Improved Planar Spiral Transformer Theory Applied to a Miniature Lumped Element Quadrature Hybrid

J. Hogerheiden, M. Ciminera, G. Jue, *Members*, IEEE

Abstract -- A MMIC compatible lumped element model is derived for symmetric width, uniformly coupled transmission lines. Lumped element values are calculated from the coupled line even and odd mode impedances and electrical length. An improved technique is presented to calculate lumped equivalent circuit values of planar spiral transformers, which improves on previous work in modeling planar spiral inductors and grounded planar spiral transformers. The improved techniques are used to realize a miniature 90 degree hybrid using planar spiral transformers. Producibility is enhanced and coupling is effectively increased with the novel use of series capacitors which cancel some of the self inductance of the transformers. Measured results are presented for both a quadrature hybrid and the individual planar spiral transformer used in the quadrature hybrid. The measured results show excellent agreement with the computer models.

I. INTRODUCTION

Lange couplers [1] are planar devices which are commonly used in MMICs at higher microwave frequencies. At frequencies below 3.0 GHz, the length approaches 1cm on GaAs which is very difficult to fabricate. A lumped element implementation is needed to solve this problem.

A coupled line section can be modeled as an equivalent lumped element circuit similar to the way a transmission line can be modeled with lumped elements. A transmission line is modeled with shunt capacitors and series

inductors derived from the characteristic impedance and phase velocity [2,3]. Coupled lines can be modeled similarly if they are assumed to be in a homogeneous media. Details for the lumped element model of a coupled line section are given in section II.

The Planar Spiral Transformer (PST) is used to realize the lumped element coupler as a planar structure compatible with MMIC technology. The PST structure consists of a pair of tightly coupled lines spiraled in a manner similar to the planar spiral inductors commonly used in MMICs [4,5].

II. LUMPED ELEMENT COUPLER MODEL

Lumped element coupler models using planar spiral transformers have been described in the literature [6,9], but the equations are not general. In this section, values for lumped element coupled sections of any electrical length are derived. This allows the designer to design a 90 degree coupler by cascading as many sections of shorter couplers as desired. For instance, two 45 degree sections or three 30 degree sections can be cascaded to achieve 90 degrees. A greater number of sections more closely approximates a distributed pair of coupled lines. Two 45 degree sections perform much better than one 90 degree section; however, little is gained with more than three sections.

Figure 1 shows the parameters of a distributed coupled line section, and Figure 2 shows the general lumped element model for a coupled line section. Starting from the characteristics of the distributed coupled line section shown in Figure 1, the values for the lumped element model in Figure 2 are derived by equating the two circuits with ports 3 and 4 grounded in each case. The other 2 ports (port 1 and port 2) are then driven first in the odd mode and then in the even mode. When the grounded coupled line section is driven in the odd mode as shown in Figure 3a, the lumped equivalent

circuit is that shown in Figure 3b. When the groundedcoupled line section is driven in the even mode as shown in Figure 4a, the lumped equivalent circuit is that shown in Figure 4b[2,3].

The values of C_g, C_c, L, and M can be determined in terms of Zoo, Zoe, and Θ. This is done by grounding ports 3 and 4 and replacing the coupled inductors of Figure 2 with their "'J'" equivalent circuit as shown in figure 5a. Driving this circuit in the odd and even modes results in the equivalent circuits in Figure 5b. [7] The lumped values are equated to the distributed values as shown below:

$$L+M = L_{oe} = \frac{Z_{oe}\sin\Theta}{2\pi f}$$

$$L-M = L_{oo} = \frac{Z_{oo}\sin\Theta}{2\pi f}$$

$$C_g = C_{oe} = \frac{\tan\Theta/2}{Z_{oe}\,2\pi f}$$

$$C_g+2C_c = C_{oo} = \frac{\tan\Theta/2}{Z_{oo}\,2\pi f}$$

Solving for L, M, Cg and Cc in terms of Zoe, Zoo, and Θ:

$$L = \frac{(Z_{oe}+Z_{oo})\sin\Theta}{4\pi f}$$

$$M = \frac{(Z_{oe}-Z_{oo})\sin\Theta}{4\pi f}$$

$$C_g = \frac{\tan\Theta/2}{Z_{oe}\,2\pi f}$$

$$C_c = \left(\frac{1}{Z_{oo}} - \frac{1}{Z_{oe}}\right)\frac{\tan\Theta/2}{4\pi f}$$

These are then the lumped element values of Figure 2 which relate to the coupled lines of Figure 1.

111. MODIFICATION OF LUMPED ELEMENT COUPLER MODEL TO IMPROVE MANUFACTURABILITY

A theoretical lumped element coupler model which covers a 3:1 bandwidth using only two 45 degree sections requires a midband coupling of 2.2 dB. Figure 6 shows the lumped circuit for such a coupler resonant at 510 MHz with equal loss at 250 MHz and 750 MHz. The coupler was derived using the techniques described in section II.

The required coupling factor (M/L) for the coupled inductors in this symmetric model is large (0.777). This is difficult to realize in practice because the gap spacing required for a PST is very small and interconnecting wire bonds and lines add to the self-inductance further decreasing M/L.

An additional inductance of 1nH (a typical value for interconnecting lines and bond wires) can be compensated for at band center by adding 98.1pF capacitors as shown in figure 7.

The compensating capacitors are large and add undesirable complexity to the coupler; therefore, the structure shown in figure 8 was developed using computer optimization (Super Compact). It has smaller series capacitors and eliminates the shunt capacitors at each end of the structure.

The structure shown in figures 7 & 8 both have an M/L ratio of 0.733 which would yield 2.7 dB of midband coupling without the series capacitance; however both circuits have a midband coupling of 2.2 dB due to these Capacitors. The structure of figure 8 has the additional advantage of better VSWR and isolation over a 3:1 bandwidth. The VSWR is under 1.22:1 and the isolation is greater than 20dB.

Figure 9 compares the performance of the circuits. The amplitude and phase balance of the circuits shown in figures 6 and 8 are very close. The? isolation of the circuit shown in figure 8 is less than that. of figure 6 but still greater than 20 dB. The circuit in figure 8 is much easier to realize because the looser coupling (M/L) between the coupled inductors allows for a larger gap in the PST.

IV. PLANAR SPIRAL TRANSFORMER (PST)

The Planar Spiral Transformer (} 'S1') analysis is based on the work done by Frlan et. al [4,5] in which frequency dependent element values are calculated for a lumped element equivalent circuit. Frlan's lumped element circuit is shown in figure 10a. The equivalent r)et. we)]-k shown in figure 1013 was used instead because it allowed the PST model to be incorporated into the hybrid model shown in figure 8. Closed form expressions are used to calculate the model element values, which are dependent on the physical parameters of the transformer. Frlan's work shows good agreement with measured data on PSTs used as a two port network, where one end of the primary and secondary are grounded.

