On the Ionospheric Impact of Recent Storm Events on Satellite-Based Augmentation Systems in Middle and Low-Latitude Sectors

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ABSTRACT

The ionospheric correction algorithms have been characterized extensively for the mid-latitude region of the ionosphere where benign conditions usually exist. The United States Federal Aviation Administration's (FAA) Wide Area Augmentation System (WAAS) for civil aircraft navigation is focused primarily on the Conterminous United States (CONUS). Other Satellite-Based Augmentation Systems (SBAS) include the European Geostationary Navigation Overlay Service (EGNOS) and the Japanese Global Navigation Satellite System (MSAS). Researchers are facing a more serious challenge in addressing the ionospheric impact on navigation using SBAS in other parts of the world such as the South American region or India. At equatorial latitudes, geophysical conditions lead to the so-called Appleton-Hartree (equatorial) anomaly phenomenon, which results in significantly larger ionospheric range delays and range delay spatial gradients than is observed in the CONUS or European sectors.

In this paper, we use GPS measurements of geomagnetic storm days to perform a quantitative assessment of WAAS-type ionospheric correction algorithms in other parts of the world such as the low-latitude Brazil and mid-latitude Europe. For the study, we access a world-wide network of 400+ dual-frequency GPS receivers. The

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network includes: 1) the Continuously Operating Reference Sites (CORS) in the United States; 2) stations in and near South America as part of the Brazilian Network of Continuous Monitoring of GPS (RBMC), operated by the Brazilian Institute of Geography and Statistics (IGBE); and (3) European and CONUS sites included in the International GPS Service (IGS) global network. Data sets have been selected to include 18 quiet and storm days upon which the CONUS threat model is based on. To provide ground-truth and calibrate GPS receiver and transmitter inter-frequency biases, we processed the GPS data using Global Ionospheric Mapping (GIM) software developed at the Jet Propulsion Laboratory to compute calibrated high resolution observations of ionospheric total electron content (TEC).

In this research, we investigated major storm events from the last few years and evaluated their impact on WAAS ionospheric model performance in Brazil, Europe and CONUS. These storms included the "worse-case" CONUS storms such as those on July 15, 2000 and March 31, 2001, and investigated their impact on SBAS in Europe and Brazil. Results indicated that lesser known storms such as the one on April 5, 2000 had a more significant impact in Europe than a near worse-case storm in CONUS. Furthermore, we provided further evidence that there is little difference between quiet and storm time behavior of the planar fit residuals in Brazil. We found that the Brazilian planar fit residuals are 2 to 4 times higher (RMS) than those for Europe and CONUS.

INTRODUCTION

The Wide-Area Augmentation System (WAAS) developed for the Conterminous United States (CONUS) is only one of the several Space-Based Augmentation Systems (SBAS) under consideration worldwide. Other SBAS developments are under way in Europe, Japan, India and Brazil.

Relatively benign ionospheric conditions in the mid-latitude CONUS region are compatible with accurate ionospheric range corrections for WAAS. Providing ionospheric corrections for Brazil is significantly more challenging, since ionospheric range delays and range delay spatial gradients are among the largest in the world even in the absence of ionospheric storms (during infrequent ionospheric storms, even mid-latitude regions present challenging conditions). In summary, the ionosphere in the Brazilian sector shows significantly different behavior from that of the mid-latitude sector.

The ionosphere has been extensively studied to support WAAS at the CONUS sector. The published literature discussing ionospheric corrections for WAAS in the CONUS is extensive; see e.g. Enge et al., [1996], WAAS MOPS [1999], Walter et al., [2000] and Sparks et al., [2002]. Various alternative ionospheric correction algorithms have been presented by e.g., Hansen et al., [1997], Sparks et al., [2000] and Blanch et al., [2002]. A potential application of WAAS algorithms to Brazil has recently been investigated by Klobuchar et al., [2001] using simulated data. The authors showed large differences between modeled slant TEC from LowLat model and values interpolated from ionospheric grid points. Klobuchar et al [2002] revealed differences between modeled slant TEC values and those obtained from interpolating from IGPs, can be up to 14 meters. The temporal and spatial variability of the low-latitude ionosphere was studied in the context of ionospheric storms by Dehel and Corbelli [2002] and Fedrizzi et al., [2001] using a network of dual-frequency GPS receivers in Brazil. Lejeune et al. [2002] investigated spatial decorrelation errors in the equatorial region which were found to be an order of magnitude larger than those found in the CONUS, computed using LowLat simulations of the equatorial ionosphere. Furthermore, several meter level errors were obtained from the standard slant-to-vertical conversion technique. In a paper by Doherty et al [2002], scintillation, depletion and large gradient TEC issues were discussed in the equatorial region. Investigating the possible application of the current WAAS algorithm using GPS-derived TEC is the natural progression of the previous studies and therefore the main focus of our subsequent papers.

