Low-Frequency Gravitational Wave Searches
Using Spacecraft Doppler Tracking

Doppler Method

Signal transfer function

Pulses and their transfer functions

Reports to date: some results from GLL/MO/MGS/Pioneer/ULS

Status

Expected sensitivity

Could do better than Cassini with earth-spacecraft Doppler tracking?

CaJAGWR-2
11/3/00
J. Anderson
B. Bertotti
F. Estabrook
R. Hellings
L. less
M. Tinto
H. Wahlquist

and many engineers and analysts from the JPL technical divisions, the Deep Space Network, and the flight projects
SOME JARGON

<table>
<thead>
<tr>
<th>DSN</th>
<th>Deep Space Network, the NASA/JPL tracking system with antennas in California, Spain, and Australia</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-band</td>
<td>radio frequency ≈ 2.3 GHz (e.g., Galileo)</td>
</tr>
<tr>
<td>X-band</td>
<td>radio frequency ≈ 8.4 GHz (e.g., Mars Observer)</td>
</tr>
<tr>
<td>Ka-band</td>
<td>radio frequency ≈ 32 GHz (e.g., Cassini)</td>
</tr>
</tbody>
</table>

\[ y(t) \]  
\[ S_y(f) \]  
\[ S_\phi(f) \]  
\[ y(t) \] time series of \( \triangle f/f \)  
\[ S_y(f) \] power spectrum of \( y(t) \)  
\[ S_\phi(f) \] power spectrum of phase
MORE JARGON

Allan variance

$\sigma_y(\tau)$, a measure of fractional frequency stability, $\Delta f/f$, as a function of integration time

$$\sigma^2_y(\tau) = \frac{1}{2} \left\langle \left| \bar{y}(t) - \bar{y}(t+\tau) \right|^2 \right\rangle$$

$$\bar{y}(t) = \frac{1}{\tau} \int_t^{t+\tau} y(t') dt'$$

$$\sigma^2_y(\tau) = 4 \int_0^\infty S_y(f) \frac{\sin^4(\pi f \tau)}{(\pi f \tau)^2} df$$

$$S_y(f) = S_\Phi(f) \cdot f^2 f_0^{-2}$$

scintillation

variation of phase of radio signals due to refractive index variations by a medium (solar wind, ionosphere, troposphere) between the source and the receiver
**JARGON (CONCLUDED)**

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>clock</td>
<td>precision frequency standard</td>
</tr>
<tr>
<td>uplink</td>
<td>radio beam transmitted from the earth to a distant spacecraft</td>
</tr>
<tr>
<td>downlink</td>
<td>radio beam transmitted from a distant spacecraft to the earth</td>
</tr>
<tr>
<td>DSS</td>
<td>Deep Space Station. Followed by a number it designates antennas within the Deep Space Network, as in &quot;DSS 25&quot;</td>
</tr>
</tbody>
</table>
REPRESENTATIVE REFERENCES

Regarding the method:

Estabrook and Wahlquist, *GRG*, 6, 439 (1975)

Wahlquist *GRG*, 19, 1101 (1987)


Regarding the noises:


Response of Spacecraft Doppler Tracking to Gravitational Wave

DOPPLER SIGNALS CORRESPONDING TO DIFFERENT TYPES OF DISTURBANCE IN THE COMMUNICATION LINK
Estimated Value of S/C's $\sigma_{Z}^{GWE}$

- Results:
  - 40 hours of data beginning 2001-DOY-152/T02:00:03.558
    (sampling time is 4 s)
  - CAPS articulation motion was active over this time span
    (articulation frequency $\approx 0.0025$ Hz)
  - Estimated value of $\sigma_{Z}^{GWE}$ is $\approx 0.071$ $\mu$m/sec. It is significantly better than the requirement (0.3 $\mu$m/sec)

- Corresponding Allan deviation
  $\approx 2.3 \times 10^{-16} < 10^{-16}$ (Requirement)

From: Won, Hanover, Baleutz & Lee 10/18/01
MAIN NOISES: FREQUENCY STANDARD STABILITY

• Spacecraft Doppler tracking is not interferometric; coherence is maintained through the frequency and timing system. Thus FTS is fundamental.