A 90 degree hybrid design requires an accurate four port. model, wit.bout- any of the ports grounded. An in-house computer modeling capability was developed, applying the analysis techniques used by Frlan to analyze PSTs used as a four port. network.

Using the developed modeling capability, the PST dimensions were determined for the element values required by the hybrid design. To validate the PST model, a frequency scaled (5x) PST was fabricated on 1.27 mm thick alumina with "/ .5 um thick Ti-W gold conductor metallization. The dimensions of the square PST were 1651 urn x 1651 urn with 56 urn line

widths and 10 urn gaps. This scaled circuit corresponds to a MMIC design on a 250 urn thick GaAs substrate with 2 um gaps and 13 urn linewidths.

Original performance predictions based on Frlan's model for grounded PSTs did not. provide a good match to the measured data. In order to produce a usable model, the model element values were varied to match the measured data. The primary source of error in the predicted model was Cm, the primary-to-secondary capacitance. Frlan's predicted Cm was too low. Frlan calculates the primary-to-secondary capacitante using 1/2*(Co-Ce)*(1-cos0), where O represents the relative phase shift. between a pair of segment-s. ~'bus, the primary-to-secondary capacitance is equal to zero at. d.c. and increases as frequency increases.

To provide a more accurate model, a physical basis' for the gap capacjt.ante was used. The gap capacit.ante per unit length was calculated using Smith's Cgap= Cdelta= Cfo'-Cfe' [8] and multiplied by the total gap length between the primary and the secondary. This yielded a non-frequency dependent value of Cm equal to 1.8 pF, which was 58 times larger at. the center frequency than the Cm calcu1ated using Frlan's method.

New predicted data for the PST was generated using the new value of Cm with all of the other model element values remaining the same. Very good agreement, between the new predicted and measured data is shown in Figures 1 la-h, with Cm being the only model element. different from Frlan's model. Also shown in Figures 11a-h is the predicted data using Frlans technique for calculating Cm.

Although this analysis and Frlan's both use a constant. Keff versus frequency, we believe this improved technique for calculating Cm is more accurate than Frlan's four port PST model when used well below resonance.

V . F'Srl' QUADRATURE HYBRI D

A quadrature hybrid using PSI'S wit h series capacitors was developed using the technique described in section 111 , Parasitics such as "T" junctions, wire bonds, gap capacitances and interconnecting lines were included in t-he model . 'l'he shunt. capacitors shown in figure 8 were realized as radial Stubs . When this circuit was optimized using Super Compact-, the series capacitors were further reduced from 49pF to 33pF to improve manufacturabi lity. This reduced the bandwidth, but still met. the design requirements. The model for the fabricated circuit is shown in figure 12.

The PST quadrature hybrid (Figure 13) was fabricated using the same process described for t-he PST in section IV. The overall size of the PST quadrature hybrid was 0.92 x 1.25 cm at the scaled frequency. This corresponds to a size of 3.048 x 5.080 mm at. the design frequency of 2.5 GHz . The layout was not. optimized for reduced size.

Good correlation between predicted vs. measured data for the PST Quadrature Hybrid is shown in Figures 14a-f. The predicted data utilize the value of Cm calculated using Smith.

In summary, the PST Quadrature Hybrid was designed for a 2.2dB maximum coupling and 3:1 bandwidth at. 1 .8 dB amplitude difference and achieved 2.1dB maximum coupling and 2.8:1 bandwidth at. 1 . '/"/ dB amplitude difference.

VI. CONCLUSIONS

We have found that Frlan's model for calculating Cm is inaccurate for transformers used as a four port network. Frlan notes that the most sensitive components of the transformer model are M and Cm, and he states that. if these equivalent- circuit components are not. calculated accurately, the resonant frequency response of the model is severely affected. Frlan's work is based on grounded transformers and the value of Cm is effectively zero at d.c. and increases with frequency. We have demonstrated a more accurate technique for calculating Cm which agrees well with measured data for frequencies well below resonance.

We have shown general design equations for the PST hybrid. In addition we have shown that highly miniaturized broadband' quadrature hybrid couplers are made more realizable using PST's with compensating series capacitors. This design offers a significant size reduction over Lange couplers and is compatible with MMIC technology, making it a valuable building block for integration into larger MMIC circuit designs.

ACKNOWLEDGMENT

The work described in this paper was performed at AIL (Airborne instruments Laboratory) Systems incorporated on internal research and development funds, and at the Jet Propulsion Laboratory of the California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

REFERENCES

[1] Lange, J. , "Interdi gitated Stripline Quadrature Hybrid", IEEE Transactions on Microwave Theory and Techniques, Vol . 17, No. 12, Dec. 1969, pp. 1150-1151

[2] Matthaei, George L., Young, L., and Jones, E.M.T. , "Microwave Filters, Impedance Matching Networks, and Coupling Structures", McGraw Hill 1964, pp. 360-361

[31 Kopp, Bruce, "Asymmetric Lumped Element Power Splitters", 1989 IEEE MTT-S Digest, pp. 333-336

[4] Frlan, E., Mesz.ares, S., Cuhaci, M., anti Wight, J.S., "Computer Aided Design of Square Spiral Transformers and Inductors", 1989 IEEE MTT-S Digest, pp. 661-664

[5] Frlan, Edward, "Miniature Hybrid Microwave Integrated Circuit Passive Component Analysis Using Computer Aided Design Techniques", Masters Thesis, Carleton University, Dept. of Electronics, August. 1989

[6] Ali, E'., and Podell, A., "Design and Applications of a 3:1 Bandwidth GaAs Monolithic Spiral Quadrature Hybrid", 1990 IEEE GaAs IC Symposium Digest, pp. 279-282

[7] Hayt, W., and Kemmerly, J., Engineering Circuit. Analysis, McGraw-Hill Inc., 1962, pg. 499

REFERENCES (Continued)

[8] Smith, J., "The Even and Odd-Mode Capacitance Parameters for Coupled Lines in Suspended Substrate", IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-19, No.5, May 1971, pp. 424-431

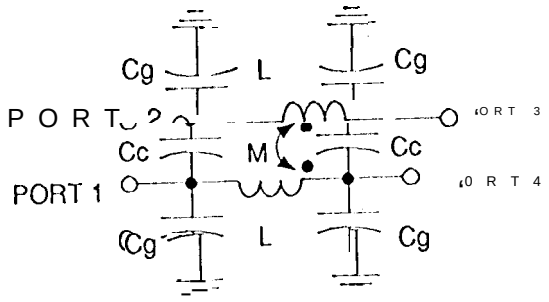
[9] Ali, F., and Podell, A., "A Wide-Band GaAs Monolithic Spiral Quadrature Hybrid anti its Circuit Applications", IEEE Journal of Solid-St-ate Circuits, vol. 26, No. 10, October 1991, pp. 1394-1398