In Komjathy et al., [2002b], we assessed the WAAS's planar fit algorithm in the equatorial region where the spatial gradients and the absolute slant TEC are known to be the highest in the world. We found that in Brazil the dominant error source for the WAAS planar fit algorithm is the inherent spatial variability of the equatorial ionosphere with ionospheric slant range delay residuals as high as 15 meters and root-mean square (RMS) residuals for the quiet day of 1.9 meters. This compares to a maximum residual of 2 meters in CONUS, and 0.5 meter RMS. We revealed that ionospheric gradients in Brazil are on average at the level of 2 meters over 100 km. Contrary to results obtained for CONUS, we discovered that a major ionospheric storm (March 31, 2001) had a small impact on the planar fit residuals in Brazil.

As a subsequent step, in Komjathy et al., [2003], we have compared the performance of the WAAS ionospheric planar-fit and the new quadratic-fit approaches in the low latitude region where the temporal and spatial variation of the ionosphere is the highest in the world. We found slant ionospheric range delays up to 55 meters for Brazil. For the quiet day, we obtained WAAS planar fit residuals less than 13 meters (1.42 meter RMS) in Brazil. When using a quadratic approach we obtained less than 9 meter (1.29 meter RMS) slant residuals. For a storm day, the planar fit residuals were as large as 25 meters (2.16 meter RMS) whereas the quadratic fit resulted in less than 11 meters (1.69 meter RMS). When considering all quiet and storm days, we found that the quadratic fit approach reduced the residuals by an average of 20 percent. We found that the Brazil planar fit residuals are a factor of three larger than those for CONUS. The quadratic fit residuals are also increased by at least a factor of 2. The main contributing factors of the large residuals in Brazil appear to be the high spatial variability of the spatial gradients, the large absolute slant TEC and the errors in the ionospheric mapping function.

In this research, we investigated a larger and more comprehensive data set including the same major storm events that the WAAS CONUS threat model is based on. To establish a background for this work, we first review the estimation method used to solve for inter-frequency biases (nuisance parameters) in the GPS satellites and receivers using a global network of 230 GPS sites in order to provide "ground truth" data for the analysis. Subsequently, we briefly review the standard WAAS planar fit algorithm used to estimate the vertical ionospheric range delay at fixed latitude/longitude locations known as ionospheric grid points (IGPs). We examine the implications of using the currently adopted WAAS algorithm in Brazil, Europe and CONUS to compare ionospheric range residuals.

GIM BIAS ESTIMATION STRATEGY

To provide ground-truth, we used the Global Ionospheric Mapping (GIM) software developed at the NASA Jet Propulsion Laboratory [Mannucci et al., 1998] to compute high precision slant ionospheric delay by removing the satellite and receiver differential biases from the ionospheric observables, generated from carrier-phase data adjusted to match the ionospheric delay based on dual-frequency pseudoranges. The estimation of the satellite and receiver biases has been published before but we will review it here briefly.

Ionospheric measurements from a GPS receiver can be modeled with the well-known single-shell ionospheric model using the following observation equation [see e.g. Mannucci et al., 1999 and Komjathy et al., 2002a]:

$$TEC = M(h,E)\sum_i C_i B_i(lat,lon) + b_r + b_s \,, \qquad (1)$$

where

- TEC is the slant Total Electron Content measured by the linear combination of the GPS dual-frequency carrier phase and pseudorange ionospheric observables, typically expressed in TEC units. One TEC Unit (10^{16}electron/m^2) corresponds to about 0.163 meter ionospheric delay at the L1 frequency,
- $M(h,E)$ is the thin-shell mapping function for ionospheric shell height h and satellite elevation angle E (for the definition of the thin-shell geometric mapping function see e.g. Mannucci et al., [1998] or Komjathy [1997],
- $B_i(lat, lon)$ are horizontal basis functions (based on, for example, bicubic splines or bilinear interpolants) evaluated at the ionospheric pierce point (IPP) – the intersection of the ray path of a signal propagating from the satellite to the receiver with a thin spherical shell – located at latitude *lat* and longitude *lon* on the thin shell,
- C_i are basis function coefficients (real numbers),
- b_r, b_s are the satellite and receiver differential biases, assumed constant over periods of 24 hours or more.

