILITY MAPS and MANIFOLDS

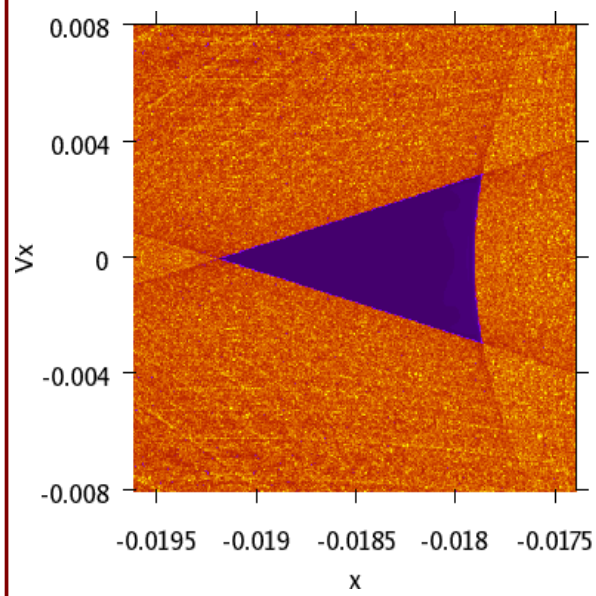


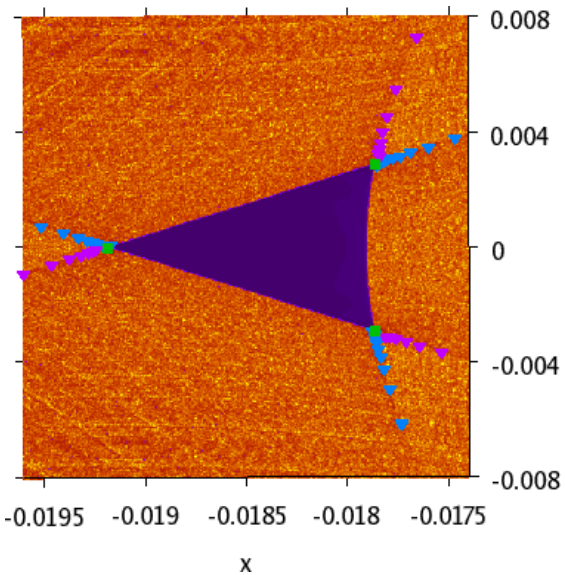
Generating FLI maps in **forward times** exhibits the traces of the **stable manifolds** associated with a b.u.p.o.

