Thrust Vector Control of the Jovian Icy Moons Orbiter Spacecraft

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Abstract

In the Jovian Icy Moons Orbiter spacecraft, attitude and orbital dynamics interactions are present due to the designed low-thrust trajectory. In order to investigate this coupling, which originates due to the thrust vectoring control, sensitivity analyses and simulation studies were carried out using complex orbital and attitude dynamics models describing the vehicle’s dynamics arising during the low-thrust spiraling maneuvers of the spacecraft. The results of these studies point out that a new approach is needed to integrate the trajectory design and the attitude control.

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Introduction

NASA is developing plans for an ambitious mission to orbit three planet sized moons of Jupiter - Callisto, Ganymede and Europa - which may harbor vast liquid oceans beneath their icy surfaces. These plans had their genesis in a study conducted in 2002 of a Jovian Icy Moon Tour (JIMT) mission. This objective of the JIMT mission study was to design a spacecraft to explore the three icy moons and investigate their makeup, their history and their potential for sustaining life. To do so, NASA looked at how a nuclear reactor could enable long-duration deep space exploration. It was found that a nuclear fission reactor could produce unprecedented amounts of electrical energy to significantly improve scientific measurements, mission design options, and telecommunications capabilities. The proposed JIMT mission would incorporate a form of electric propulsion called ion propulsion, which would be powered using a nuclear fission reactor and a system for converting the reactor's heat to electricity. Figure 1 depicts the representative thrust vector angular rate throughout a sample Jupiter tour mission developed as part of the JIMT study in 2002. For this type of mission, attitude and orbital dynamics interactions are present due to the low-thrust trajectory design. This is a significant challenge to the dynamics and controls analyst because additional degrees of freedom have to be included in the control design, and because the timing of the attitude control maneuvers ends up affecting the overall mission timeline on account of the long duration of the thrust vectoring maneuvers. Attitude and orbital dynamics interactions are present due to the low-thrust trajectory design. The low-thrust propulsion system results in orbital trajectories which tend to be open spirals, rather than pure Keplerian orbits. To achieve these non-Keplerian trajectories, the thrust vector control system must drive the spacecraft attitude dynamics through the appropriate ion engine pod gimbal angles, which in turn must be operated in a way that minimizes the translation-rotation coupling and the dynamic back-reaction onto the rest of the vehicle. Since this coupling cannot be eliminated, an interaction exists between the orbital and the attitude dynamics of the vehicle. A simulation model is described in this paper which is capable of handling the coupled orbital and attitude dynamics arising during the spiraling maneuvers of the spacecraft.

![Figure 1. Thrust angular rate for sample Jupiter tour mission (courtesy of Greg Whiffen, JPL).](image)

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6 Within Project Prometheus, the Jovian Moon Tours study has evolved into what is now referred to as the “Jupiter Icy Moons Orbiter”, or JIMO, mission.
Thrust Vector Control

Figure 2 depicts two types of articulated ion pods being considered. Two boom mounted Ion Engine arrays (pods) of 3 (4) thrusters each will be articulated with 2 dof gimbals (Figure 3) to produce Roll, Pitch, and Yaw torques that null the spacecraft body rates and drive the resultant thrust vector through the spacecraft’s center of mass. The electric propulsion (EP)-based TVC system will have the capability of performing continuous coplanar spiral pitch turns during planetary escape and capture maneuvers, and uncoupled turns for Plane change Delta-V maneuvers. Trajectory Path Guidance Control Laws will perform the combined functions of Delta-V and Thrust Vector Control during Powered Flight.

Figure 2. Articulated Ion Engine Pod Platforms.

The ion thruster pod gimbals are driven by two-axis gimbals powered by DC-brushless electric motors. A gimbal travel of each pod of up to 120 degrees in 2 axes is proposed and analyzed. The zero degree center of travel reference will be aligned parallel with the S/C Roll Axis and normal to the Pitch axis. The large JIMO (350 mN peak each) Xenon electric propulsion ion thrusters do not have any throttle capability. For comparison with another NASA EP spacecraft, the Deep Space One ion engine was about 90mN peak and had over 18 throttle steps. Additionally, the ion engine generated plasma at under 20% of peak thrust is not stable, so thruster operation under 70 mN will be possible for the JIMO EP thrusters if they are similar in plasma behavior to the Deep Space One engine. The TVC (thrust vector control) closed loop control law and performance will be severely limited and over-constrained without throttle steps above the 20% thrust.
level. Only very low bandwidth EP pod gimbal servos will be available to control the TVC if no throttle capability is provided. The massive EP pod gimbals, drives, and Xenon feed lines will have substantial non-linearities and impact the controls-structure interaction. These problems can be strongly mitigated with throttle modulation of the EP thrusters for thrust vector steering, particularly in the maneuvering during the moon tour that requires much greater TVC agility than simple Earth escape and cruise.