• Transfer function in two-way Doppler: $\delta(t) - \delta(t - T_2)$

• Cassini era LITS/SCO has excellent stability on integration times 1–10,000 seconds (see Allan deviation plot, due to L. Maleki)
LINEAR ION TRAP STANDARD (LITS)
FRACTIONAL FREQUENCY STABILITY

LOG $\sigma_y$ vs. LOG $\tau$ (SECONDS)

- Passive H-Maser
- Best H-Maser
- NIST 7
- SCMO (JPL)
- Linear Hg Ion (JPL)
- PSR 1937 + 21
- Rb
- Cs (HP)
- rf Hg Ion (HP)

CaJAGWR-14
11/3/00
MAIN NOISES: PLASMA SCINTILLATION

- Dispersive, refractive index fluctuations proportional to $\lambda^2$

- Transfer function in two-way Doppler: $\delta(t) + \delta(t - T_2 + 2x/c)$

- Plasma scintillation is dominant noise in S-band observations (even at opposition), but a secondary noise source for Ka-band observations at opposition
INTERPLANETARY PHASE SCINTILLATION

S-band Plasma

X-band Plasma

Approx GTS Summer Troposphere

Approx GTS Winter Troposphere-Moderate Elevation Angle

Approx GTS Winter Calibrated Tropo

Approximate Uncalibrated Troposphere (One-Way)

$S_n (10^{-3} \text{Hz}) (\text{ps})$ at S-band

$N_{\text{BER}} (10^{10})$

Sun-Earth-S/C Angle (Deg)
MAIN NOISES: TROPOSPHERIC SCINTILLATION

- Refractive index fluctuations at microwave frequencies are dominated by fluctuations of the water vapor along the LOS.

- Transfer function in two-way Doppler: \( \delta(t) + \delta(t - T_2) \)

- Independent measures of the effect available using WVRs (e.g., Keihm *TDA Prog. Rep. 42-122, 1 (1995)*)

- Operational X-band data can be approximately decomposed into tropospheric and plasma scintillation; results consistent with Keihm's observations (*Armstrong Radio Sci. 33, 1727 (1998)*)

- Cassini-era Advanced Media Calibrations System will calibrate and allow removal of \( \approx 80\% \) of the wet component; dry component + residual wet component will have transfer function \( \delta(t) + \delta(t - T_2) \)
WVR-CEI Comparison
DOY 138, 2000

Path Delay (cm)

UT (SEC)
MAIN NOISES: ANTENNA MECHANICAL STABILITY

- Differential measurements (under controlled conditions) indicate
  \[ \sigma_y(1000\text{sec}) \approx 1 \times 10^{-15} \] (Otoshi and Franco, *TDA Prog. Report 42-10*, 151 (1992))

- Transfer function in two-way Doppler is \( \delta(t) + \delta(t - T_2) \)

- Measurements at X-band under operational conditions are confused with tropospheric scintillation and produce only poor limits \(< 1 \times 10^{-14} \) at \( \tau = 1000 \text{ sec} \) (Armstrong *Radio Sci.* 33, 1727 (1998))

- Infrequent large events—almost certainly antenna mechanical—are observed, however (see example)
Model of Doppler Time Series

\( y(t) = \Delta f/f_0 = \) gravity waves + unmodeled spacecraft motion
+ propagation noise + antenna mechanical noise
+ clock noise + thermal noise
+ systematic effects

\[
g(t) = \left(1 - \mu^2\right)^{-1}\{n \cdot [h_+ (t)e_t + h_x (t)e_x] \cdot n\}
\]

\( L = \) earth-s/c distance; \( \mu = k \cdot n; * = \) convolution
SUMMARY OF SIGNALS AND NOISE

- Signals: "three-pulse" response in the Doppler of the GW excitation
  - Depends on direction to source and s/c two-way light time
  - Not shift-invariant if direction or distance depend on time-of-observation
  - Bandpass: low-frequency signals attenuated due to pulse cancellation; high frequencies cut off by thermal and clock noise. Typical wave duration for best sensitivity depends on $T_2$ but $\sim 10$-10,000 seconds
  - Unlike LISA and other detectors: antenna size/wavelength $\sim 1$ to 100

- Noise sources: various "2-pulse" transfer functions for clock instability, propagation noise (solar wind, ionosphere, troposphere), thermal noise
  - Noise nonstationarity
  - Systematic errors
HOW TO DO A DOPPLER TRACKING EXPERIMENT

• Need two separated test masses—the earth and a spacecraft in cruise as operationally quiet as possible (need to be far from perturbing masses and need to minimize unmodeled motion of the spacecraft)

• Spacecraft should be as close to anti-solar direction as practical (minimize charged particle scintillation due to solar wind)

• Spacecraft-earth separation should be large (maximize band of Fourier frequencies to which the experiment is sensitive)

• Highly-stable Doppler system to measure relative velocity of the earth and spacecraft (excellent frequency standard; careful signal distribution, etc.)

