

OBSERVING A GRAVITATIONAL WAVE BACKGROUND WITH LISA

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Abstract

LISA (Laser Interferometer Space Antenna) is a proposed space mission which will use coherent laser beams exchanged between three remote spacecraft, to detect and study low-frequency cosmic gravitational radiation^[1]. The multiple Doppler readouts available with LISA, which incorporate frequency standards for measuring phase differences between the received and transmitted laser beams, permit simultaneous formation of several observables^[2,3,4]. All are independent of lasers and frequency standards phase fluctuations, but have different couplings to gravitational waves and to the various LISA instrumental noises. Comparison of the conventional Michelson interferometer observable with the fully-symmetric Sagnac data-type allows unambiguous discrimination between a gravitational wave background and instrumental noise. The method presented here can be used to detect and measure a confusion-limited gravitational wave background.

INTRODUCTION

The Laser Interferometer Space Antenna (LISA) is a space mission, jointly proposed by NASA and ESA, aimed to detect and study gravitational radiation in the millihertz frequency band. With its three spacecraft, each carrying lasers, beam splitters, photodetectors and drag-free proof masses on each of their two optical benches, LISA will have the capability of measuring six time series of Doppler shifts of the one-way laser beams between spacecraft pairs, and six shifts between adjacent optical benches on each spacecraft. By linearly combining, with suitable time delays, these twelve data sets, it will be possible to cancel the otherwise overwhelming phase noise of the lasers ($\Delta\nu/\nu \simeq 10^{-13}$) to a level $h \simeq \Delta\nu/c \simeq 10^{-23}$. This level is set by the buffeting of the drag-free proof masses inside each optical bench, and by the shot noise at the photodetectors^[4].

LISA is expected to detect monochromatic radiation emitted by galactic binary systems. Particularly at low Fourier frequencies (say 0.1 – 8 mHz), however, there will be many galactic binaries radiating within each Fourier resolution bin^[1]. These latter signals will not be detectable individually, forming a continuum which could be confused with instrumental

noise. The level of this stochastic background is uncertain but could be in the range 10^{-20} – 10^{-23} . Since these galactic binary populations are virtually guaranteed, the detection of their signals could be the first direct detection of gravitational waves.

For this measurement it is very desirable that competing proof-mass and other instrumental noises be both characterized and calibrated before flight, and measured in the actual flight configuration while data are being taken. In contrast to Earth-based, equal-arm interferometer detectors of gravitational radiation, LISA will have multiple readouts, and the Doppler data they generate can be combined differently to give measurements not only insensitive to laser phase fluctuations and optical bench motions, but also with different sensitivities to gravitational waves and to the remaining system noise^[3,4].

In this article we discuss two laser-and-optical-bench-noise-free combinations of the LISA readouts, previously denoted ζ (Sagnac) and X (Michelson), that have very different responses to the gravitational wave background but comparable responses to instrumental noise sources^[4,5].

THE SAGNAC AND MICHELSON INTERFEROMETERS

The six Doppler beams exchanged between the LISA spacecraft imply the six Doppler readouts y_{ij} ($i, j = 1, 2, 3$) recorded when each transmitted beam is mixed with the laser light at the receiving optical bench. Delay times for light travel between the spacecraft must be carefully accounted for when combining these data. Six further data streams, denoted z_{ij} ($i, j = 1, 2, 3$), are generated internally to monitor both lack of rigidity and laser synchronization between the independent optical benches at each spacecraft. The combination ζ uses all the Doppler data symmetrically^[4,5]

$$\begin{aligned} \zeta = & y_{32,2} - y_{23,3} + y_{13,3} - y_{31,1} + y_{21,1} - y_{12,2} \\ & + \frac{1}{2}(-z_{13,21} + z_{23,12} - z_{21,23} + z_{31,23} - z_{32,13} + z_{12,13}) \\ & + \frac{1}{2}(-z_{32,2} + z_{12,2} - z_{13,3} + z_{23,3} - z_{21,1} + z_{31,1}) . \end{aligned} \quad (1)$$

The comma notation indicates time-delays along the arms of the 3-spacecraft configuration

$$y_{32,2} \equiv y_{32}(t - L_2) , \quad (2)$$

and so forth (units in which $c = 1$).

The transfer functions of ζ to instrumental noises and to gravitational waves were calculated in references [3, 4]. The resulting instrumental noise power spectrum for ζ is shown in Figure 1. Also shown there is the computed power spectrum of ζ , averaged over the sky

and elliptical polarization states, that would result from a stochastic background originated by an ensemble of galactic binary systems^[1].