LIST OF FIGURES

Figure	Description
1	Coupled Line Section
2	Lumped Element Model
3	Shunt Coupled Line Driven in t-he Odd Mode and the Lumped Equivalent
4	Shunt Coupled Line Driven in the Even Mode and the Lumped Equivalent
5a	"T" Equivalent Transformer Model
5b	Equivalent Circuits When Driven in the Odd and Even Modes
6	2.2 dB Coupler Model Resonant at, 510 MHz
7	Coupler Model with Series Capacitors
8	Improved Circuit Model
9	Coupler Performance Comparison
10a	Frlan's Lumped Element Circuit
10b	Network Equivalent to Frlan's Circuit-
10c	Planar Spiral Transformer
11a	PST S21 Magnitude
11b	PST S21 Phase
11c	PST S31 Magnitude
11d	PST S31 Phase
11e	PST S41 Magnitude
11f	PST S41 Phase
11g	PST S11 Magnitude
11h	PST S11 Phase
12	Model for Fabricated Hybrid
13	PST Quadrature Hybrid
14a	Hybrid Coupled Port Magnitude
14b	Hybrid Coupled Port Phase

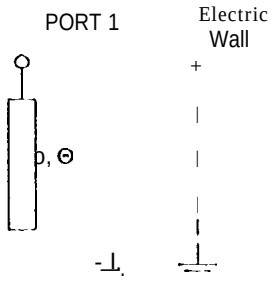
LIST OF FIGURES (Continued)

Figure	Description
14C	Hybrid Thru Port Magnitude
14d	Hybrid Thru Port. Phase
14e	Hybrid S11 Magnitude
14f	Hybrid Isolation Magnitude

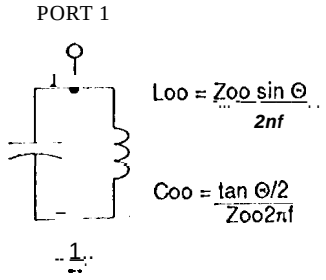


PARAMETERS: C9, CO, L, M

Figure 2. Lumped Element Model

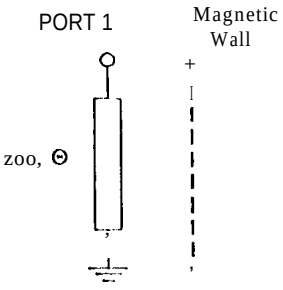


3a. Coupled Line

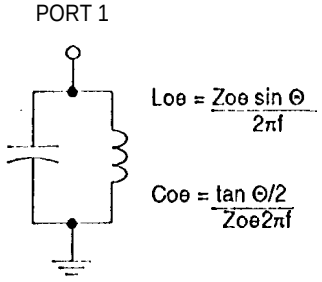


3b. Lumped Element Model

Figure 3. Shunt Coupled Line Driven in the Odd Mode and the Lumped Equivalent



4a. Coupled Line



4b. Lumped Element Model

Figure 4. Shunt Coupled Line Driven in the Even Mode and the Lumped Equivalent

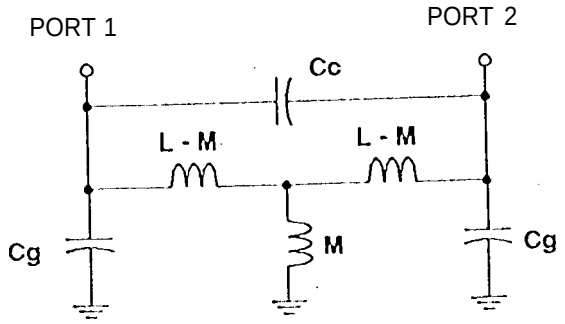
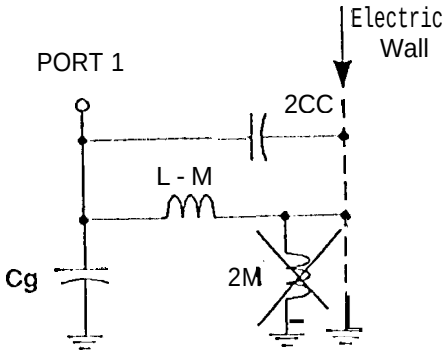
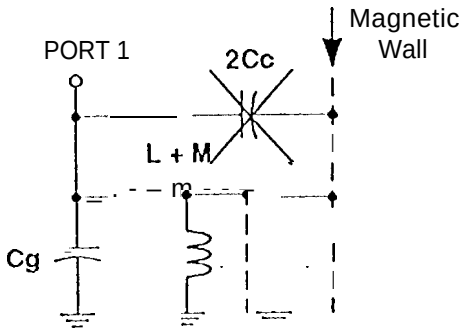