**Interaction between orbital and attitude dynamics**

In this section, we describe the effect of the attitude dynamics on the orbital dynamics and of the orbital dynamics on the attitude dynamics of the vehicle. Defining the skew-symmetric operator by a superposed tilde, the kinetic energy of an extended rigid body in a general orbit around a planet can be written as:

\[
T = \frac{1}{2} m \left( \dot{R}^2 + \frac{1}{2} R^2 \tilde{\omega} \tilde{\Omega} \tilde{\omega} - 2 R \tilde{\dot{R}} \tilde{\omega} \tilde{\Omega} \tilde{\omega} \right) + \frac{1}{2} \omega \cdot J \omega
\]  

(1)

where \( \omega \) represents the unit vector in the nadir direction (vertical down), \( \Omega = \Omega(\dot{\Omega}_L, \dot{i}, \dot{\omega}_p) \) is the vector of orbital angular velocity dependent on the time rates of the longitude of the ascending node \( \dot{\Omega}_L \), the mean anomaly \( M \), and the argument of perigee \( \omega_p \). Similarly, the gravitational potential energy of the spacecraft can be written, to second order in the displacements, as:

\[
U = -\frac{\mu m}{R_0} - \frac{\mu}{2 R_0^3} \left[ \text{trace}(J) I - 3 \omega_3 \cdot J \cdot \omega_3 \right]
\]  

(2)

where \( J \) is the spacecraft moment of inertia tensor. The perturbation force on an extended body orbiting around a spherical primary, caused by the extended inertia distribution of the body, can be written to second order in displacements as:

\[
f_{\text{pert}} = \frac{3 \mu}{2 R_0^4} \left[ \text{trace} \left( J \right) I + 2 J \right] \cdot \omega_3 - 5 \left( \omega_3 \cdot J \cdot \omega_3 \right) o_3
\]  

(3)

This perturbation force results in an acceleration applied to the vehicle’s center of mass. The result is that, even if the vehicle is orbiting a perfectly spherical primary, the mass center does not follow a truly Keplerian orbit, on account of the extended body inertia distribution. Conversely, when the spacecraft orbits a planet with a non-spherical gravitational distribution (i.e., resulting in asphericity measurable in terms of \( J_2 \) and \( J_3 \) effects), the orbit in general will be elliptic, and the line of nodes will in general move in space. In particular, the longitude of the ascending node, the mean anomaly, and the argument of perigee will change with time. The result is that the attitude dynamics of the vehicle is coupled to the orbital dynamics. The modeled time variations of mean anomaly, longitude of ascending node, and argument of perigee are well known and can be written as follows:

\[
\dot{M} = \Omega \left[ 1 + 1.5 \left( J_2 (R_p/a)^2 \right)(1-1.5 \sin^2 i)/(1-e^2)^{3/2} \right]
\]

\[
\dot{\Omega}_L = -1.5 J_2 \Omega (R_p/a)^2 \cos i/(1-e^2)^2
\]

\[
\dot{\omega}_p = +1.5 J_2 \Omega (R_p/a)^2 (2-2.5 \sin^2 i)/(1-e^2)^2
\]  

(4)
where $R_p$ is the planet’s radius, $e$ is the orbit’s eccentricity, and $a$ the semimajor axis. At each time step, the eccentric anomaly $E$ is found by solving Kepler’s equation $E-M=e \sin(E)$. The true anomaly $v$ can then be found as $v=\text{atan2}(\sin(E)\sqrt{1-e^2}, \cos(E)-e)$. The current radius is $R=a[1-e \cos(E)]$. The components of $R_0$ in the inertial frame defined by the vectrix $\Phi_t$ are then:

$$\mathbf{R}_0=\Phi_t^T \mathbf{R}_0 \begin{bmatrix} \cos(\omega_p+v) \cos\Omega_L-\sin(\omega_p+v) \sin\Omega_L \cos i \\ \cos(\omega_p+v) \sin\Omega_L+\sin(\omega_p+v) \cos\Omega_L \cos i \\ \sin(\omega_p+v) \sin i \end{bmatrix}$$ (5)