The dependence of vertical TEC on latitude and longitude is parameterized as a linear combination of the two-dimensional basis functions B_i which are functions of solar-geomagnetic longitude and latitude [Mannucci et al., 1998] (We note that the summation in Equation 1 is over all basis functions B_i). Using the carrier phase-leveled ionospheric GPS observables, a Kalman filter simultaneously solves for the instrumental biases and the coefficients C_i which are allowed to vary in time as a random walk stochastic process [Iijima et al., 1999]. The basis functions currently used are based on a bicubic spline technique developed at JPL [Lawson, 1984].

Although the main focus of this research is the comparison between CONUS, Europe and Brazilian sectors, we decided to use a global network of some 230 stations to solve for high precision satellite and receiver differential biases that are used to correct the measurements. Research has shown that the most reliable satellite bias estimates can be achieved when using the data strength of a global network of GPS receivers instead of regional GPS networks [Komjathy, 1997]. We note that the WAAS system itself uses a similar estimation scheme for biases applied over the regional WAAS network.

WAAS PLANAR FIT IONOSPHERIC MODEL

In the currently implemented WAAS ionospheric real-time correction algorithm, the vertical ionospheric delay is estimated at each ionospheric grid point (IGP) by constructing a planar fit to a set of (bias-corrected) slant measurements projected to vertical:

$$TEC = M(h,E)\left[a_0 + a_1 d_E + a_2 d_N\right] , \qquad (2)$$

where

a_0, a_1, a_2	are the planar fit parameters,
d_E, d_N	are the distances from the IGP to the IPP in the eastern and northern directions, respectively.

Each least squares fit includes all IPPs that lie within a minimum fit radius surrounding the IGP. If the number of IPPs within this minimum radius is less than N_{min}, the fit radius R_{fit} is extended until it encompasses N_{min} points. In this study we do not tabulate data when the fit radius is increased to its maximum value of R_{max} without having reached N_{min} points. R_{max} was chosen to be 2100 km which is the value in the current WAAS implementation. We chose this value to take into consideration of the spacing of reference stations in Brazil and the number of parameters we need to estimate for model comparisons.

In our WAAS estimation scheme (see Equation 2), we did not solve for the satellite and receiver differential biases. Instead, we used the GIM approach, outlined in Equation 1 to solve for high precision differential biases and calibrated the ionospheric range measurements before applying Equation 2. This is similar to the approach used in the WAAS.

DATA ANALYSIS STRATEGY

In our data analysis, we treated every IPP data point as if it were collocated with a WAAS IGP (so-called "pseudo IGP" approach). Subsequently, we applied the WAAS planar fit ionospheric model algorithm to estimate the vertical ionospheric delays at each of these IPPs, treated as pseudo IGP values. Starting with the set of measurements that contributed to the planar fit, we then computed the residual difference between the slant measurements and the estimated slant delays based on the planar fit, projecting the vertical TEC from the planar estimate into the line-of-site using the WAAS thin-shell obliquity factor. This residuals analysis provides a measure of the performance of the planar fit algorithm in reproducing slant TEC for the user.

DATA SETS

For our test data set, we chose a quiet and a storm days, respectively, using GPS receivers from the Continuously Operating Reference Stations (CORS) network, maintained by the US National Geodetic Survey [CORS, 2002], the International GPS Service [IGS, 2003], and the Brazilian Network for Continuous Monitoring of GPS (RBMC).

In Figure 1 we show the global distribution of the GPS reference stations for a typical day of the data set. The small filled circles represent the 230 sites that were used to provide unbiased line-of-sight TEC ground-truth data. The larger circles in blue indicate the CONUS, European and Brazilian sectors for which stations were used for the data analysis.

