ABSTRACT - This paper is the first comprehensive spacecraft formation flying guidance (FFG) survey. Here by the term guidance we mean both path planning (i.e., reference trajectory generation) and optimal, open loop control design. FFG naturally divides into two areas: Deep Space (DS), in which relative spacecraft dynamics reduce to double integrator form, and Planetary Orbital Environments (POE), in which they do not (e.g. libration point formations). Both areas consider optimal formation reconfigurations. In addition, DS FFG addresses optimal $u,v$-coverages for multiple spacecraft interferometers and rest-to-rest rotations. The main focus of the POE literature, however, is “passive apertures.” These are periodic and fuel-efficient relative spacecraft trajectories that accomplish scientific objectives (e.g. synthesizing an aperture).

1 - INTRODUCTION

In 1969, data from European Space Research Organization and US and Soviet satellites were correlated to study how large solar flares interacted with the Earth—thereby achieving the first contemporaneous spatial sampling by a group of separated spacecraft [Mann 69]. Less than a decade later, Labeyrie of CERGA proposed forming a stellar interferometer from free-flying telescopes [Labe 78]. Today, there are dozens of missions either flying, under development or proposed [Bris 00] that use spacecraft flying in formation. For example: Terrestrial Planet Finder (TPF) and Darwin will look for extra-solar Earth-like planets [Beic 99; Frid 00]; XEUS, the Constellation X-Ray Mission and MAXIM will explore high-energy astrophysical sources with unequaled resolution [Batt 00]; and Grace, EO-1/L-7 and CloudSat/Picasso-Cena, looking homeward, will study our own Earth [Kirs 01; Folt 02; Keen 01].

Previous definitions of formation flying have not clearly differentiated it from constellations. We define formation flying as a set of more than one spacecraft whose dynamic states are coupled through a common control law. In particular, at least one member of the set must 1) track a desired state relative to another member, and 2) the tracking control law must at the minimum depend upon the state of this other member. The second point is critical. For example, even though relative positions are being actively maintained, GPS is a constellation since orbit corrections only require an individual satellite’s position and velocity (state).

This paper is the first comprehensive survey of the guidance aspects of spacecraft formation flying. Formation flying guidance (FFG) is defined as the generation of any reference trajectories used as an input for a formation member’s relative state tracking control law. This FFG definition includes open-loop control design (i.e., an optimal control profile that only depends on time and initial conditions).

The FFG literature can be divided into two main areas based on the ambient dynamic environment. In Deep Space (DS) relative spacecraft dynamics reduce to the standard double integrator form (i.e., no state dependent forces in open loop) [Scha 02]. The second main area is Planetary Orbital Environments (POE), where spacecraft have significant orbital dynamics.

Both DS and POE FFG consider optimal formation reconfigurations. The DS literature also addresses formation rotations and planning $u,v$-coverages for multiple spacecraft interferometers (or MSIs). In a POE, the dynamics are the dominant consideration. Since tracking

[1]Recently, [Bout 02] made a similar DS/POE distinction, instead naming it Inertial/Central Potential.
arbitrary trajectories generally requires prohibitive amounts of fuel,\(^2,3\) the POE literature focuses on developing fuel-efficient, periodic relative spacecraft trajectories that are useful for synthesizing scientific instruments. As many of these trajectories are thrust-free and are used to form synthetic apertures, these trajectories are referred to as \textit{passive apertures}.

Due to the dynamical environment inherent to POE guidance, this area has a larger number of papers. However, this imbalance is a matter of perspective—when one also considers the research in formation flying control, the literature is more evenly divided between DS and POE. It is worthwhile to note that, due to its mission focus, JPL has been the most active contributor to the DS FFG area. For example, Wang and Hadaegh [Wang 99] first addressed formation reconfiguration, precisely defining it (see §2) and reducing the problem to a study of permutation groups. Also, in a series of papers, Beard, his students and Hadaegh analyzed DS formation rotations and highlighted the need to not only minimize fuel use, but also to \textit{balance} fuel use across a formation.