The laser-and optical-bench-noise-free combination, X , only requires four data streams. This combination is equivalent to an (unequal arm) Michelson interferometer. Its expression is equal to^[4,5]

$$\begin{aligned}
X &= y_{32,322} - y_{23,233} + y_{31,22} - y_{21,33} + y_{23,2} - y_{32,3} + y_{21} - y_{31} \\
&\quad + \frac{1}{2}(-z_{21,2233} + z_{21,33} + z_{21,22} - z_{21}) \\
&\quad + \frac{1}{2}(+z_{31,2233} - z_{31,33} - z_{31,22} + z_{31}) .
\end{aligned} \tag{3}$$

The expected instrumental noise power spectrum in X is shown in Figure 1. Also shown is the anticipated galactic binary confusion spectrum^[1], which would be observed in X . Comparison of X and ζ allows the background to be discriminated from instrumental noise.

DETECTING THE GALACTIC STOCHASTIC BACKGROUND

The flight configuration of the three spacecraft forming LISA will be essentially equilateral, with $L_1 = L_2 = L_3 = L = 16.67$ sec. In the frequency band of interest (0.1 – 8 mHz), the expressions for the Fourier transforms of the gravitational wave signals $\tilde{X}^{gw}(f)$, $\tilde{\zeta}^{gw}(f)$ and the power spectral densities of the system noises in X and ζ , $S_{Xnoise}(f)$, $S_{\zeta noise}(f)$, can be Taylor-expanded in the dimensionless quantity fL . The first non-zero terms are equal to

$$\tilde{X}^{gw}(f) \simeq 2 (2\pi i f L)^2 \left[\hat{n}_3 \cdot \tilde{\mathbf{h}}(f) \cdot \hat{n}_3 - \hat{n}_2 \cdot \tilde{\mathbf{h}}(f) \cdot \hat{n}_2 \right] , \tag{4}$$

$$\begin{aligned}
\tilde{\zeta}^{gw}(f) \simeq \frac{1}{12} (2\pi i f L)^3 &\left[(\hat{k} \cdot \hat{n}_1)(\hat{n}_1 \cdot \tilde{\mathbf{h}}(f) \cdot \hat{n}_1) + (\hat{k} \cdot \hat{n}_2)(\hat{n}_2 \cdot \tilde{\mathbf{h}}(f) \cdot \hat{n}_2) \right. \\
&\quad \left. + (\hat{k} \cdot \hat{n}_3)(\hat{n}_3 \cdot \tilde{\mathbf{h}}(f) \cdot \hat{n}_3) \right] ,
\end{aligned} \tag{5}$$

$$\begin{aligned}
S_{Xnoise}(f) &\equiv S_{Xproofmass}(f) + S_{Xopticalpath}(f) \\
&\simeq 16 [S_1(f) + S_{1^*}(f) + S_3(f) + S_{2^*}(f)] (2\pi f L)^2 \\
&\quad + 4 [S_{32}(f) + S_{23}(f) + S_{31}(f) + S_{21}(f)] (2\pi f L)^2
\end{aligned} \tag{6}$$

$$\begin{aligned}
S_{\zeta noise}(f) &\simeq [S_1(f) + S_2(f) + S_3(f) + S_{1^*}(f) + S_{2^*}(f) + S_{3^*}(f)] (2\pi f L)^2 \\
&\quad + [S_{32}(f) + S_{23}(f) + S_{31}(f) + S_{21}(f) + S_{13}(f) + S_{12}(f)] ,
\end{aligned} \tag{7}$$

where we have denoted by $S_{Xproofmass}(f)$, and $S_{Xopticalpath}(f)$ the aggregate contributions to the power spectrum of the noise in the response X from the proof mass and optical path noises respectively. The expressions in square brackets in equations (4, 5) incorporate LISA

antenna responses^[3,4], and are of the same order of magnitude. The proof mass Doppler noise spectra $S_i(f)$, $S_{i^*}(f)$ ($i = 1, 2, 3$) will be designed to a nominal power spectral level^[4] $S^0(f) = 2.5 \times 10^{-48}[f/1Hz]^{-2} \text{ Hz}^{-1}$, while the optical path noise spectra $S_{ij}(f)$, ($i, j = 1, 2, 3, i \neq j$), which include shot noises at the photo detectors and beam pointing noise^[1], are expected to be equal to a nominal spectrum $S^1(f) = 1.8 \times 10^{-37}[f/1Hz]^2$. Both these noise sources will be estimated before launch, but could be larger when the in-orbit data will be taken.

