# $N$-qubit quantum memory through a well-defined sequence of optical pulses


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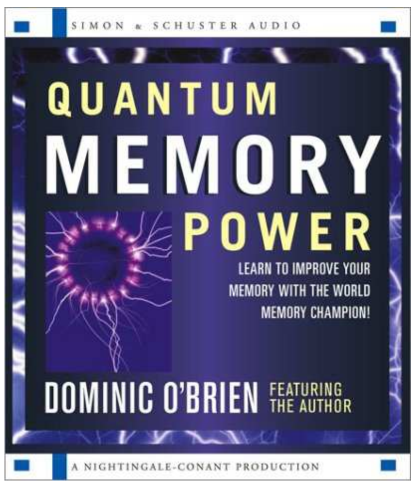
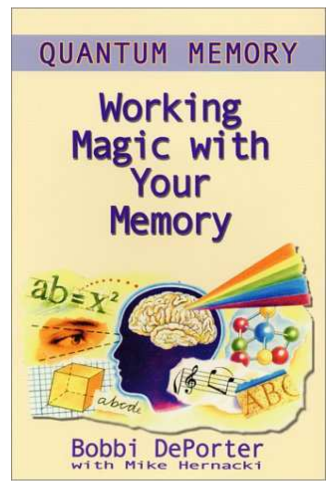
## Quantum memory in the literature



# What is “Quantum Memory”?

- Just as the conventional computer memory stores classical bits of information for long times, quantum memory is would do the same for a quantum bit of information–“qubit”.
- Quantum memory would form an integral part of future quantum information processing systems, immaterial of actual physical systems used for implementation.
- Quantum memory should have following characteristics:
  - ⋆ Fast access: easy write and read processes.
  - ⋆ Long storage times: should not decohere.
  - ⋆ Scalibility: large N-qubit systems should be equally easy to store.
- We propose a simple scheme for scalable quantum memory of atomic qubits using quantum control theory.
- Our proposal can be implementated in variety of systems including, ion trap, cavity QED, Bose-Einstein Condensates and atoms trapped in optical lattices.

# Overview of existing approaches to quantum memory

Loss of quantum information is attributed to unwanted coupling of the system of interest to its surrounding, termed as decoherence. Various approaches like "decoherence-free subspaces" and "quantum error-correcting codes" have been developed to battle this decoherence.

## Decoherence-free subspaces

Special encoding of the kind, $|0\rangle_L \equiv |01\rangle$ and $|1\rangle_L \equiv |10\rangle$, can be shown to be immune to collective dephasing of the type – $|0\rangle \to |0\rangle$ and $|1\rangle \to e^{i\phi}|1\rangle$.

## Quantum error-correcting codes

An idea derived from conventional computing uses redundancy, *syndrome measurement* and error-correcting procedures for protecting the loss of quantum information.
Example: Consider random bit-flip errors: $|0\rangle \leftrightarrow |1\rangle$.
**Encoding:** $|0\rangle_L \equiv |000\rangle$ and $|1\rangle_L \equiv |111\rangle$
**Two measurements:**
First in basis $\{|000\rangle + |111\rangle + |001\rangle + |110\rangle, |010\rangle + |101\rangle + |100\rangle + |011\rangle\}$, and
second in basis $\{|000\rangle + |111\rangle + |010\rangle + |101\rangle, |001\rangle + |110\rangle + |100\rangle + |011\rangle\}$.
**Error Correction:** If the result of the two successive projection measurements is

00, do nothing
01, flip the rightmost spin
10, flip the middle spin
11, flip the leftmost spin.

This operation corrects the superposition state $c_a|0\rangle_L + c_b|1\rangle_L$ from one bit-flip error.

# Experimental and theoretical proposals so far

- Photonic qubits:
  - ⋆ Light storage in atomic systems[a]
  - ⋆ Decoherence free subspaces[b]
  - ⋆ Error correcting codes[c]
- Atomic qubits:
  - ⋆ Decoherence free subspaces[d]

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- All implementations have been demonstrated only for single-qubit states so far.
- Scalability of all the above approaches requires more and more resources, reduces speed and would require complicated timing issues to be resolved.
- We provide an alternative approach, which is naturally scalable and fast and needs marginal increase in resources.

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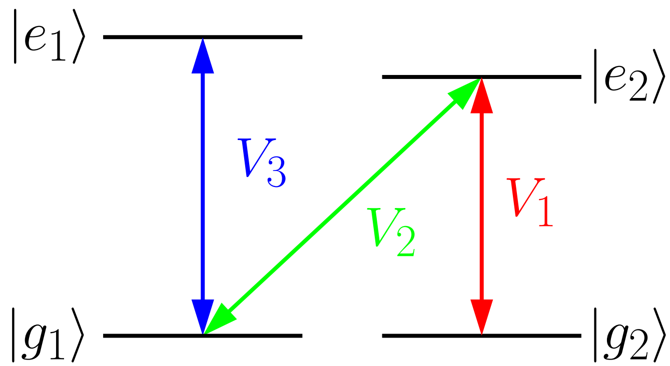
[a] C. Liu *et al.*, Nature **409**, 490 (2001); D.F. Phillips *et al.*, Phys. Rev. Lett. **86** 783 (2001).