Figure 5a. "T" Equivalent Transformer Model



Odd Mode



Even Mode

Figure 5b. 'Equivalent Circuits When Driven in the Odd and Even Modes

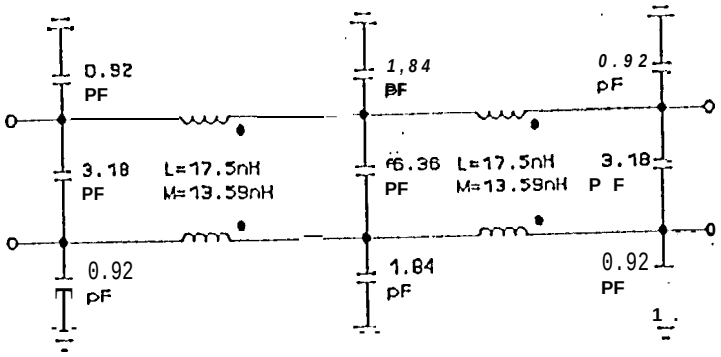


Figure 6. 2.2 dB Coupler Model Resonant at 510 MHz

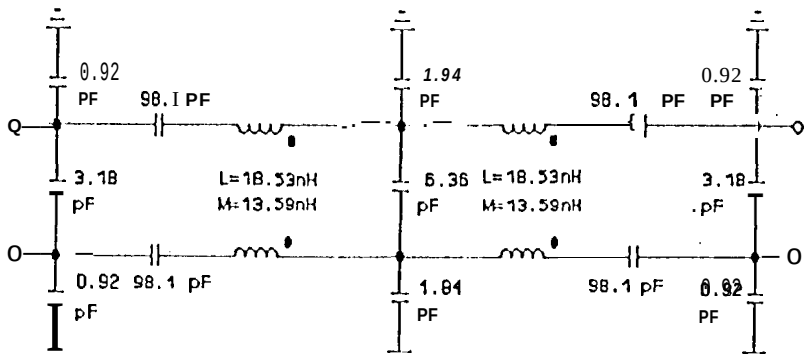


Figure 7. Coupler Model with Series Capacitors

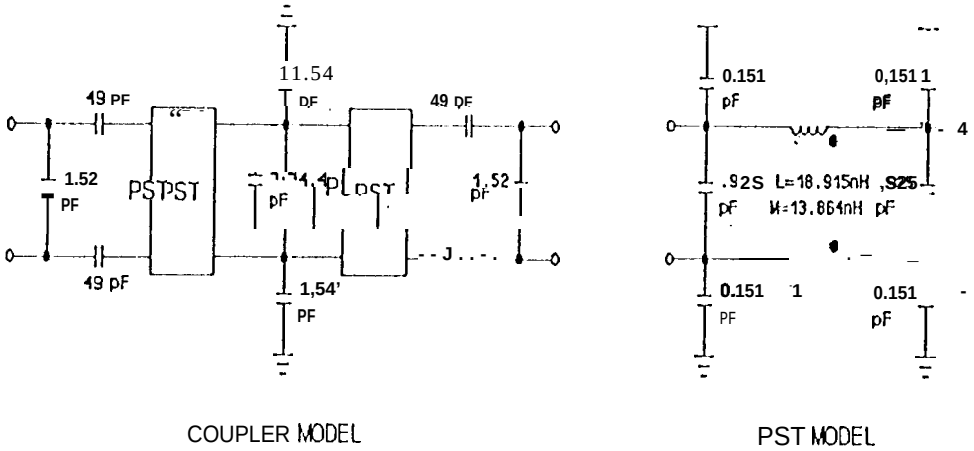


Figure 8. Improved Circuit Model

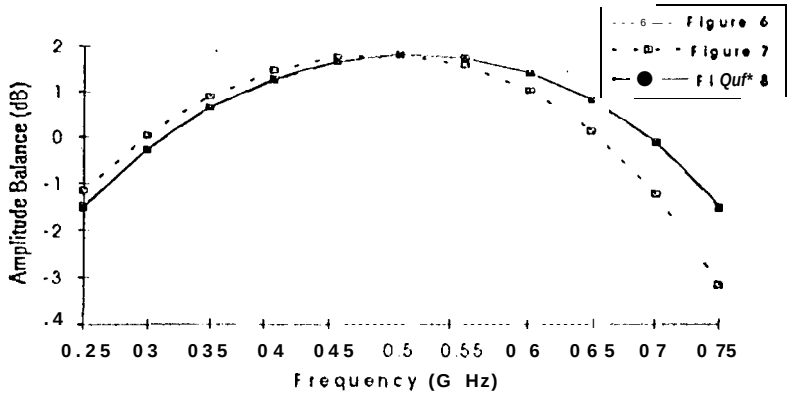


Figure 9a. Amplitude Balance

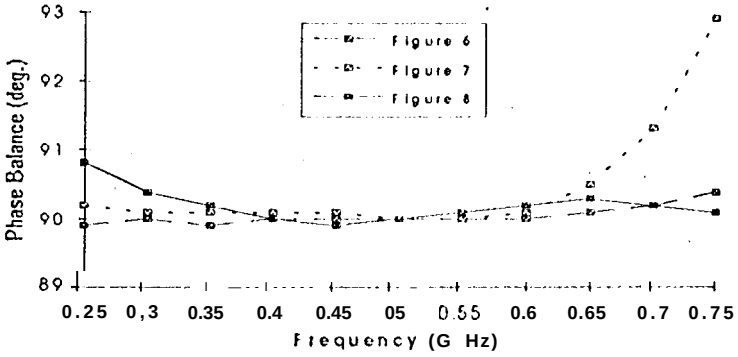


Figure 9b. Phase Balance

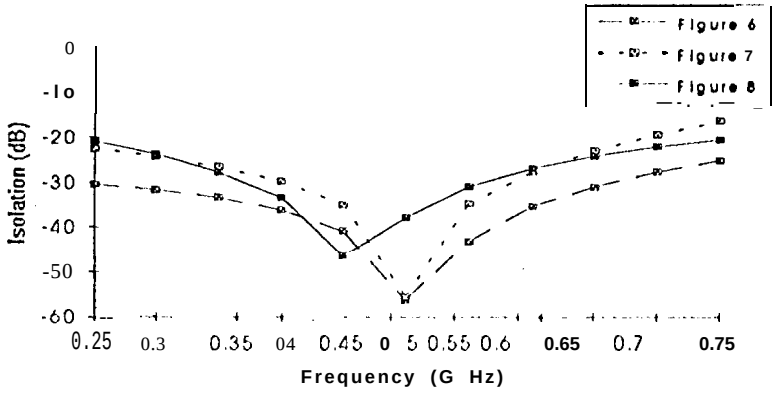
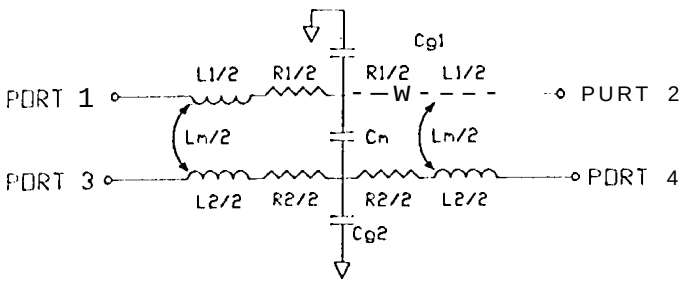
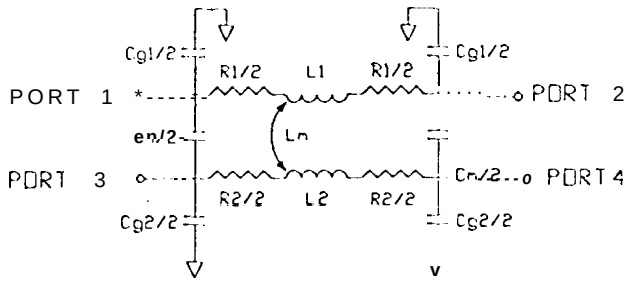