First consider the responses to the gravitational wave signal, given in equations (4, 5). At $f = 10^{-3} \text{ Hz}$, for instance, (where $2\pi fL \simeq 10^{-1}$) the absolute value of the coefficient in front of the squared-bracket in the ζ response (Eq. 5) is about three orders of magnitudes smaller than the corresponding coefficient given in the expression for X (Eq. 4). The power spectral densities of the noises due to the proof masses and the optical-path noise (Eqs. 6, 7) will only differ at most by an order of magnitude. We conclude that in this lower frequency range the LISA Sagnac response, ζ , can be used as a *gravitational wave shield*. In what follows we will ignore the gravitational wave background contribution to ζ .

To take quantitative advantage of this property of ζ , consider the observed power spectral densities of X and ζ

$$S_X^{obs}(f) = S_{Xgw}(f) + S_{Xproofmass}(f) + S_{Xopticalpath}(f) \quad (8)$$

$$S_\zeta^{obs}(f) = \frac{1}{16} \left[S_{Xproofmass}(f) + \frac{S_{Xopticalpath}(f)}{(\pi fL)^2} \right] + [S_{13}(f) + S_{12}(f)] \\ + [S_2(f) + S_{3^*}(f)] (2\pi fL)^2, \quad (9)$$

where in equation (9) we have written the power spectra of the noises in ζ in terms of the power spectra of the noises in X and of some remaining terms that are not present in X , to emphasize commonality of some noise sources. We suppose that the noise contributed by any one of the proof masses and optical-path noise sources will be greater than or equal to the design values, $S^0(f)$ and $S^1(f)$ respectively. From equation (9), if the magnitude of the measured power spectral density of the response ζ is at its anticipated level $S_\zeta^{obs}(f) = 6 S^0(f)(2\pi fL)^2 + 6 S^1(f)$, then the level of the power spectral density of the noise entering into X is known. The spectrum

$$S_{Xgw}(f) = S_X^{obs}(f) - 64 S^0(f)(2\pi fL)^2 - 16 S^1(f)(2\pi fL)^2, \quad (10)$$

should then be attributed to a galactic binary background of gravitational radiation. In any event, the RHS of equation (10) is an upper bound to S_{Xgw} .

On the other hand, if the measured spectrum of ζ is above its anticipated design level, consider the following combination of the measured spectra

$$\begin{aligned}
S_X^{obs}(f) - 16 S_\zeta^{obs}(f) &= S_{Xgw} - 16 [S_2(f) + S_{3^*}(f)] (2\pi fL)^2 \\
&\quad - 16 [S_{13}(f) + S_{12}(f)] \\
&\quad - 16 [S_{32}(f) + S_{23}(f) + S_{31}(f) + S_{21}(f)] \times \\
&\quad \times [1 - (\pi fL)^2].
\end{aligned} \tag{11}$$

The coefficient of S_ζ^{obs} has been chosen so that the noise terms on the right hand side are all now negative-definite and can thus be bounded from above by their design, or nominal, values $S^0(f)$ and $S^1(f)$ respectively. The result is a lower bound for observational discrimination of the gravitational wave background spectrum

$$\begin{aligned}
S_{Xgw}(f) &\geq S_X^{obs}(f) - 16 S_\zeta^{obs}(f) + 32 (2\pi fL)^2 S^0(f) \\
&\quad + 16 [6 - (2\pi fL)^2] S^1(f).
\end{aligned} \tag{12}$$

Equations similar to (11) and (12) can be written for the other two interferometer combinations, Y and Z^[4]. In those equations, there will be different mixes of canceled and bounded noise sources, resulting, in general, in different gravitational wave spectrum lower bounds.

CONCLUSIONS

The response of the Sagnac interferometer to a gravitational wave signal is several orders of magnitudes smaller than that of the Michelson interferometer. In the frequency band of interests (0.1 – 8) mHz, however, the Sagnac response to the noise sources is of the same order of magnitude as that of the Michelson interferometer. As a consequence of these facts we have shown that it is possible to estimate the magnitude of the noise sources affecting the Michelson interferometer response in the low-frequency region of the accessible band by using the Sagnac interferometer. This in turn allows us to discriminate a gravitational wave background of galactic origin from instrumental noise affecting the Michelson interferometer response.