Finally, spacecraft rendezvous guidance is also FFG, but has already been surveyed [Jeze 91].\(^4\)

\section{DEEP SPACE FFG}

DS FFG is simplified by the fact that arbitrary rigid formations can be maintained with no fuel penalty. Optimal u,v-coverages and reconfigurations then reduce to Traveling Salesmen problems. Since formation rotations can be used in u,v-coverages, optimal methods for rotating a rigid formation are also studied.

MSI's interfere the electromagnetic waves from an observational target collected by spacecraft at various relative positions \((u,v\)-points).\(^5\) Given a desired \(u,v\)-set, algorithms for finding fuel-optimal \(u,v\)-coverages (i.e., paths through the corresponding relative spacecraft positions) have been developed [Kong 98; Bail 02]. [Mesb 01] derives optimal MSI mission-fuel/mission-time trade-offs assuming each target's \(u,v\)-coverage is the same. If targets are close as compared to the extent of their \(u,v\)-sets, then the optimal \(u,v\)-coverage for multiple targets is not a series of individual target-optimal \(u,v\)-coverages, but a target-combined \(u,v\)-coverage [Bail 02].

One method for sampling a \(u,v\)-set is to rotate the entire formation. Given an axis of rotation and angle to rotate a rigid formation through, authors have found the fuel-optimal point about which to rotate the formation [Bear 98; Bear 99b; Bear 99a; Bear 01]. The objective function weights total formation fuel consumption as well as \textit{unbalanced} fuel consumption—it is vital not to deplete one spacecraft's fuel before the others.

In a rotating formation, spacecraft on the outside of the formation will consume more fuel since they are traveling faster. It may then be necessary to periodically switch spacecraft positions within the formation to balance their fuel consumption. A reconfiguration is essentially a reassignment of spacecraft positions within a given formation geometry. From [Wang 99], let \(\mathcal{I}\) be a set of spacecraft identifiers (e.g. \(\{a, b\}\)) and \(\mathcal{R}^d\) be a set of time-varying desired spacecraft trajectories (e.g. \(\{r_1^d(t), r_2^d(t)\}\)). A \textit{configuration} is a mapping \(C : \mathcal{I} \rightarrow \mathcal{R}^d\). A \textit{reconfiguration} is a change of this mapping, including adding and deleting elements from each set (e.g. merging two sub-formations). \textit{Reconfiguration trajectories} are used to move spacecraft to new desired trajectories. This definition applies to POE formations as well. [Wang 99]

\(^2\)For example, [Kong 99] shows that spacecraft placed in an arbitrarily oriented, 20 km diameter circular formation in a geostationary orbit require a \(\Delta v\) of approximately 7 m/s per orbit (assuming five year lifetime).

\(^3\)Exceptions are XEUS and the MSI described in [Stac 84a]. Both missions would use space station refueling.

\(^4\)Traditional constellation designs (e.g. Walker) can be used as a reference trajectory for Mean Constellation Control (MCC) [Lamy 93]. MCC first fits a constellation template to spacecraft positions. Then spacecraft track the resulting desired "mean" locations. Since spacecraft states are coupled through the fitting step, MCC is formation flying. Traditional constellation design, however, has already been surveyed (e.g. [Lans 98]). See [Guzm 02; Schi 00] and references therein for tetrahedral constellation design.

\(^5\)In synthetic aperture imaging, spacecraft are generally restricted to a plane, and the critical variable is not the physical positions, \((x_1, y_1)\) and \((x_2, y_2)\), but relative positions. Scaling by the wavelength observed \((\lambda)\) and the distance to the target \((z)\) results in \((u,v) = (x_1 - x_2, y_1 - y_2)/(z\lambda)\). A \(u,v\)-set is then a set of ordered pairs representing planar relative spacecraft locations. A \(u,v\)-coverage is an ordered \(u,v\)-set. See [Mesb 01].
considers fuel-optimal reconfigurations between rigid formations. Given a new configuration, [Li 00; Sing 01] find optimal, collision avoidance-constrained reconfiguration trajectories.

3 - PLANETARY ORBITAL ENVIRONMENT FFG
Since the dynamics are significant, POE FFG concentrates on finding passive apertures. The passive aperture papers divide into three categories: 1) passive apertures designed via linearized models, 2) passive apertures designed via nonlinear models and 3) passive apertures that mitigate disturbances, whether designed via linear or nonlinear models. POE FFG also considers reconfigurations.