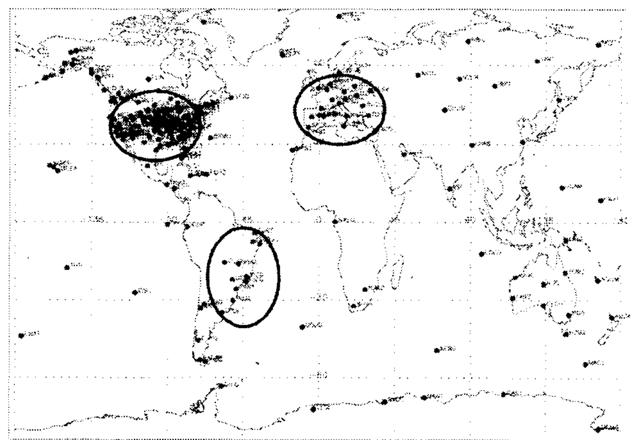

Figure 1. Network of CORS, IGS and RBMC stations processed for the 18 day focus period.

The 18 days we selected coincide with the focus period for which the threat model for CONUS was derived. In Figure 2, we show the 3-hour Kp indices for a period between January 2000 and September 2002 encompassing major storm evens.

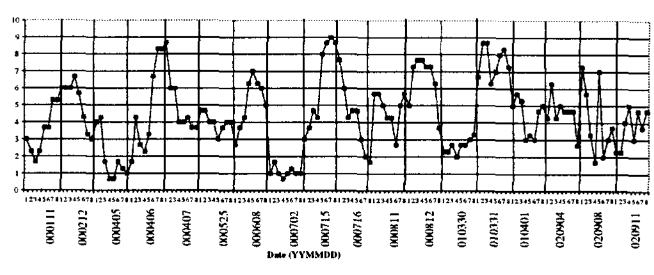

Figure 2. Behavior of Kp index during the selected 18 days between January 2000 and September 2002.

ANALYSIS OF RESULTS

First we calibrated the satellites and receiver differential biases using the GIM method and data from the CONUS, European and South American sectors. We selected a quiet (April 5, 2000) and two subsequent storm days (April 6-7, 2000) to illustrate residuals using the traditional planar fit approach.

Comparison of European and WAAS CONUS residuals for quiet day. For the comparison we used planar fit residuals between 30 and 60 degrees geomagnetic latitudes making sure that we used about the same number of observations covering the same geomagnetic latitude band. In Figure 3, we plotted the European and CONUS planar fit residuals in the slant domain for the quiet day as a function of local time. Note that to compute residuals, the fitted vertical TEC value at an IPP location was converted to slant and differenced with the slant TEC measurement. It is clear that the European and CONUS residuals are very similar for the quiet day of April 5, 2000. For the quiet day, we obtained WAAS planar fit residuals less than 7.2 meters (0.52 meter RMS). For Europe, we achieved less than 9.8 meter ionospheric residuals (0.66 meter RMS). In the figure, we also displayed the 68, 95 and 99.9 percentiles both for Europe and CONUS approaches. It is shown that the WAAS CONUS has smaller residuals at the tails of the distribution.

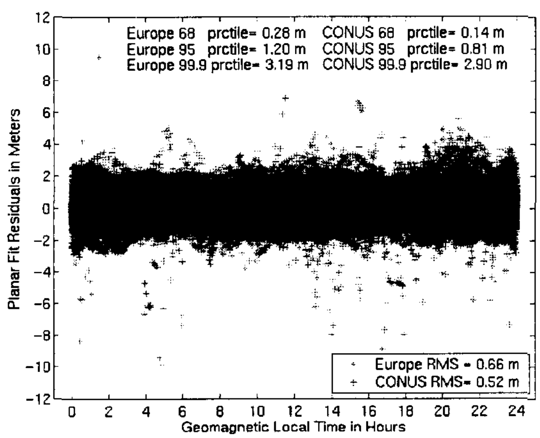

Figure 3. Planar fit residuals for WAAS CONUS and Europe for the *quiet* day of April 5, 2000.

Comparison of European and WAAS CONUS residuals for storm days. In Figure 4, we show the planar fit residuals for the subsequent storm day both for the European and CONUS regions. The WAAS CONUS residuals demonstrate that the storm impacted the ionosphere during day-time hours. On the other hand, it is indicated that the storm impacted the European sector during post-sunset hours. For the entire day, the European fit RMS residuals resulted in 0.9 meter compared to the 0.68 meter for the WAAS CONUS fit. It appears that for April 6th, the CONUS fit has smaller residuals at the tails of the distribution.