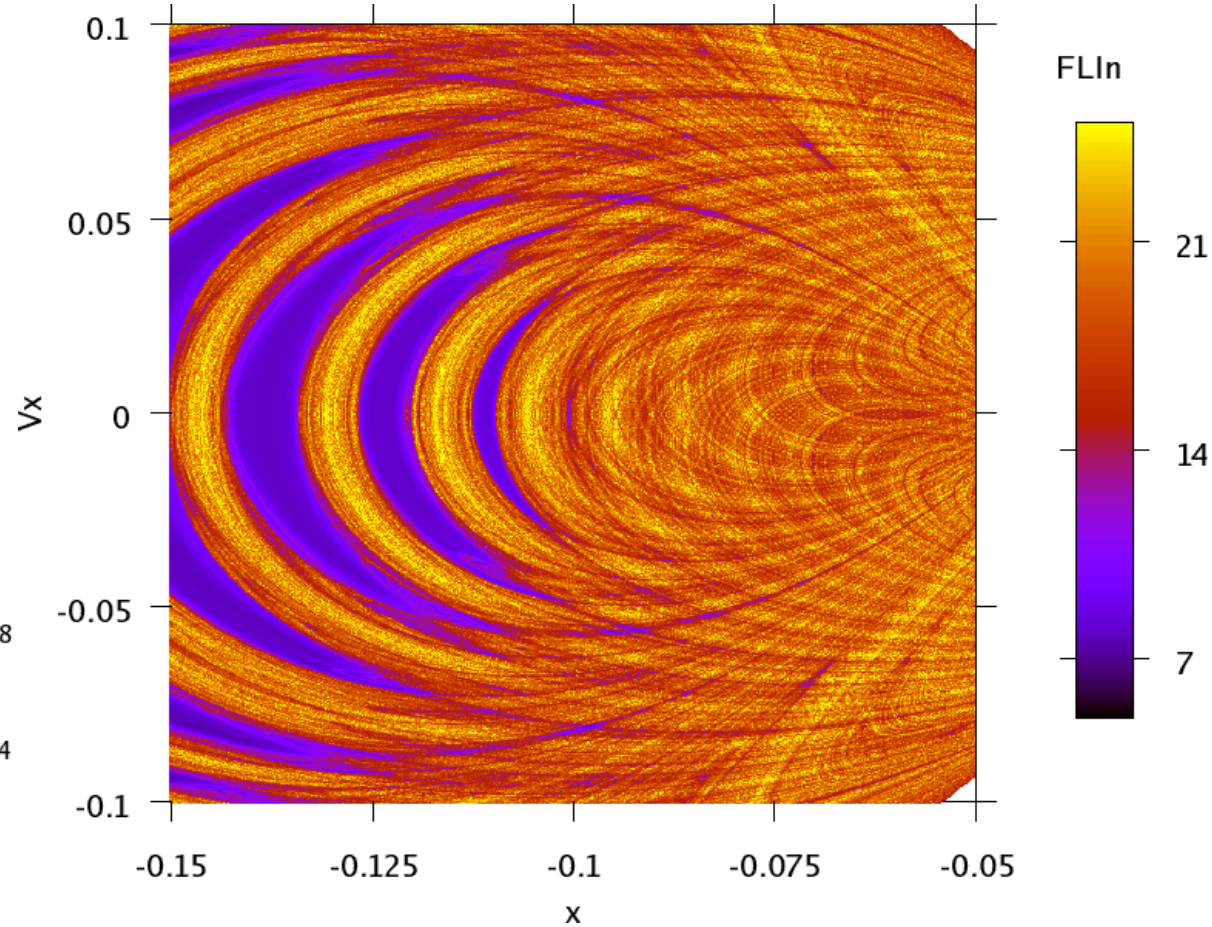


Generating FLI maps in **backward times** reveals the traces of the **unstable manifolds** associated with a b.u.p.o.