For reconfiguration, bidding algorithms and nonlinear optimization have been used to determine the new configuration (see the discussion on reconfigurations in the DS FFG section for definitions) [Mort 99; Inal 00]. Authors have also considered a more general problem where there may be a number of final formations that satisfy the mission constraints (i.e., there may be multiple $R^d$'s to choose from) [Yang 01]. In this case, there are three stages to optimize: 1) the optimal reconfiguration trajectory to move a spacecraft from its current trajectory to a new desired trajectory, 2) the optimal assignment of new desired trajectories to individual spacecraft, and 3) the optimal set of new trajectories that satisfy mission objectives. [Till 01] and [Camp 02] address Steps 1 and 2. Linear optimal control methods [Wang 99; Kong 01; D’Soo 02; Till 02], and Lambert’s solution and Gauss’ variation of parameters equations [Foit 98; Vada 99; Mail 00; Scha 01] have been applied to Step 1. Also for Step 1, [Rich 02] includes collision avoidance and thruster plume impingment constraints, and [Mila 01] uses nonlinear programming.

The most common linear passive apertures are thrust-free, periodic solutions to the Hill-Clohessy-Wiltshire (HCW) Equations, referred to in [Kong 99] as Free Elliptical Trajectories (or FETs). [Kong 99; Sabo 01; Alf 00b; Koga 01; Swin 01] emphasize two particular types of FETs: the circular FET (CFET), and the circular-projection FET (CPFET). The CPFET has elliptical relative orbits$^6$ that project circles onto a plane perpendicular to reference orbit$^6$ plane. The interferometric cartwheel FET is useful for synthetic aperture radar [Mass 01].

The FETs rotate with the local-vertical, local-horizontal frame and are useful for looking at the Earth. For astronomical targets there are also relative orbits that remain in inertially fixed planes [John 90; DeCo 91]. The relative orbit plane may be arbitrarily oriented, but the eccentricity of the relative orbit depends on the target direction. Also using a linear model, [Inal 02] and [Baoy 02] derive energy-matching-based constraints for relative orbits to exist about an eccentric reference orbit.

Turning to nonlinear models, [Yan 00] derives a similar initial condition constraint for the existence of relative orbits about an eccentric reference orbit, while [Vada 99] numerically searches for relative orbits. The energy-matching method is also used to design formations. First, a point in the reference orbit is selected and spacecraft are put in the desired relative positions. Then their velocities are directed parallel to the reference orbit’s and their velocity magnitudes are selected to match the energy of the reference orbit [Chao 99; Schi 00].

Another common approach, pioneered in [Fall 84], is to expand the formation geometry parameters (e.g. angular extent of formation) in a series based on eccentricity and then select relative orbital elements to eliminate first order terms [Vinc 87; Chic 99; Hugh 00]. Using this approach the CFET is recovered with the addition of a second order term in the series that quantifies the variation from the exactly circular HCW solution [Fall 84; Melt 99]. Even without invoking a series expansion, geometrical arguments can be used to obtain one dimensional MSIs ([Stac 84b]) and constant inter-spacecraft distances for eccentric reference orbits ([Tan 00]).

$^6$We adopt the following terminology to avoid confusing three types of “orbits.” An orbit is the periodic motion of a spacecraft about a planetary center or libration point. A relative orbit is the periodic motion of one spacecraft with respect to a reference point tracing out an orbit. The reference orbit is the orbit of this reference point. A spacecraft may or may not occupy the reference orbit.
Still another approach is to formulate a formation performance metric, such as the number of \( u, v \)-points sampled in one orbit, and numerically search for optimum spacecraft orbital elements [Mall 98; Hugh 99; Hugh 01].

Libration points have also been proposed as low-disturbance parking orbits for MSIs and as another location for passive apertures. In the former case, libration point dynamics do not affect the formation significantly over the time scales involved in formation maneuvers (e.g. 8 hours) [Góme 01]. In the latter case, passive apertures are designed where one relative orbit takes approximately 6 months [Bard 98; Howe 99].