Defining the colatitude $\delta=\text{asin}(z/|R_0|)$, the inertial vectors of the acceleration of the vehicle due to higher order gravitational potential terms ($J_2, J_3$) can also be written as:

$$\mathbf{a}_j=\frac{3}{2} \frac{\mu J_2}{|\mathbf{R}_0|^6} \begin{bmatrix} x(1-5\sin^2\delta) \\ y(1-5\sin^2\delta) \\ z(3-5\sin^2\delta) \end{bmatrix} \Phi_t$$ (6)

$$\mathbf{a}_j=\frac{5}{2} \frac{\mu J_3}{|\mathbf{R}_0|^6} \begin{bmatrix} x(3-7\sin^2\delta) \sin\delta \\ y(3-7\sin^2\delta) \sin\delta \\ z(6-7\sin^2\delta) \sin\delta -3|R_0|/5 \end{bmatrix} \Phi_t$$ (7)

Next, we describe how the orbit-attitude coupling arises, by looking at a simplified scenario. In the simplified case in which the vehicle is approximated as a rigid dumbbell constrained to librate in the orbital plane only, the fully coupled nonlinear equations of motion in the orbital radius $R_0$, the true anomaly $v$, and the pitch angle $\sigma$ become as follows:

**Radius**: $\ddot{R}_0 - \dot{v}^2 R_0 = -\frac{\mu}{R_0^2} \left[ 1 + \frac{3}{2mR_0^2} \left( \text{trace}(J) - 3J_1 \sin^2 \sigma - 3J_3 \cos^2 \sigma \right) \right] - \frac{f}{m} \cos(\sigma + \theta)$ (8)

**True Anomaly**: $\dot{v} + 2 \frac{\dot{R}_0}{R_0} \dot{v} = -\frac{3\mu}{mR_0^3} (J_3 - J_1) \sin \sigma \cos \sigma - \frac{f}{mR_0} \sin(\sigma + \theta)$ (9)

**Pitch Angle**: $\dot{\sigma} - \dot{v} = \frac{3\mu}{J_2} \left( \frac{J_3 - J_1}{J_2} \right) \sin \sigma \cos \sigma + \frac{f}{J_2} (d_3 \sin \theta - d_1 \cos \theta)$ (10)

where $f$ is the applied thrust, $d_1$ and $d_3$ are the components of the thrust application point in the spacecraft body frame, $\theta$ is the thrust direction angle in body frame. We have assumed that the spacecraft body frame is a principal axis frame. From the last three equations, one can observe that:

- the equations of motion are nonlinear and non-homogeneous.
- the equations of motion apply to any type of orbit, including spiral-in/out phases.
- the attitude dynamics and the orbital dynamics are, indeed, coupled through the pitch angle $\sigma$, which is not necessarily small, and through the true anomaly.
- the fact that the orbit is eccentric is reflected in the true anomaly rates. When the orbit is circular, the attitude dynamics is uncoupled from the orbital dynamics.
the thrust direction and magnitude affect both the orbital and attitude dynamics. The applied thrust is a “follower” force (i.e., it follows the motion of the vehicle in its own body frame), and the thrust vector is not necessarily directed along one of the body axes only.

- the gravity gradient effect (represented by the terms in \( \sin \sigma \) and \( \cos \sigma \)) appears in all the equations.

These interaction effects are shown in Figures 4 to Figure 9 where, for a typical 152 km altitude eccentric and regressive science orbit around Europa (hence \( J_2 \) and \( J_3 \) effects are taken into account), the pitch response is shown to depend slightly on the orbital inclination, more strongly on the eccentricity, and even more strongly on the orbital altitude, as expected. Similarly, the yaw and roll responses are also shown to depend on eccentricity, inclination, and altitude to various degrees. From an engineering standpoint, the implication of this interaction is that additional requirements related to the induced orbit change need to be defined and imposed on the design of the vehicle. To conclude this section, one might think that the attitude-orbit coupling is a totally negligible effect in most situations. However, because of the low-thrust dynamics, all the attitude maneuvers have durations comparable with the orbital period of a low altitude science orbit around one of Jupiter’s moons (~ 2 hours). This fact alone singles out the need to further investigate this complex interaction.