[b] P. G. Kwiat *et al.* Science **290**, 498 (2000); T. B. Pittman and J. D. Franson, Phys. Rev. A **66**, 062302 (2002)

[c] R. M. Gingrich *et al.* Phys. Rev. Lett. **91**, 217901 (2003).

[d] D. Kielpinski *et al.*, Science **291**, 1013 (2001).

# Single-qubit quantum memory for an atomic qubit[a]




- A short-lived working qubit $|g_1\rangle - |e_1\rangle$ is to be transferred to the long lived degenerate memory qubit $|g_1\rangle - |g_2\rangle$.
- We assume the transition $|e_1\rangle - |g_2\rangle$ to be forbidden to make sure we are not adding extra decoherencing channels.
- The memory write operation is nothing but the unitary transformation $U_1 \rho^{(\mathrm{in})} U_1^\dagger$:

$$\rho^{(\mathrm{out})} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} a & 0 & b & 0 \\ 0 & 0 & 0 & 0 \\ c & 0 & d & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Here the matrix representations are in the basis $\{|g_1\rangle, |e_1\rangle, |g_2\rangle, |e_2\rangle\} \equiv \{1, 2, 3, 4\}$.

[a] A. D. Greentree *et al.*, quant-ph/0103118.

# Implementation of $U_1$ through a sequence of SU(2) transformations [a]

- Any unitary transformation can be reduced to a product of transformations which have only a 2 X 2 non-trivial block of the SU(2) form apart from an identity matrix.
- Thus, it can be shown that $U_1 = V_1 V_2 V_3 V_2 V_1$.
- Here,

$$V_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, V_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, V_3 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

- These transformation matrices $V_1$, $V_2$ and $V_3$ can be easily seen to be achievable through transformations on transitions $|g_2\rangle - |e_2\rangle$, $|e_2\rangle - |g_1\rangle$ and $|g_1\rangle - |e_1\rangle$ respectively.
- Once again, the matrix representations are in the basis $\{|g_1\rangle, |e_1\rangle, |g_2\rangle, |e_2\rangle\} \equiv \{1, 2, 3, 4\}$.

[a] V. Ramakrishna *et al.*, Phys. Rev. A **61**, 032106 (2000).

# Implementation of SU(2) transformations through well-defined optical pulses

- Consider an optical pulse with electric field $E(t) = E_0(t)\cos(\omega_{e_1 g_2} t + \phi)$ applied to a general state $\psi(t) = a(t)\,|a\rangle + b(t)\,|b\rangle$.
- In the interaction picture, this interaction can be shown to be governed by

$$\begin{pmatrix} \dot{a}(t) \\ \dot{b}(t) \end{pmatrix} = \begin{pmatrix} 0 & i\gamma \\ i\gamma^* & 0 \end{pmatrix} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

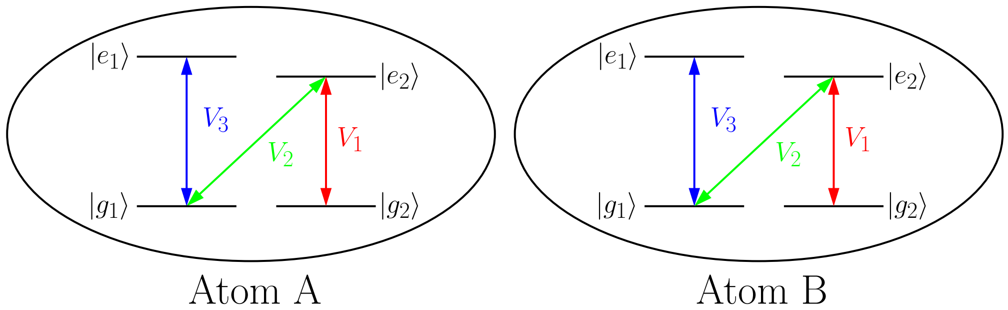
where $\gamma = \mu_{ab} \mathrm{e}^{i\phi} \int_0^t E_0(t) \mathrm{d}t$.