Figure 9c. Isolation

Figure 9. Coupler Performance Comparison

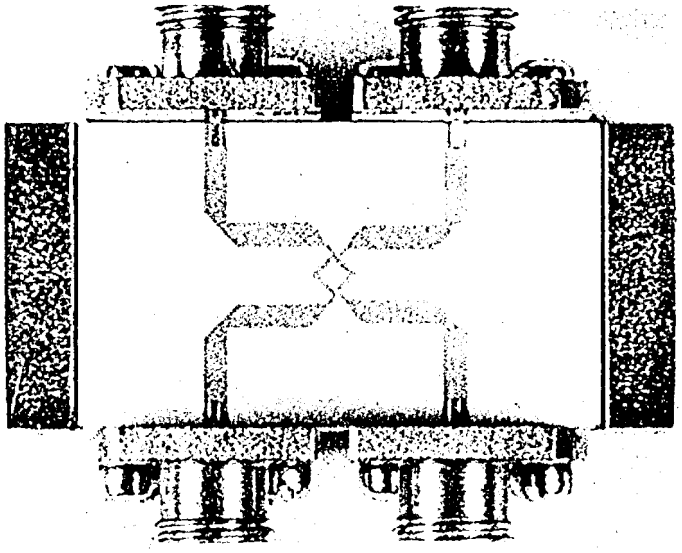


10a. Frlan's Lumped Element. Circuit



10b. Network Equivalent to Frlan's Circuit

Figure 10. Lumped Element Equivalent Circuit Model for PST



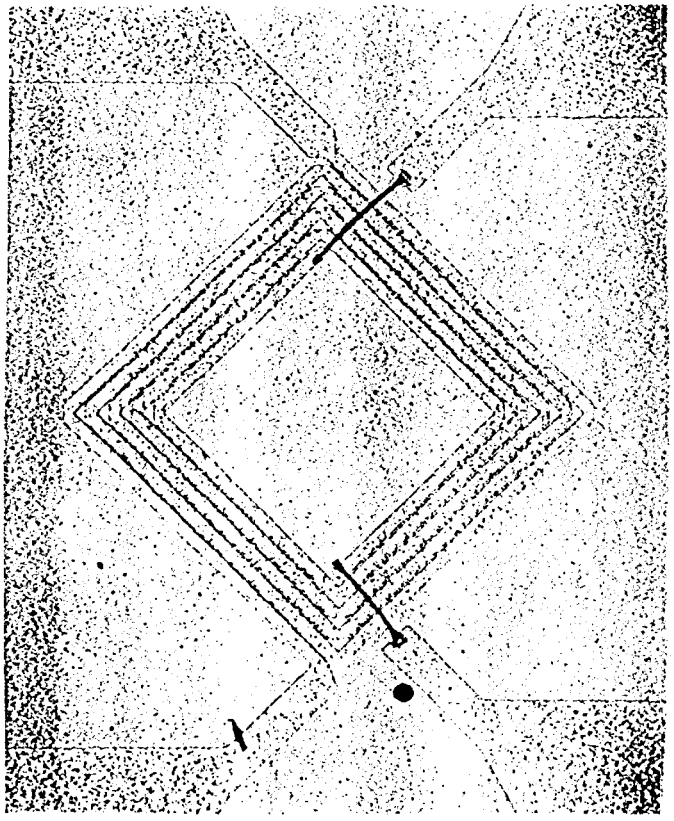


Figure 10c. Planar Spiral Transformer

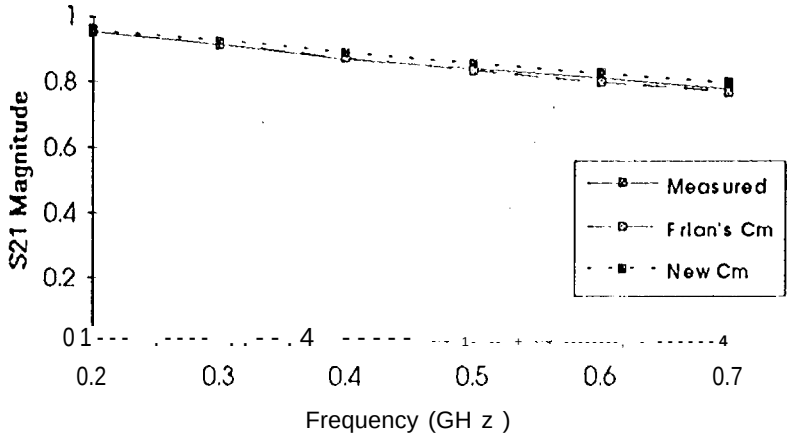