MEAN FLI MAPS and CONNECTIONS

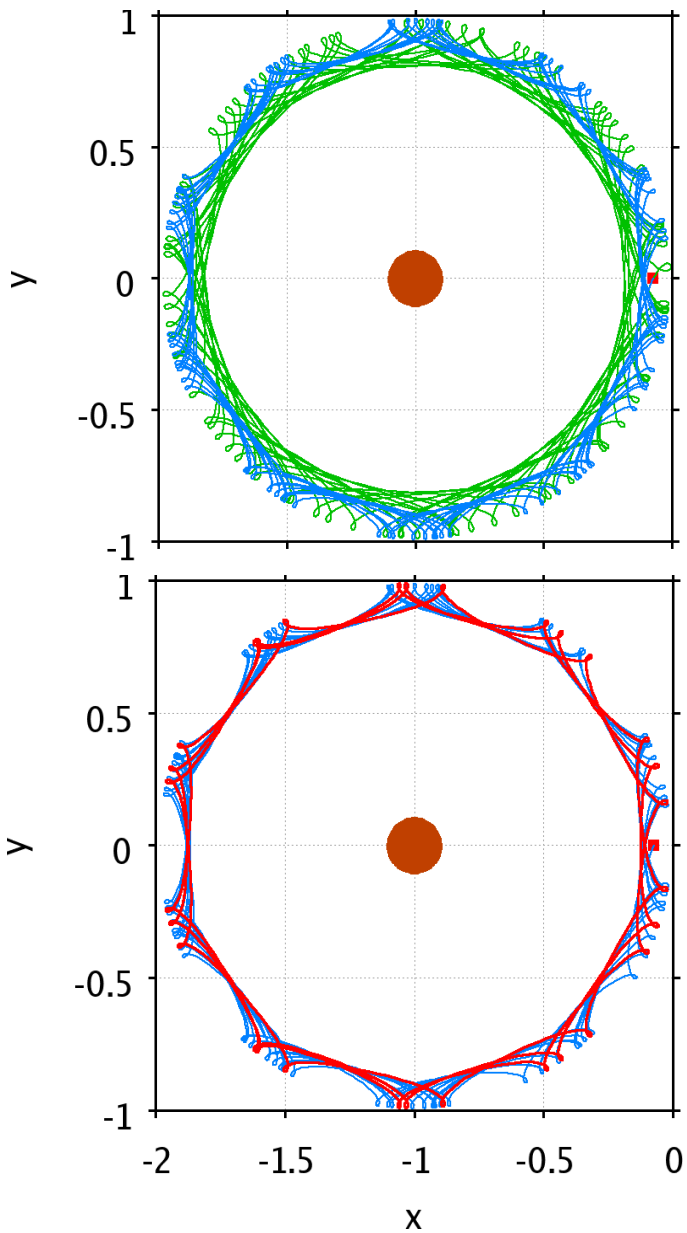


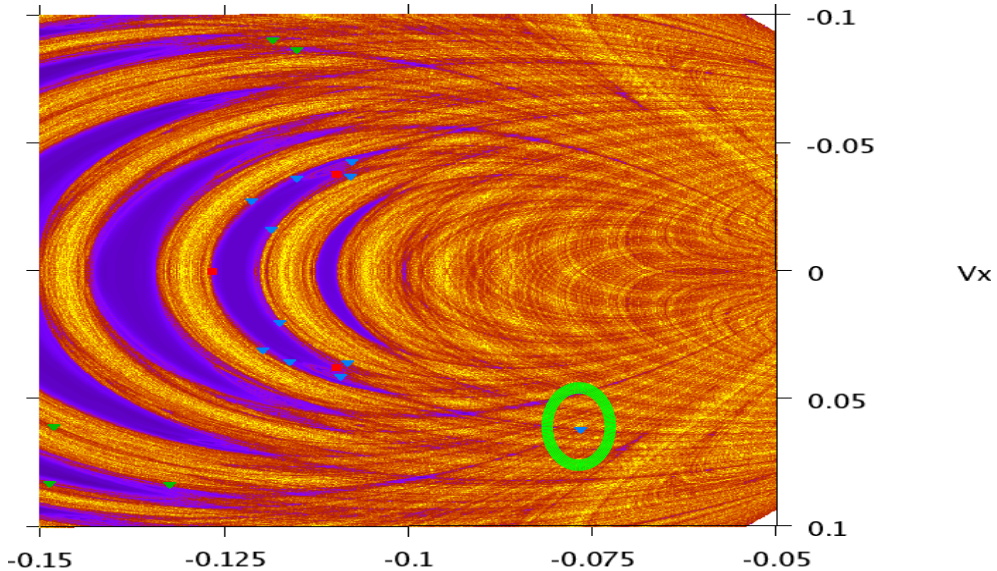


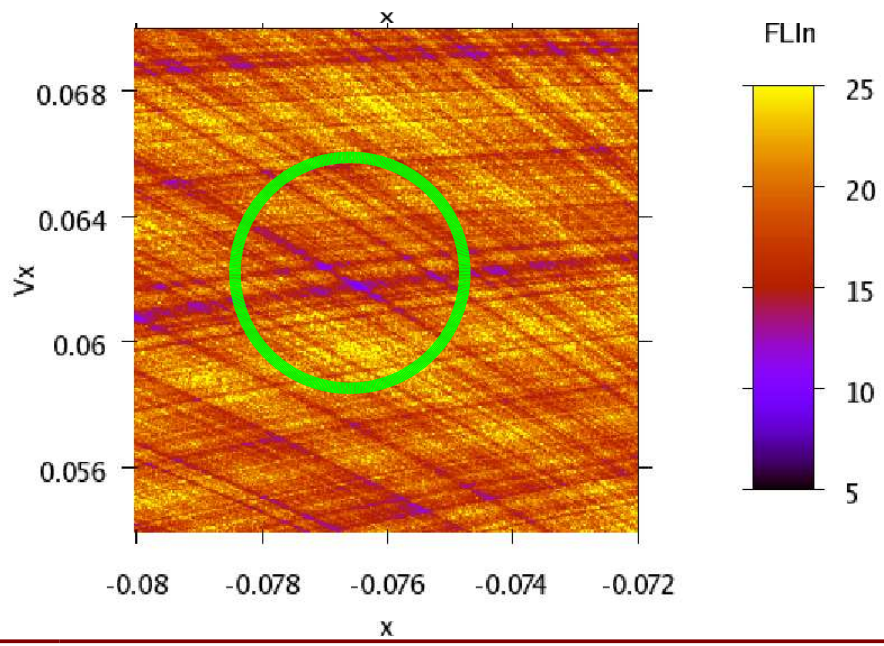


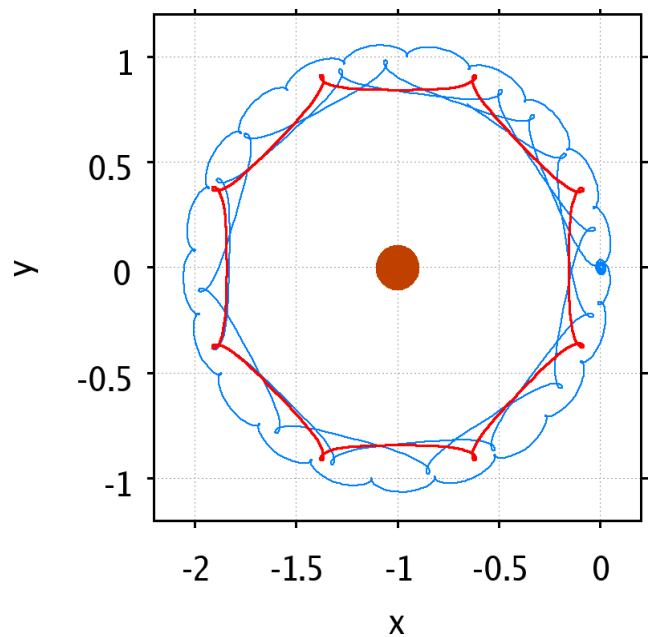
$$FLI_{mean}(T) = \frac{1}{2}\left(FLI_{forward}(T) + FLI_{backward}(T)\right)$$

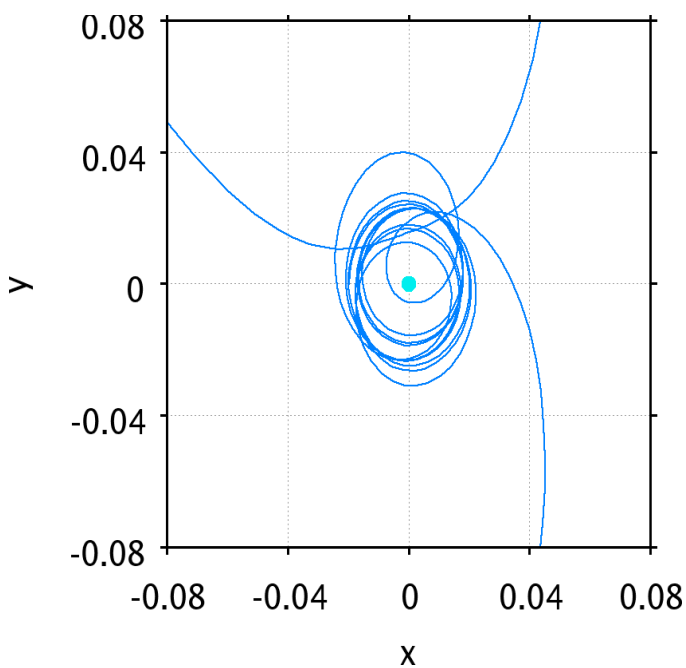
TRANSFER BETWEEN the 10:9 and 7:6 RESONANT ISLANDS











ESCAPE FROM a DRO REGION

- **DRO transfers:**
  - The introduction of a fly-by result in scattering of the manifold structures.
  - Manifold computations with FLI maps enabled the design of a transfer, by placing the transfers into their dynamical context.

- **Potential extensions:**
  - Extend these ideas to 3D/ other systems.
  - Analyze the chaining of several coast arcs/ the effect of thrust.
  - Consider frequency based chaoticity indicators.

A FEW REFERENCES

- Villac, B., Lara, M., “Stability maps, Global dynamics and Transfers”, paper AAS 05-378 presented at the 2005 AAS/AIAA Astrodynamics Specialists Conference, Lake Tahoe, California.
- Froeschle, C., Guzzo, M., Lega. E., “Graphical evolution of the Arnold web: From order to chaos”, SCIENCE 289 (5487): 2108-2110, SEP 22 2000.