Figure 4. Left: Spacecraft linear orbital acceleration induced by attitude dynamics for a typical 152 km altitude orbit around Europa. Right: Spacecraft Pitch angle vs. orbital inclination for a typical 152 km altitude orbit around Europa.

Figure 5. Left: Spacecraft Pitch angle vs. orbital eccentricity for a typical 152 km altitude orbit around Europa. Right: Spacecraft Pitch angle vs. orbital altitude for an orbit around Europa.
Figure 6. Left: Spacecraft Roll angle vs. inclination for a typical 152 km altitude orbit around Europa. Right: Spacecraft Roll angle vs. eccentricity for an orbit around Europa.

Figure 7. Left: Spacecraft Roll angle vs. orbital altitude for a typical 152 km altitude orbit around Europa. Right: Spacecraft Yaw angle vs. inclination for an orbit around Europa.

Figure 8. Left: Spacecraft Yaw angle vs. eccentricity for a typical 152 km altitude orbit around Europa. Right: Spacecraft Yaw angle vs. orbital altitude for an orbit around Europa.
Figure 9. Left: Spacecraft Pitch Rate vs. eccentricity for a typical 152 km altitude orbit around Europa. Right: Spacecraft Pitch Acceleration vs. eccentricity for an orbit around Europa.

**Attitude Control Strategy Using Thrust Vectored Engines**

Thrust vectoring, if used for attitude control, ends up creating spurious lateral deltaV’s, resulting in navigation error. Pure torque couples are needed to avoid navigation error. A reaction control system (RCS) instead is best suited to provide pure couple control torques. Figure 10 shows the geometry and forces used for trade studies involving thrust vector control.

Figure 10. Geometry and Forces using the ion pods for TVC.
Figure 11 shows that pitch control can be achieved by articulating both pods about the X-axis. Figure 12 shows that roll control can be achieved by articulating both pods differentially about the X-axis. For a yaw turn, several options exist:

- Turn off one Ion Engine Pod (Figure 13).
- Turn both Ion Engine Pods (but this option presents potential plume impingement issues).
- Turn only one Ion Engine Pod outward (Figure 14).
- Shift gimbal range from -5 to 118 degrees, which increases torque authority. (Figure 15).
Figure 13. Yaw Turn using one engine only.

Figure 14. Yaw Turn - One Ion Engine Pod Turning Outward.
Pod rotation for maximum torque 13.5 Nm

Figure 15. Yaw Turn - One Ion Engine Pod Turning Outward with a range - 5 to 118 degrees, for maximum torque.

Attitude Control Laws Using Thrust Vectored Engines

Figure 16 depicts the ion engine Thrust Vector Control Strategy. The TVC control strategy assumes that sensor information is available from Sun sensors, star tracker, and gyros and accelerometers in the inertial measurement units (IMU) to the basebody system identification (SID) and attitude estimator, which provides accurate estimates of the basebody attitude, and attitude rates, with respect to an inertial frame, together with estimates of the vehicle’s position and velocity from the inertial vector propagator. These estimates are made available to the attitude commander and attitude controller, as well as the TVC controller. The next stage is the conversion of the commands to the gimbal articulation controllers using the gimbaled electric propulsion (EP) engines. An update rate of 10 Hz is expected for these on-board functions.

Figure 16. Ion Engine Thrust Vector Control Strategy for Delta-V.
The absolute angular velocity of the ion pod is

\[ \omega^C = \omega^B + (\beta \sin \alpha - \beta \cos \alpha) F_b \]  

(11)

where \( \omega^B \) denotes the angular velocity of the base body, \( \alpha \) and \( \beta \) denote the ion engine pod azimuth and elevation angles, measured in the body-fixed frame \( F_b \). The gimbal control torque vector is:

\[ \tau = (\tau_\beta \sin \alpha, -\tau_\beta \cos \alpha, \tau_\alpha) F_b \]  