Figure ha. PST S.21 Magni tude

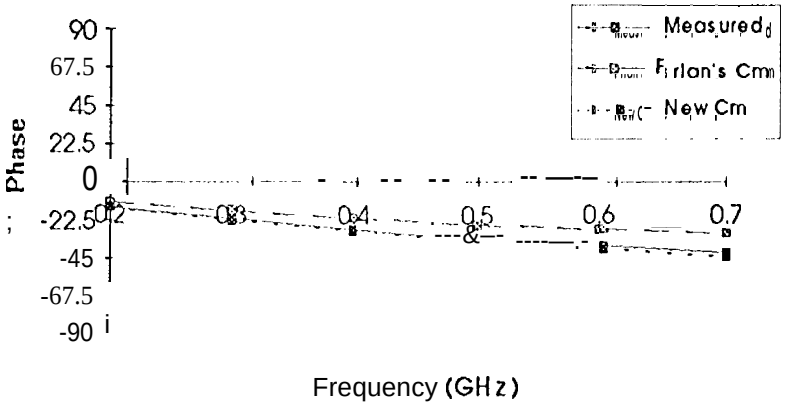


Figure 11b. PST S.21 Phase

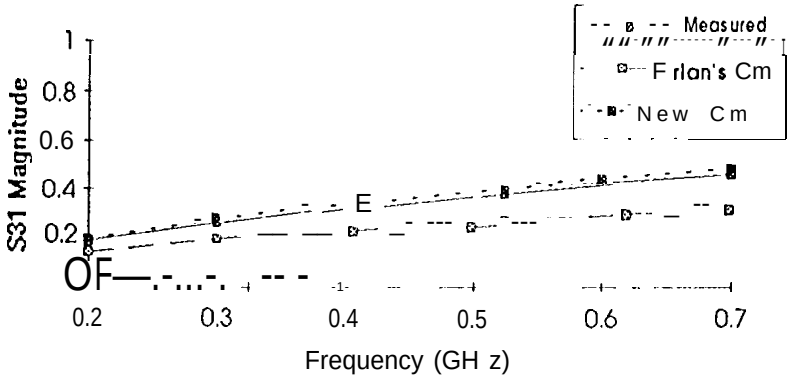


Figure 11c. PST S31 Magnitude

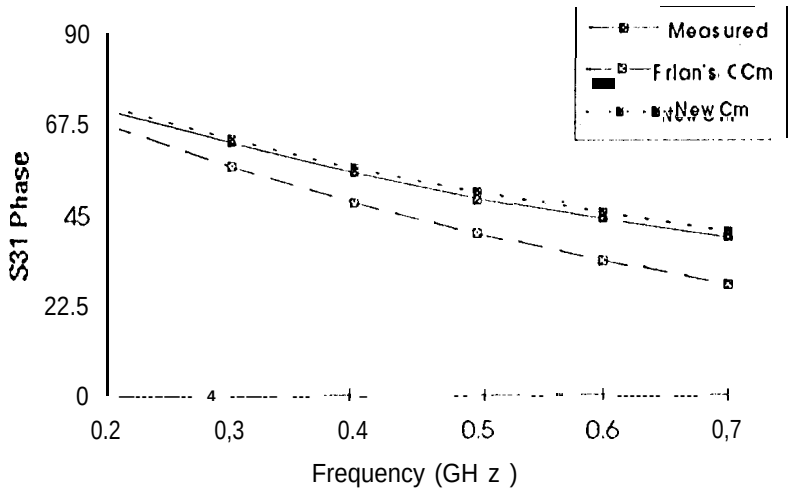


Figure 11d. PSTS31 Phase

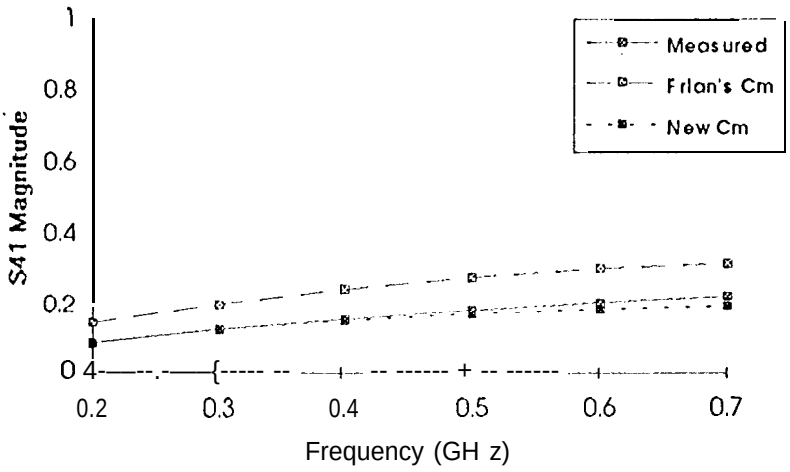


Figure he. PST S41 Magnitude

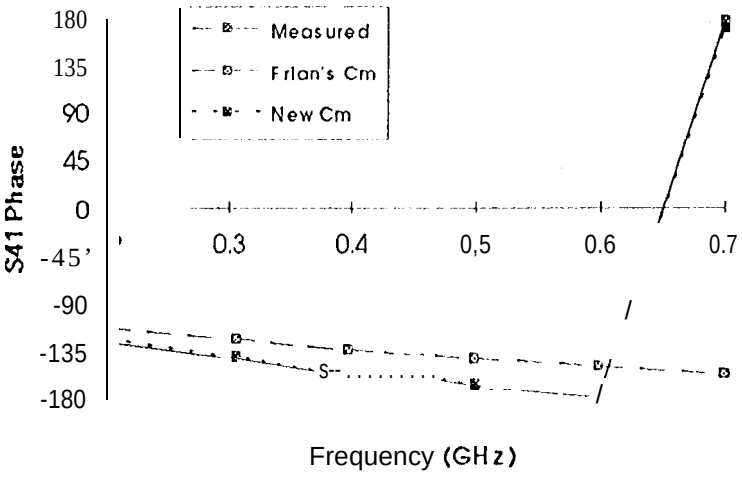


Figure 11f. PST S41 Phase