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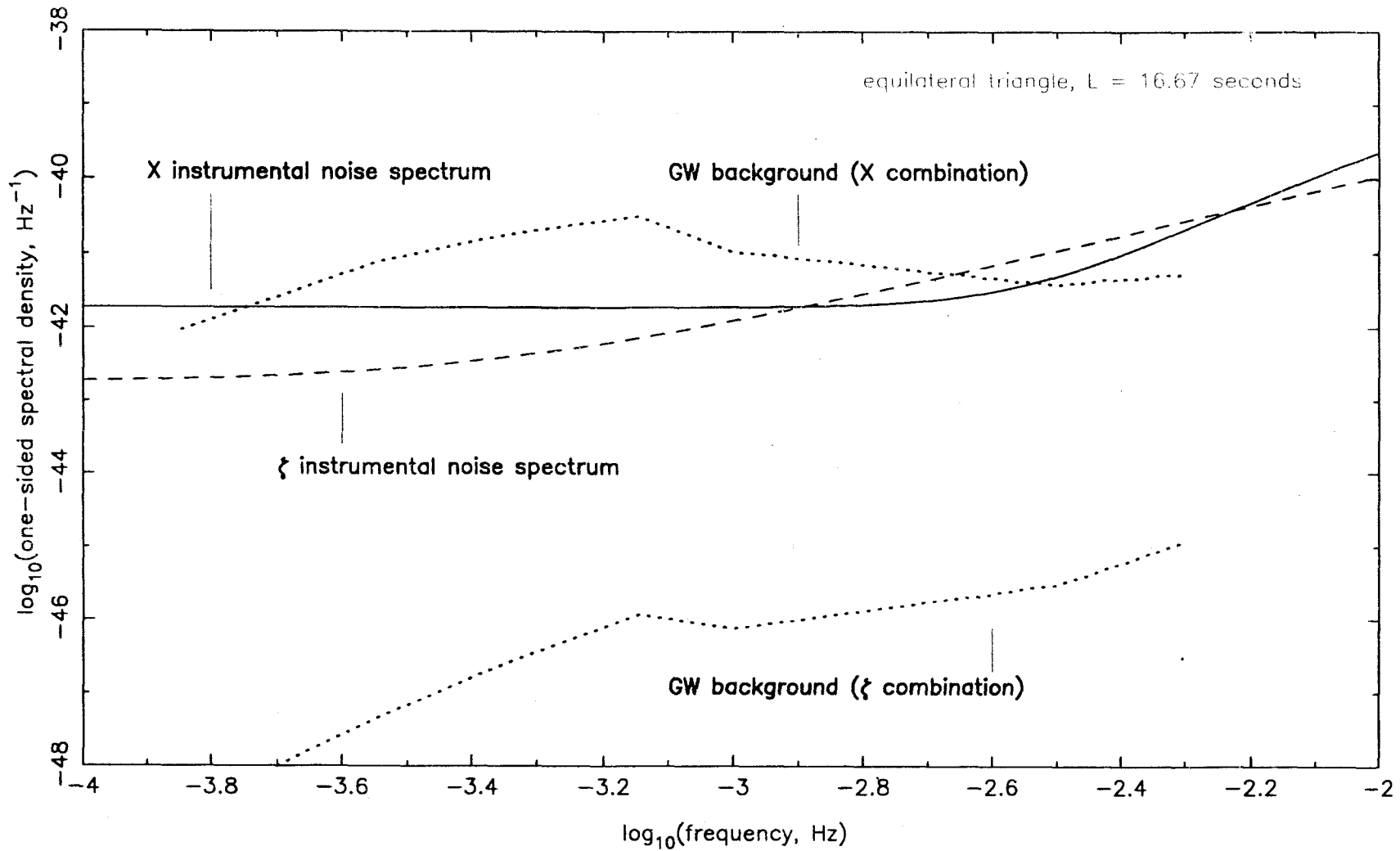


Figure 1. Fractional Doppler frequency instrumental noise power spectra for data combinations X (solid line) and ζ (dashed line). These are derived from the transfer functions for X and ζ to instrumental noises and the nominal LISA spectra for individual proof mass noise [$3 \times 10^{-15} (m/sec^2)/\sqrt{Hz}$] and one-way optical path noise [$20 \times 10^{-12} m/\sqrt{Hz}$], converted to fractional Doppler spectra. Also plotted are the spectral responses of X and ζ to the expected stochastic gravitational wave background as discussed by Bender & Hils [3], and Hils [4]. Using ζ to measure on-orbit instrumental noise allows a gravitational wave background in X to be either uniquely determined or bounded.