(12)

where and \( \tau_\alpha \) and \( \tau_\beta \) are the ion engine pod gimbal command torques in the \( \theta \) and \( \phi \) directions. The payload is hinged at a bottom location on the basebody, via a universal-joint connection, or a two degrees of freedom gimbal. This implies that there are two independent axes which need to be controlled to provide declination and azimuth of the payload, about the reference body triad represented by the bus body axes. With the dynamics described above (i.e., payload and bus dynamics) the equations of motion are strongly coupled through centrifugal and Coriolis terms (gyroscopic forces). These forces are of small magnitude if sufficiently slow maneuvers take place, and providing the attitude of the bus is only slightly perturbed from an equilibrium inertial orientation. Therefore, with standard sensing equipment located on board the bus, i.e. three-axis accelerometers, gyro unit, and a global attitude determination system such as an on-board star tracker, both inertial position (in planetocentric coordinates), inertial attitude (with respect to the planetocentric reference frame, which is being propagated through ephemeris in the on-board computer) and their rates can be determined. Some estimation procedure is necessary when the full dynamic state cannot be measured. With this information, the nonlinear gyroscopic terms can be cancelled from the equations. This approach results in a feedback linearized equation of motion in the direction of the controlled axes, namely we achieve near perfect state decoupling, and we can design the local controllers assuming independent control loops. The pointing control algorithms developed for this study rely on a feedback linearization of the dynamics to derive a globally, exponentially stable controller for the pointing dynamics. An attitude estimator on board the bus provides real-time estimates of the attitude quaternion and angular velocity. A command profiler specifies the command to be tracked, in the form of a constant or a step versus time. These commands are provided to the controller in the form of desired attitude, angular velocity, and angular acceleration. It is desired to cancel all possible dynamic nonlinearities arising from gyroscopic and centrifugal terms, as derived from the equations of motion. The rotational control torque vector \( \tau \) for each axis is then of the following form

\[ \tau = \Gamma \left[ \lambda (\theta_{err})_{Cmd} - \lambda (\theta_{err})_{Est} \right] + \Gamma (N_\omega^P_{Cmd} - N_\omega^P_{Est}) + J P \omega_{Cmd} + h_{cancel} \]  

(13)

where \( \Gamma \) and \( \lambda \) are rotational control gain matrices, \( J \) is the engine pod moment of inertia matrix, \( \lambda \) is the unit eigenaxis of rotation, \( \theta_{err} \) is the magnitude of rotation corresponding to the difference between the commanded \( (\cdot)_{Cmd} \) and the estimated \( (\cdot)_{Est} \) quaternion. \( h_{cancel} \) is the vector of the centrifugal and Coriolis nonlinear terms to be cancelled, which can be obtained from the appropriate terms in the equation of motion, and \( N_\omega^P \) and \( N^\alpha \) are the angular velocity and acceleration vectors of the ion engine pod respectively with respect to the inertial frame \( N \).

**Impact of JIMO Thrust Vectoring on AACS**

In order to quantify the influence of the thrust vectoring approach onto the attitude dynamics, we conducted a study to determine the sensitivity of the thrust vector to thrust magnitude, thrust pod gimbal angles; and thrust moment arms. Once a trajectory had been designed as a function of mission time, the trajectory and thrust vector profile were used as inputs to derive quantities of interest for the attitude dynamics, such as orbital and body-fixed frames. This is the first step of the control allocation problem, which has the
objective of converting the optimized thrust used in designing the trajectory into the gimbal articulation angles needed for attitude control of the vehicle.

**Mapping thrust from orbiting frame to body frame**

Next, we derive a procedure to convert thrust vector in orbital frame into thrust vector in body frame and ion pod frame, and also derive the sensitivity of the thrust vector to: 1) thrust magnitude; 2) thrust pod gimbal angles; 3) thrust moment arms. The way to extract the body-frame thrust components from the orbiting frame thrust vector components, assuming the spacecraft is rigid and the gimbals are locked, is shown in Figure 17. Defining as in Figure 17 the following variables:

- \( F_0 = (f_1, f_2, f_3) \) = thrust vector in ORF
- \( F_b \) = thrust vector in body frame
- \( |f| \) = thrust magnitude
- \( f_{23} \) = thrust projection in \( O_2-O_3 \)
- \( O_1 \) (radial R), \( O_2 \) (along velocity V), \( O_3 \) (orbit normal N)
- \( \theta = \operatorname{atan2}(f_3,f_2) \) = thrust in-plane angle
- \( \varphi = \operatorname{atan2}(f_1,f_{23}) \) = thrust out-of-plane angle
- \( \alpha, \beta \), azimuth and elevation of \( i \)-th ion engine pod angles
- \( C_{bo} \) = transformation from orbital axes to body-fixed axes
- \( f_b = C_{bo} f_0 \) = thrust in body frame
- \( \theta_1, \theta_2, \theta_3 \) = Roll, pitch, yaw angles of vehicle’s body axes with respect to orbiting frame (large angles)

and introducing the abbreviations \( s=\sin \) and \( c=\cos \), we can obtain the transformation from orbital axes to body-fixed axes as follows:

\[
C_{bo} = C_1(\text{roll})C_2(\text{pitch})C_3(\text{yaw}) = C_1(\theta_1)C_2(\theta_2)C_3(\theta_3) = \begin{bmatrix} c2c3 & c2s3 & -s2 \\ s1s2c3 - c1s3 & s1s2s3 + c1c3 & s1c2 \\ c1s2c3 + s1s3 & c1s2s3 - s1c3 & c1c2 \end{bmatrix}
\]
The way to extract the body-frame thrust components from the orbiting frame thrust vector components, assuming the spacecraft is rigid and the gimbals are locked, is shown in Figure 18.

\[
\begin{bmatrix}
    h_1 \\
    h_2
\end{bmatrix} = \begin{bmatrix}
    1 & 1 \\
    \vec{r}_L & \vec{r}_R
\end{bmatrix}^\dagger \begin{bmatrix}
    f_b \\
    \tau_\phi
\end{bmatrix}
\]

The geometry of the ion engine thrust components, as shown in Figure 17, can be derived as:
\[(h_1)_1 = h_1 | c \beta_i s \alpha_i \]
\[(h_1)_2 = h_1 | s \beta_i \]
\[(h_1)_3 = h_1 | c \beta_i c \alpha_i \]
\[(h_1)_{13} = \sqrt{(h_1)_1^2 + (h_1)_3^2} = h_1 | c \beta_i \]

where \(\alpha_1 = \text{atan2}((h_1)_1, (h_1)_3)\) is the ion engine pod azimuth angle, and \(\beta_1 = \text{atan2}((h_1)_2, (h_1)_{13})\) is the ion engine pod elevation angle.

**Thrust Vector Control Sensitivity Study**

Figure 18 depicts the thrust vectoring geometry, which allows us to write the vector balance of forces and torques about the instantaneous center of mass of the vehicle, in the body frame, as:

\[
\mathbf{f} = f_1 + f_2 = F_b^T \begin{pmatrix}
\begin{pmatrix} c \beta_i s \alpha_i \\ s \beta_i \\ c \beta_i c \alpha_i \end{pmatrix} + f_2 \begin{pmatrix} c \beta_2 s \alpha_2 \\ s \beta_2 \\ c \beta_2 c \alpha_2 \end{pmatrix}
\end{pmatrix}
\]

(16)

\[
\mathbf{\tau} = \tau_1 + \tau_2 = F_b^T \begin{pmatrix}
\begin{pmatrix} d_1 s \beta_i \\ -d_1 c \beta_i s \alpha_i - l_1 c \beta_i c \alpha_i \\ l_1 s \beta_i \end{pmatrix} + f_2 \begin{pmatrix} d_2 s \beta_i \\ -d_2 c \beta_i s \alpha_i - l_2 c \beta_i c \alpha_i \\ l_2 s \beta_i \end{pmatrix}
\end{pmatrix}
\]

(17)

Linearizing about reference ion engine pod angles as follows:

\[
s \alpha_i \approx s \alpha_{0i} + c \alpha_{0i} \alpha_i
\]
\[
c \alpha_i \approx c \alpha_{0i} - s \alpha_{0i} \alpha_i
\]

(18)

one gets the linearized force balance as:

\[
f_X \approx \sum_{i=1}^{2} \left\{ f_i \left[ c \beta_{0i} s \alpha_{0i} + (c \beta_{0i} s \alpha_{0i}) \alpha_i - (s \beta_{0i} s \alpha_{0i}) \beta_i \right] \right\}
\]

\[
f_Y \approx \sum_{i=1}^{2} \left\{ f_i \left[ s \beta_{0i} + (c \beta_{0i}) \beta_i \right] \right\}
\]