• Ground system and spacecraft telemetry (correct for or veto data based on known systematics of the apparatus)

• Good weather and media calibration data
<table>
<thead>
<tr>
<th>Year</th>
<th>Mission</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>Voyager</td>
<td>Hellings et al. (1981) (few passes; bursts)</td>
</tr>
<tr>
<td>1981</td>
<td>Pioneer 10</td>
<td>Anderson et al. (1984) (3 passes, long $T_2$; no GW from Geminga)</td>
</tr>
<tr>
<td>1988</td>
<td>Pioneer 10</td>
<td>Anderson et al. (1993) (10 days; chirps and coalescing binaries)</td>
</tr>
<tr>
<td>1992</td>
<td>Ulysses</td>
<td>Bertotti et al. (1995) (1 month; sinusoids and chirps)</td>
</tr>
<tr>
<td>1993</td>
<td>MO/GLL/ULS</td>
<td>jGWE collaboration (19 days; X-band on MO; only LF coincidence experiment)</td>
</tr>
<tr>
<td>1994-5</td>
<td>Galileo</td>
<td>Estabrook et al. (40 days; long $T_2$)</td>
</tr>
<tr>
<td>1997</td>
<td>Mars Global Surveyor</td>
<td>Armstrong et al. (3 weeks; X-band)</td>
</tr>
</tbody>
</table>
Frequency-Time for Various Waveforms

- Spectrum localizes in frequency to good effect (minimal noise strip)

- Generalizations (e.g., chirp analysis) can localize in other regions of this phase space
Sinusoids

- If phase is unknown, use power spectral analysis. Appropriate if change in signal frequency < 1/integration time

- In absence of a signal, real and imaginary parts of Fourier transform are gaussian and uncorrelated, thus sum-of-squares = power is exponentially distributed:

  \[ p(x) = \exp(-x) \]

- In presence of a signal of amplitude "c", pdf of power is "Rice-squared":

  \[ p(x) = \exp(-(x + c^2)) I_0(2c/x^{0.5}) \]

- Since frequency is unknown, and since Fourier bins are approximately independent, joint pdf of power in "n" Fourier bins is product of individual bin pdf's--this can be used to set confidence limits for broadband observations (Armstrong, Estabrook, Wahlquist 1987)

- Examples follow
Other Ideas We Have Tried

• Wavelet transforms
  • A time-frequency localization procedure
  • To date for denoising only—attempt to filter out the "high frequency noise" while retaining the "edges" of bursts. Useful particularly with Galileo (low-gain antenna means lower signal-to-thermal-noise)

• Karhunen-Loeve expansion
  • Let the data determine their own basis: basis functions derived from the autocovariance matrix
  • Problem: in simulations at low SNR (the practical case), modes found are never the physical modes. Disappointing.
Other Ideas We Have Tried (Continued)

- Bispectral analysis
  - Fourier decomposition of third statistical moment--look for non-gaussian component to time series
  - Problem: difficult to estimate accurately at level of any putative non-gaussian signals in Doppler data

- Multi-taper spectral analysis
  - Another way of partitioning spectrum into a continuum + "lines"
  - Achieved recent notoriety with claim of spectral lines in flux of keV electrons observed in solar wind (Thomson et al. 1995)
  - Main advantage over what we have done to date is when lines are present with a choppy continuum--if continuum is smooth, it reduces to what we already do.
  - A "neat idea", but data gaps may be a problem
galactic center
twit = 3333. sec
mu = -0.90 (154.3 deg)
range (kpc) = 8.
GLL 1994; f_0 = 0.04500 Hz

\[ \text{twlt corrected} \]

\[
\begin{align*}
twit & = 3494. \text{ sec} \\
mu & = -0.84 \text{ (146.7 deg)} \\
theta \text{ (degrees)} & = 146.7
\end{align*}
\]
FIRST EVER RECEIPT OF 2-WAY COHERENT KA-BAND SIGNAL