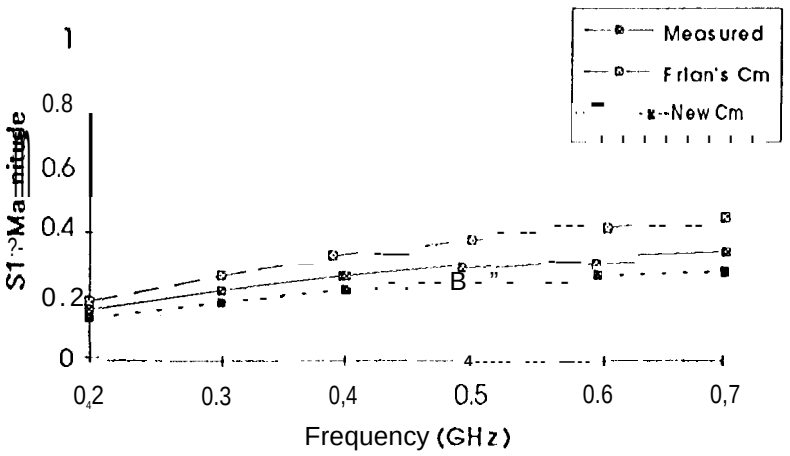


Figure 11g. PST S11 Magnitude

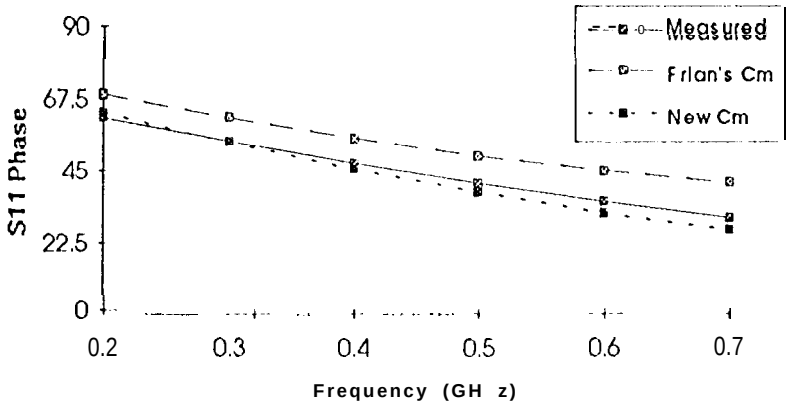


Figure 11h. PST S11 Phase

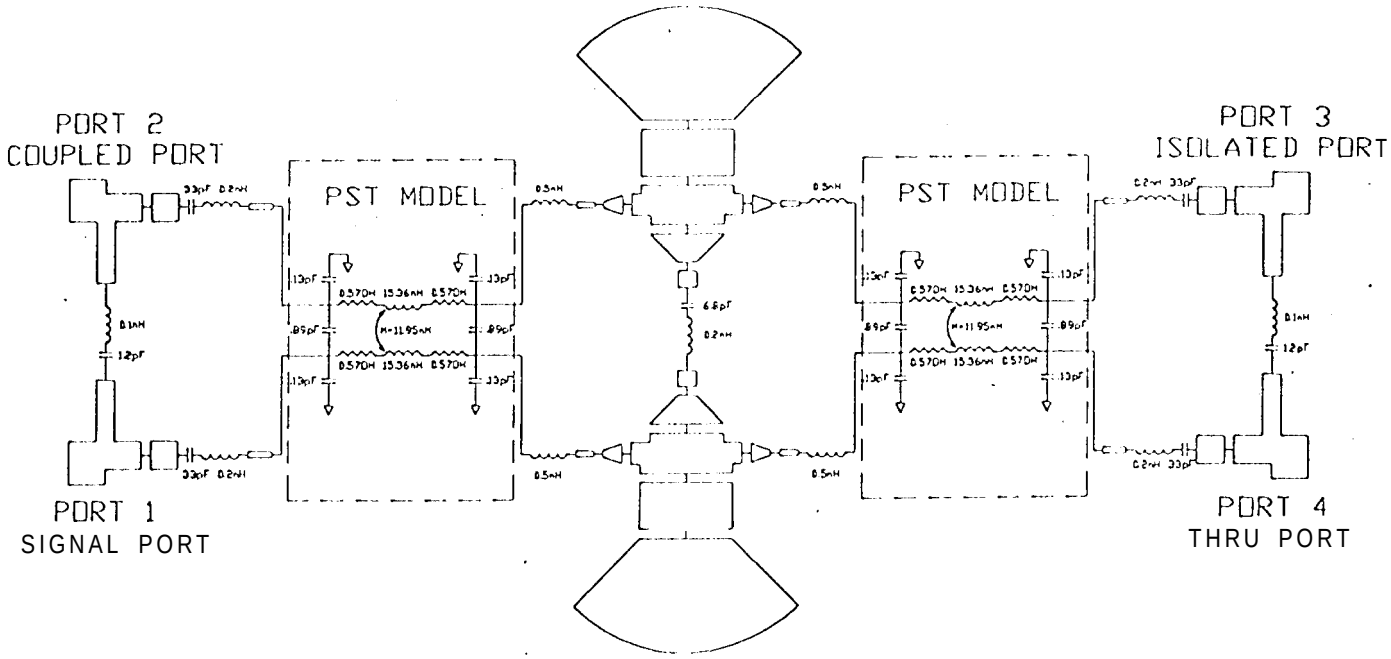


Figure 12 Model for Fabricated Hybrid

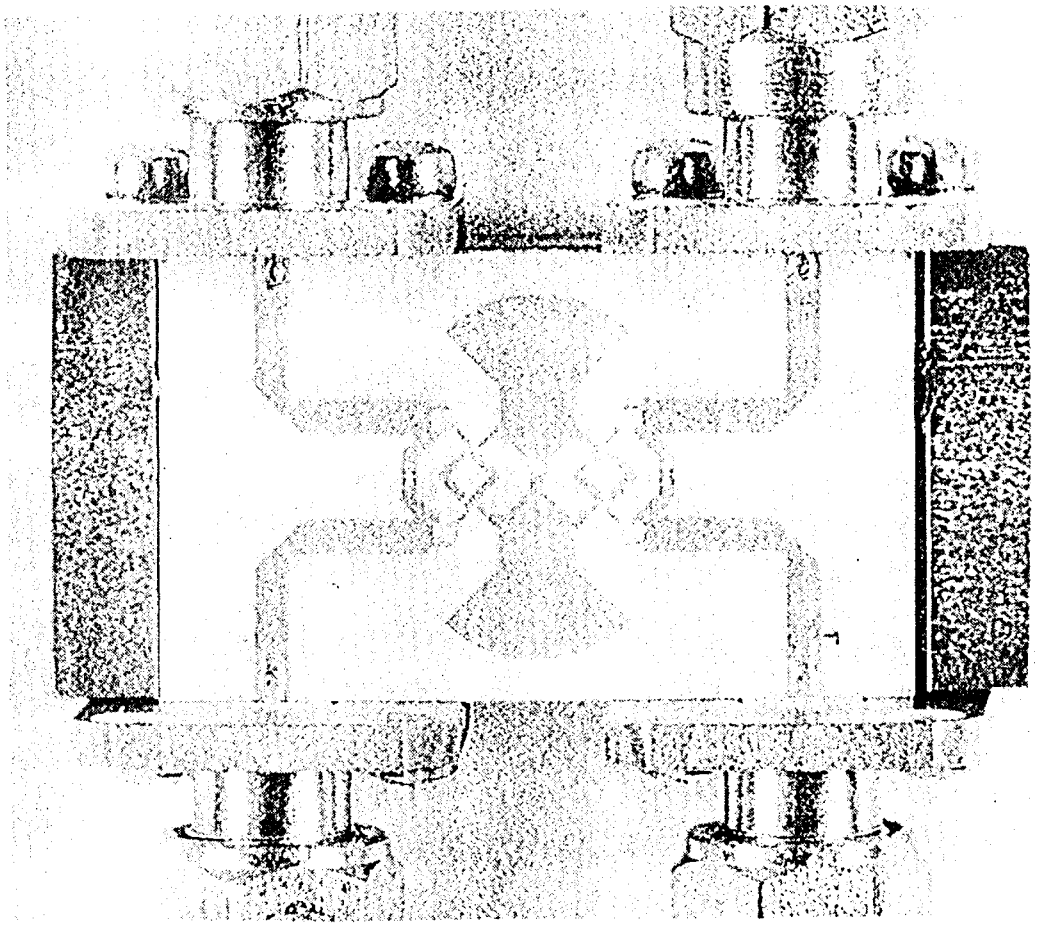


Figure 13. PST Quadrature Hybrid

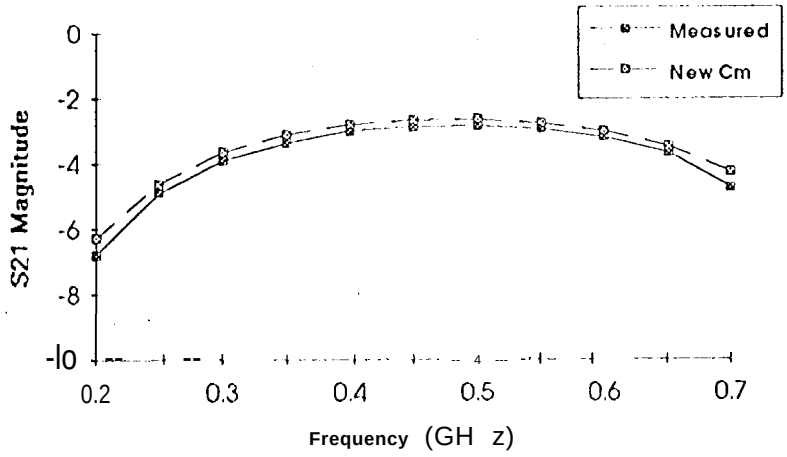


Figure 14a. Hybrid Coupled Port Magnitude