(19)

\[
f_Z \approx \sum_{i=1}^{2} \left\{ f_i \left[ c \beta_{0i} c \alpha_{0i} - (c \beta_{0i} c \alpha_{0i}) \alpha_i - (s \beta_{0i} c \alpha_{0i}) \beta_i \right] \right\}
\]

and the linearized torque balance as:
\[\tau_X = \sum_{i=1}^{2} \left\{ f_i d_i \left[ s \beta_{0i} + (c \beta_{0i}) \beta_i \right] \right\}\]

\[\tau_Y = -\sum_{i=1}^{2} \left\{ f_i \left[ (d, c \beta_{0i} s \alpha_{0i} + l, c \beta_{0i} c \alpha_{0i}) - (d, c \beta_{0i} c \alpha_{0i} - l, c \beta_{0i} s \alpha_{0i}) \alpha_i + (d, s \beta_{0i} s \alpha_{0i} + l, s \beta_{0i} c \alpha_{0i}) \beta_i \right] \right\}\]

\[\tau_Z = \sum_{i=1}^{2} \left\{ f_i l_i \left[ s \beta_{0i} + (c \beta_{0i}) \beta_i \right] \right\}\]

(20)

Table 1 summarizes the sensitivity of body forces and torques to thrust magnitude and moment arm components. Table 2 summarizes the sensitivity of body forces and torques to the azimuth and elevation angles. Table 3 shows the allowable azimuth and elevation ranges, excluding pluming and hardware constraints.

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<th>(l_i)</th>
<th>(d_i)</th>
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<td>(s \beta_{0i})</td>
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Table 1. Sensitivity of body forces and torques to thrust magnitude and moment arm components.
**Table 2. Sensitivity of body forces and torques to engine pod azimuth and elevation angles.**

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<tr>
<td>$+\tau_y$</td>
<td>0 $&lt; \alpha_i &lt; \pi$</td>
</tr>
<tr>
<td>$-\tau_y$</td>
<td>$\pi &lt; \alpha_i &lt; 2\pi$</td>
</tr>
<tr>
<td>$-\tau_z$</td>
<td>$\pi &lt; \alpha_i &lt; 2\pi$</td>
</tr>
<tr>
<td>$+\tau_z$</td>
<td>0 $&lt; \alpha_i &lt; \pi$</td>
</tr>
<tr>
<td>$-\tau_z$</td>
<td>$\pi &lt; \alpha_i &lt; 2\pi$</td>
</tr>
</tbody>
</table>

**Table 3. Allowable azimuth and elevation ranges, excluding plumbing and hardware constraints.**

<table>
<thead>
<tr>
<th>Azimuth angle</th>
<th>Elevation angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+\tau_x$</td>
<td>0 $&lt; \alpha_i &lt; \pi$</td>
</tr>
<tr>
<td>$+\tau_y$</td>
<td>0 $&lt; \alpha_i &lt; \pi$</td>
</tr>
<tr>
<td>$-\tau_y$</td>
<td>$\pi &lt; \alpha_i &lt; 2\pi$</td>
</tr>
<tr>
<td>$-\tau_z$</td>
<td>$\pi &lt; \alpha_i &lt; 2\pi$</td>
</tr>
<tr>
<td>$+\tau_z$</td>
<td>0 $&lt; \alpha_i &lt; \pi$</td>
</tr>
<tr>
<td>$-\tau_z$</td>
<td>$\pi &lt; \alpha_i &lt; 2\pi$</td>
</tr>
</tbody>
</table>

Figure 19 shows sensitivity plots of body forces and torques with equal ranges of azimuth and elevation angles, when $\alpha_1 = \alpha_2$ in the range $[-20 \ldots +100]$ deg and $\beta_1 = \beta_2$ in the range $[-40 \ldots +40]$ deg. Figure 20 shows sensitivity plots of body forces and torques with different ranges of azimuth and elevation angles, when $\alpha_1 = 0$; $\alpha_2 = 0$; $\beta_1 = [-40:1:40]^{\text{deg}}$; $\beta_2 = [-40:1:40]^{\text{deg}}$; $\beta_1 = 0$; $\beta_2 = 0$. Figure 21 shows sensitivity plots of body forces and torques with different ranges of azimuth and elevation angles, when $\alpha_1 = [-20:1:100]^{\text{deg}}$; $\alpha_2 = [-20:1:100]^{\text{deg}}$; $\beta_1 = 0$; $\beta_2 = 0$. Figure 22 shows sensitivity plots of body forces and torques with different ranges of azimuth and elevation angles, when $\alpha_1 = \alpha_2 = [-20 \ldots +100]$ deg, and $\beta_1 = -\beta_2 = [-40 \ldots +40]$ deg.
Figure 19. Sensitivity plots of body forces and torques with equal ranges of azimuth and elevation angles.
Figure 20. Sensitivity plots of body forces and torques with different ranges of azimuth and elevation angles (azimuth equal to zero).
Figure 21. Sensitivity plots of body forces and torques with different ranges of azimuth and elevation angles (elevation equal to zero).
Figure 22. Sensitivity plots of body forces and torques with different ranges of azimuth and elevation angles (left azimuth equal to right azimuth, left elevation equal to right elevation).
**Interface between output of Navigation Simulation Tools and Attitude Control Simulation Tools.**