DOY 20, 1999
(10:22 PM PST, 1/19/99)
quick-look noise statistics of Cassini GWE1

red is two-way plasma at Ka-band, scaled from \( \chi - \left( \frac{880}{3344} \right) K \)
green is 2-way Ka at des25; high-elevation, no-corrections
blue dot-dash is average of Mars Observer (1993)
stars are 2-way tropo, from AMC, elevation > 20 deg
black dot-dash is pre-experiment estimate of raw 2-way tropo

allan deviation at 1000 sec

DOY in 2001

330 335 340 345 350 355 360 365 370
Discussion

- Ka-band up/down, as it had to, knocked the plasma scintillation noise out of the error budget

  - X – (880/3344)K independently estimates the downlink plasma

  - Two-way Ka-band plasma consistent with pre-experiment expectation

- 2-way Ka-band (uncorrected, selected for high elevation angles) limited by nondispersive process (e.g. some combination of troposphere, antenna mechanical, FTS, KaT instability, s/c motion noise, ...)

  - Level is consistent with independent estimate of the troposphere, therefore should be able to correct for this to about the target sensitivity level

- Potential problem: significant fraction of AMC data flagged—liquid water in AMC beam may degrade AMC correction of the data
relative energy response for Cassini 2001 December 16

circular-pol: $\sin(2\pi \times 0.001 \text{ Hz} \times t) \times \exp(-t/1000 \text{ sec})$

right ascension

hammer-alt off equal-area projection (center of plot is RA = 0, dec = 0)
relative energy response for Cassini 2004 January 4

\text{circular-pol: } \sin(2 \pi (0.001 \text{ Hz}) t) \exp(-t/1000 \text{ sec})

hammer-alt off equal-area projection (center of plot is RA = 0, dec = 0)
sensitivity for $\text{SNR} = 1$, $\tau = 40$ days

Cassini opposition $\#1$, $T_2 = 5720$ seconds
Can We Do Better than Cassini?

**Problems Are**

- tropospheric scintillation
- plasma scintillation
- antenna mechanical noise
- frequency standard noise
- spacecraft position noise

**Possible Fixes**

- better calibrations and/or Estabrook/Hellings idea
- higher radio frequency and/or Cassini-style multi-frequency links
- look in nulls of transfer function (?)
- 30X better clocks are "straightforward"
- very careful design (?)

**Conclusion:** *Maybe 10-fold improvement—to ~3 x 10^{-18} for periodic sources at selected Fourier frequencies—is possible using spacecraft Doppler tracking from an Earth-based station. However the cost to achieve this would be very high.*
Concluding Ideas

- Doppler tracking of Cassini can be used as a broadband gravity wave detector
  - Apparatus is large compared with GW wavelength; thus detector properly described in terms of three pulses GW response
  - Low-frequency band edge is \( \approx \frac{1}{(\text{two-way-light-time})} \) set by pulse-overlap
  - High-frequency band edge is \( \approx 10^{-1} \) to \( 10^{-2} \) Hz, set by combination of downlink SNR, FTS, ability to calibrate troposphere
  - Not an interferometer: coherence maintained by excellent frequency standard on the ground

- Main noise sources
  - FTS stability
  - Plasma scintillation (dominates S-band; secondary at Ka-band)
  - Tropospheric scintillation (nondispersive)
  - Antenna mechanical stability
Concluding Ideas (continued)

- Signals and noises enter with different transfer functions—a very useful discriminator

- Cassini experiment will be ≈10-fold more sensitive than previous observations
  - Ka-band lowers plasma noise at opposition to below troposphere noise
  - Sophisticated tropospheric scintillation calibration

- Sensitivity:
  - $\approx 3 \times 10^{-15}$ for bursts (i.e., $\sigma_y(\tau \approx 1000 \text{ sec})$)
  - $\Omega \lesssim 10^{-2}$ for backgrounds ($f_c \approx 10^{-4} \text{ Hz}$)
  - $\approx 3 \times 10^{-17}$ for periodic waves (at selected Fourier frequencies);
    $\approx 1.5 \times 10^{-16}$ averaged over the band