- The formal solution can be shown to be:

$$\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = V \begin{pmatrix} a(0) \\ b(0) \end{pmatrix} = \exp \begin{pmatrix} 0 & i\gamma \\ i\gamma^* & 0 \end{pmatrix} \begin{pmatrix} a(0) \\ b(0) \end{pmatrix} = \begin{pmatrix} \cos(|\gamma|) & i\mathrm{e}^{i\phi}\sin(|\gamma|) \\ i\mathrm{e}^{-i\phi}\sin(|\gamma|) & \cos(|\gamma|) \end{pmatrix} \begin{pmatrix} a(0) \\ b(0) \end{pmatrix}$$

- Thus, with appropriate choice of $\phi$ and $|\gamma|$ variety of SU(2) transformations can be obtained.
- For example, for $\phi = -\pi/2$ and $|\gamma| = \pi/2$ we obtain $V = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ which is nothing but the nontrivial component of $V_1$, $V_2$, and $V_3$ as applied to corresponding transition.

# Quantum memory for an entangled two-qubit system




- We need to identical four-level atoms as shown above.
- We consider a two-qubit state $|0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B$ apart from the normalization, $i.e.$, the state is: $|g_1\rangle_A |e_1\rangle_B + |e_1\rangle_A |g_1\rangle_B$.
- Therefore,

$$\rho^{(\text{in})} = |g_{1A} e_{1B}\rangle \langle g_{1A} e_{1B}| + |g_{1A} e_{1B}\rangle \langle e_{1A} g_{1B}| + |e_{1A} g_{1B}\rangle \langle g_{1A} e_{1B}| + |e_{1A} g_{1B}\rangle \langle e_{1A} g_{1B}| .$$

- Now, the expected density matrix of the system after memory-write operation would be

$$\rho^{(\text{out})} = |g_{1A} g_{2B}\rangle \langle g_{1A} g_{2B}| + |g_{1A} g_{2B}\rangle \langle g_{2A} g_{1B}| + |g_{2A} g_{1B}\rangle \langle g_{1A} g_{2B}| + |g_{2A} g_{1B}\rangle \langle g_{2A} g_{1B}| .$$

- We propose that we can achieve this with the unitary transformation: $U_2 = U_1 \otimes U_1$, $i.e.$, $U_1$ applied individually to each atom.

# Quantum memory for an entangled two and $N$-qubit system

- By choosing the basis: $\{|g_{1A} g_{1B}\rangle, |g_{1A} e_{1B}\rangle, |g_{1A} g_{2B}\rangle, |g_{1A} e_{2B}\rangle, \cdots\} \equiv \{1, 2, 3, \cdots, 16\}$, we can obtain the matrix representation for $\rho^{(\mathrm{in})}$ and $\rho^{(\mathrm{out})}$.
- Thus, $\rho_{i,j}^{(\mathrm{in})} = 1/2$ with $i, j = 2, 5$ and $\rho_{i,j}^{(\mathrm{in})} = 0$ for the rest.
- And the expected result is :$\rho_{i,j}^{(\mathrm{out})} = 1/2$ for $i, j = 3, 9$ and $\rho_{i,j}^{(\mathrm{out})} = 0$ for the rest.
- It can be easily shown that for the two-qubit state:

$$\rho_{\mathrm{expected}}^{(\mathrm{out})} = (U_1 \otimes U_1)\rho^{(\mathrm{in})}(U_1 \otimes U_1)^\dagger$$

- Note that we have not assumed decomposition of the two-qubit density matrix into its one-qubit components.
- To note, this approach works trivially for two-qubit product states.
- It is very straightforward to extend this approache to a general N-qubit state because of the simplicity of the above transformation.
- Thus,

$$\rho_{N-\mathrm{Qubit}}^{(\mathrm{memory})} = (U_1 \otimes U_1 \otimes U_1 \cdots N \text{ terms})\rho_{N-\mathrm{Qubit}}^{(\mathrm{in})}(U_1 \otimes U_1 \cdots N \text{ terms})^\dagger .$$

# Various advantages of the proposal

- Our proposal is naturally scalable and is very easy to implement and applies to any N-qubit states of the product form or entangled.
- As the memory qubit is encoded into two degenerate states of an atom, there is no decoherence and the memory is extremely long-lived.
- The scheme is general enough, so that it can be applied to variety of systems including ion-trap, cavity QED and atoms in optical lattices or BEC on chips.
- No atom-atom interaction is required to store the entangled states into memory.
- With quantum state transfer protocols, photonic qubit states could be transferred to atoms first and then stored for long times. Let us note that this is not similar to light storage proposals, wherein the pulse profile is stored in the atomic coherence. For example, out scheme would work even with polarization encoded single photon qubits after the state-transfer to atomic qubits.
- Also note that memory read and write operations are in principal the same, except for the phase required for the light pulses. Thus they are distinguishable and do not pose any extra challanges for implementations.
- We are currently working towards reducing the number of pulses required for the memory read and write operation and increasing coherence times even further.