Given the passive apertures based on linear and nonlinear models, authors next explored trajectory robustness in the presence of disturbances [Sabo 01; Alfr 00a; Inal 02]. The disturbance most commonly addressed is the first zonal harmonic \( J_2 \) of the central body potential field, followed by aerodynamic drag and solar pressure. [Sedw 99] applies dimensional analysis to estimate the magnitudes of these disturbances and, in particular, divides the \( J_2 \)-induced motion into bulk and differential parts. The bulk portion may be removed by carefully selecting the semi-major axis [Sedw 99; Poll 99; Koga 01]. Two other strategies to mitigate the effects of \( J_2 \) are 1) to set the secular drift rates of two orbits equal and derive constraints on the orbital elements [Vada 99; Alfr 00a; Hugh 01; Scha 01; Alfr 02a], and 2) to use dynamical system theory to select initial conditions [Koon 01]. [Alfr 01] balances fuel consumption in the presence of \( J_2 \).

Rather than selecting spacecraft orbits to mitigate the effects of disturbances, linear programming methods can be used to find optimal, model-based open loop control profiles for disturbance rejection [Lass 97; Palm 99; Robe 99; Camp 00; Inal 00; Till 01] (see [Mila 01] for nonlinear programming to reject \( J_2 \)). Also using a linear model, a drag compensation strategy for spacecraft with different ballistic coefficients was developed that maximizes the drift time between correction maneuvers [Math 88; Scol 91; Folt 98; Keen 01]. The proposed strategy consists of starting the spacecraft with a greater ballistic coefficient at a larger semi-major axis. As a result, it initially drifts one way due to a longer period, but its greater orbital decay shortens its period until the direction of drift reverses.

In many cases, passive apertures designed using linear models were not robust to disturbances and nonlinearities [Alfr 00a; Alfr 00b; Vada 00; Sabo 01]. To improve the robustness of linear passive apertures, the HCW Equations have been modified to include the effect of \( J_2 \) [Vada 00; Schw 01]. Also, [Inal 02] shows that for an eccentricity of 0.005, the error induced in the HCW Equations due to ignoring eccentricity dominates the error due to ignoring \( J_2 \). Consequently, linearized models that incorporate \( J_2 \) and eccentric reference orbits have been developed [Gim 01; Vadd 02; Alfr 02b]. Finally, [Kech 01] derives the full, nonlinear equations of motion of a spacecraft subjected to drag and \( J_2 \) with respect to an eccentric reference orbit.

**4 - CONCLUSIONS AND FUTURE DIRECTIONS**

FFG was shown to divide naturally into Deep Space (DS) and Planetary Orbital Environments (POE). The DS literature developed algorithms for finding optimal \( u,v \)-coverages, formation rotations and reconfigurations. The POE literature developed passive apertures (thrust-free trajectories that achieve formation objectives) in both linear and nonlinear models, and has also treated reconfigurations. POE reconfigurations have the added complication of significant orbital dynamics. With regards to passive apertures, disturbances such as \( J_2 \) tend to disperse a formation. To increase the robustness of passive aperture designs, disturbance mitigating passive apertures were designed using nonlinear and augmented linear models.

In DS and POE, fuel optimal reconfiguration algorithms have been developed based on linear models. The next fundamental step is to include collision-avoidance, plume-avoidance (attitude dependent), pointing-constraints (e.g. Sun constraints) and fuel-balancing. Also, a promising new avenue is the exploitation of disturbances for fuel-efficient maneuvers [Bout 02].

An undeveloped area in FFG is coupled attitude and translation planning [Hada 00]. An
important application is DS formation initialization. In deep space, positions are known at best to within tens of kilometers. After initial deployment, spacecraft must search for each other with limited field-of-view (FOV) sensors (that generally require simultaneous viewing to make a relative state measurement) before formation control can take place. Subsequently, spacecraft sensor FOV occultations should be avoided during formation maneuvers. Also, for spacecraft who do not have separate translational and rotational actuation, linear and angular acceleration limits are not independent.

Finally, POE formations are built upon passive apertures. While there are existence conditions for relative orbits about eccentric reference orbits, formation design for other than nearly circular orbits is still largely an art.

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