Under the assumption of rigid spacecraft, with articulations locked so that the thrust vector always coincides with one of the body axes, it is possible to assess the impact of the trajectory design on the attitude control strategy.

Figures 23 to Figure 28 show representative thrust vector data extracted from a trajectory segment of a spiral-in maneuver around Ganymede. Figure 23 shows the thrust in-plane angle and the thrust out-of-plane angle, assuming a keplerian osculating orbit at each time instant of thrust application. Figure 24 shows the thrust magnitude and the inertial thrust X component. Figure 25 shows the magnification of inertial thrust X component and the thrust Y component. Figure 26 shows the thrust Z component and the magnitude of the spacecraft range vector. Figure 27 shows the spacecraft velocity and the three-dimensional plot of the Ganymede spiral-in trajectory. Figure 28 shows the three-dimensional plot of Ganymede spiral-in with the computed orbiting reference frame. These plots show that the thrust is kept constant for about 1 hour. The thrusting period is about 24 hours.

Figure 29 shows the computed thrust magnitude on the right pod and the computed thrust magnitude on the left pod. Figure 30 shows the computed right pod azimuth angle and the computed right pod elevation angle. Figure 31 shows the computed left pod azimuth angle and the computed left pod elevation angle. These figures also show that, during this particular spiral-in phase, the ion engine thrust needs to be modulated up to 75%. This may indicate the need for throttling the ion engines. Also, the ion engine gimbal angles need to cover $2\pi$ rotation in 24 hours. We would like to point out that the results of this AACS study have prompted an immediate change of navigation strategy, with more benign implications on the gimbal design.

![Figure 23. Left: Thrust in-plane angle. Right: Thrust out-of-plane angle.](image-url)
Period is about 24 hours.

Figure 24. Left: Thrust magnitude. Right: Inertial thrust X component.

Figure 25. Left: Magnification of Inertial thrust X component. Right: Thrust Y component.

Figure 26. Left: Thrust Z component. Right: Spacecraft Range.
Figure 27. Left: Spacecraft Velocity. Right: Three-dimensional plot of Ganymede spiral-in.

Figure 28. Three-dimensional plot of Ganymede spiral-in with computed orbiting reference frame.
Figure 29. Left: Computed thrust magnitude on right pod. Right: Computed thrust magnitude on left pod.

Figure 30. Left: Computed right pod azimuth angle. Right: Computed right pod elevation angle.

Figure 31. Left: Computed left pod azimuth angle. Right: Computed left pod elevation angle.
Conclusions

The current JIMO Attitude and Articulation design reveals a significant conceptual problem in that the thrust vectoring capability needed for trajectory guidance is intimately coupled with the torque vectoring capability needed for attitude control. This problem arises because the electric thrusters for attitude control are placed on the same articulated pods where the propulsion ion engines are also located. Therefore, any maneuver intended to implement an attitude correction will affect the trajectory, and vice versa. Options for attitude thruster technology and placement to mitigate this coupling are under trade efforts at this time. One solution is a set of thrusters with adequate continuing authority for attitude control on the spacecraft bus, thereby decoupling the attitude control from the trajectory thrust vector control. In order to quantify this complex coupling, a sensitivity analysis was carried out as a first step to solving the control allocation problem. The thrust vector control sensitivity study indicates that we need to continue developing the modeling and simulation tools to appropriately include: 1) coupling of attitude dynamics and orbital dynamics during low thrust trajectories; 2) capability of simulating high fidelity articulation dynamics during the maneuvers.

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References