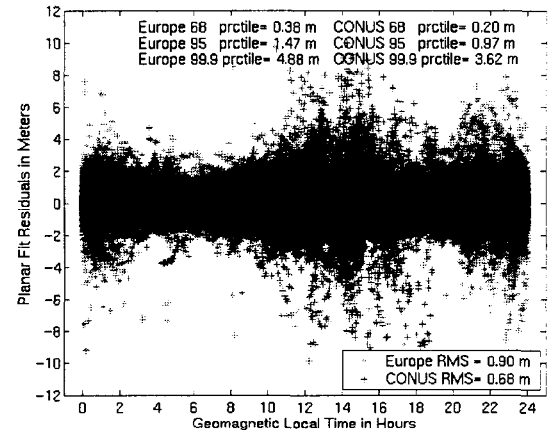

Figure 4. Planar fit residuals for WAAS CONUS and Europe for the *storm* day of April 6, 2000.

Figure 5 indicates an opposite trend. The subsequent storm day appears to have impacted the WAAS CONUS sector more than the European sector. We demonstrate that in the CONUS sector, the temporal variability of the ionosphere is high throughout the day. For the storm day, we obtained WAAS CONUS planar fit residuals of 0.93 meter RMS compared to the corresponding European value of 0.61 meter (RMS). The percentiles indicate that the tails of the residual distribution is now wider for the CONUS sector.

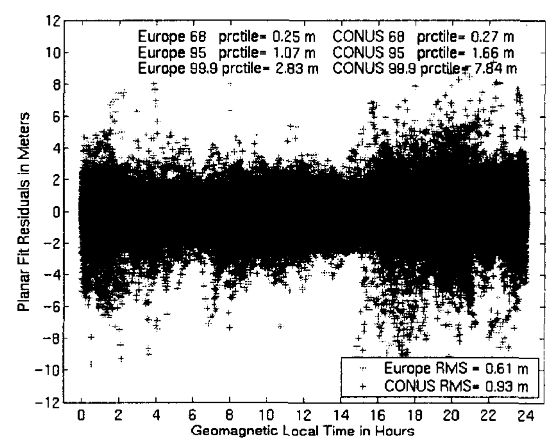

Figure 5. Planar fit residuals for WAAS CONUS and Europe for the *subsequent storm* day of April 7, 2000.

Comparison of Brazilian residuals between quiet and storm days. In Figure 6, we plotted the Brazilian planar fit residuals for the quiet (April 5) and and storm days (April 6). We demonstrate that during day-time hours we obtained planar fit residuals at the 8 meter level. During post-sunset hours we obtained residuals as large as 20 meters. It is striking that the quiet and storm time equatorial ionospheric planar fit residuals behave similarly. We do not see a major impact of the storm compared to a quiet time behavior. This finding corresponds to our earlier finding where we investigated the March 31, 2000 storm event. The RMS of planar fit residuals are somewhat higher for the storm days (2.4 meter) compared to the quiet day (2.07 meter). The distribution of the residuals is wider for the storm day for all three percentiles we investigated.

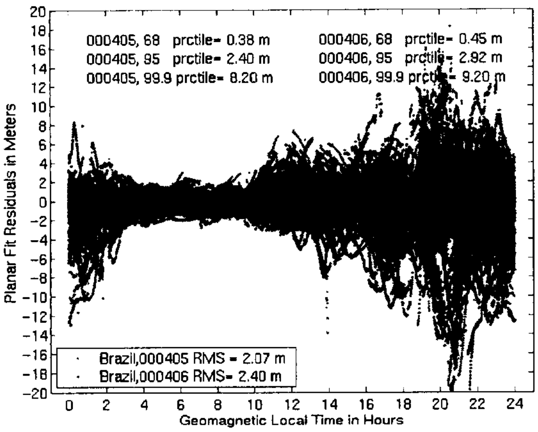

Figure 6. Brazilian planar fit residuals for quiet and storm days.

Figure 7 displays the Brazilian planar fit residuals as a function of elevation angles. The slight difference in the RMS of residuals between the quiet and storm days in Brazil can be explained by the more complex spatial structures in the ionosphere. As a consequence, large residuals even at high elevation angles are seen during the storm days.