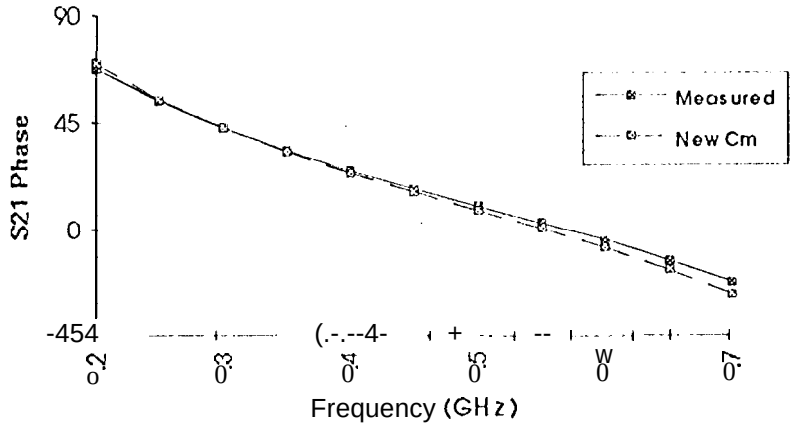


Figure 14b. Hybrid Coupled Port Phase

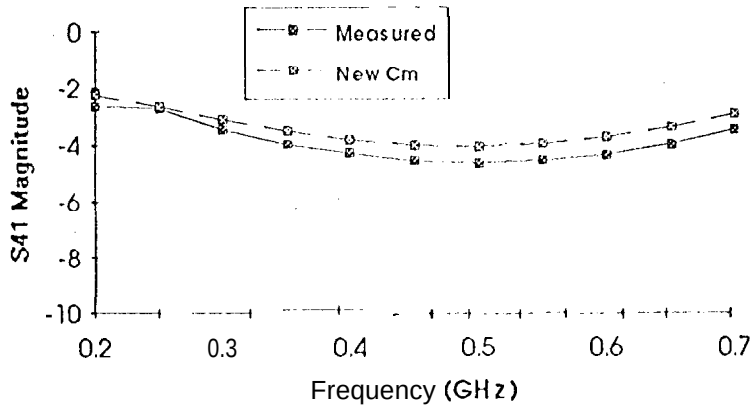


Figure 14c. HybridThruPort Magnitude

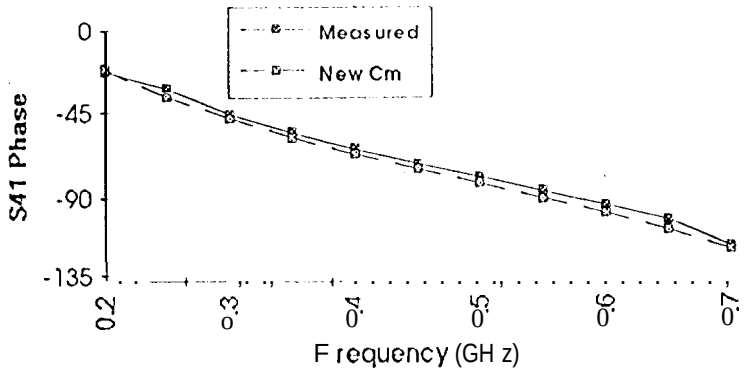


Figure 14d. Hybrid Thru Port Phase

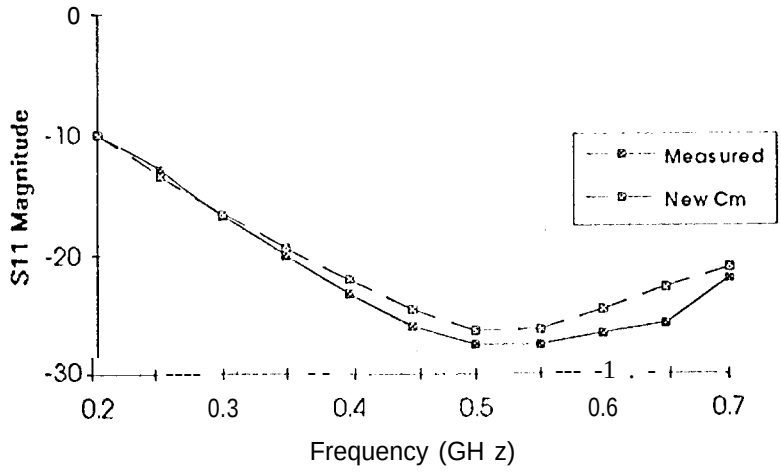


Figure 14e. Hybrid S11 Magnitude

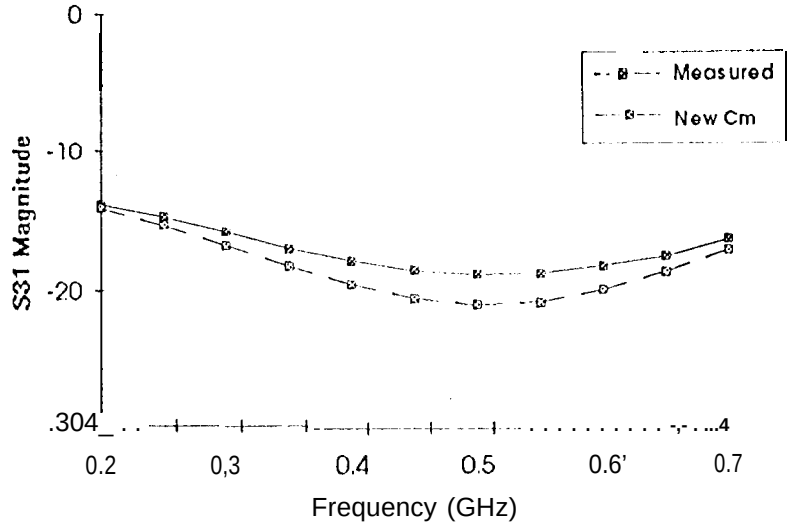


Figure 14f. Hybrid Isolation Magnitude