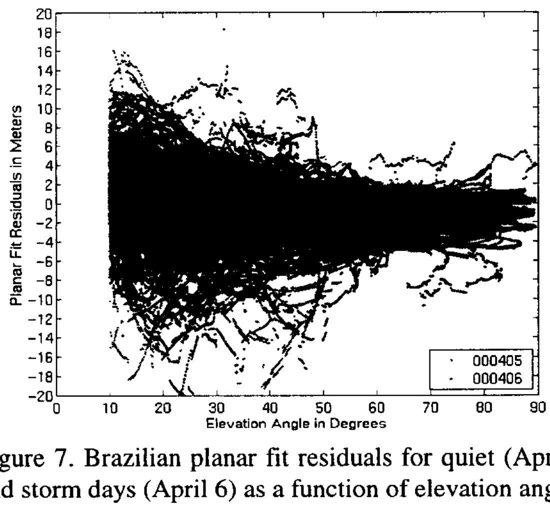

Figure 7. Brazilian planar fit residuals for quiet (April 5) and storm days (April 6) as a function of elevation angle.

In Figure 8, a summary plot is presented showing the overall RMS of planar fit residuals separately for CONUS, Europe and Brazilian sectors and for different days. It is demonstrated that 4 out 18 quiet and storm days, the European RMS of planar fit residuals were higher than those for the WAAS CONUS residuals. We also found that the Brazilian planar fit residuals are at the 3 meter (RMS) regardless if it is a quiet or storm day. It is also demonstrated that the Brazilian planar fit residuals are on average a factor a 3 larger than the WAAS CONUS or European residuals.

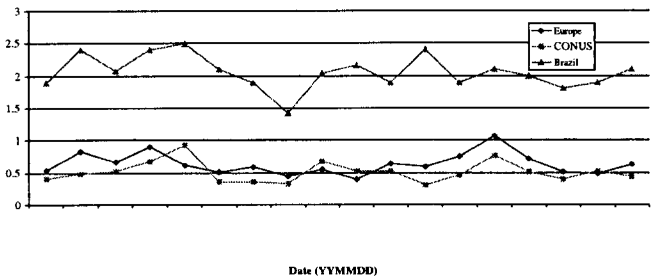

Figure 8. Summary of results for CONUS, Europe and Brazil data processing.

Implications for LNAV/VNAV availability in the low latitude sector. In this research, we have investigated ionospheric range errors in the low-latitude Brazilian sector for quiet and storm days. Based on this data set, we were able to achieve 1.4 meter RMS ionospheric slant delay planer fit residuals for a quiet and 2.5 meters for storm day. Based on our current and earlier results, it appears that the planar fit residuals in Brazil are a factor of 3 to 4 larger than in the CONUS or Europe.

At low latitudes, the GIVEs must be increased to cover the larger ionospheric range errors expected. Increased planar fit residuals by a factor of three are likely to result in a substantial number of GIVEs above 6 meters. Due to GIVE quantization in the broadcast message, computed GIVEs above 6 meters are transmitted as 15-meter bounds to the user. It is clear that LNAV/VNAV service will be unavailable if several of the user's satellite links are associated with GIVEs of 15 meters or more.

We expect that the WAAS planar fit algorithm applied to Brazil will result in significantly reduced availability of LNAV/VNAV service, particularly near solar maximum during daytime and evenings.

CONCLUSIONS AND FUTURE RESEARCH

In this paper, we have compared the performance of the WAAS CONUS, European and Brazilian ionospheric planar-fit residuals. In the case of the low latitude region, it is widely accepted that the temporal and spatial variation of the ionosphere is here the highest in the world. We used data from the network of dual-frequency GPS receivers using IGS, CORS and RBMC stations in CONUS, Europe and Brazil. Unbiased line-of-site TEC ground-truth data were generated using JPL's Global Ionospheric Mapping (GIM) software. Using the truth data, the planar fit was evaluated by treating each observation as representing a WAAS ionospheric grid point (IGP) and computing the planar fit estimates for that IGP after excluding it from the fit.

In the comparison of the European and WAAS CONUS planar fit residuals, we found that the European ionosphere shows as much storm-time variations as it is displayed in the CONUS region. 4 out of 18 cases, we registered larger RMS of planar fit residuals for Europe than for CONUS.

We investigated 18 days of quiet and storm-time data from Brazil. From this larger data set, we confirmed that there is little difference between quiet and storm time behavior of the planar fit residuals. We also found that on average the Brazilian planar fit residuals are 2 to 4 times higher (RMS) than those for the CONUS or Europe. The main contributing factors of the large residuals in Brazil appear to be the high spatial variability of the spatial gradients, the large absolute slant TEC and the errors in the ionospheric mapping function. This will likely result in a substantial number of GIVEs about 6 meters, significantly reducing LNAV/VNAV availability service in Brazil.

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