AN ALGORITHM FOR STATE-SPACE FREQUENCY DOMAIN IDENTIFICATION WITHOUT WINDOWING DISTORTIONS

David S. Bayard
Jet Propulsion Laboratory
California Institute of Technology
4800 Oak Grove Drive
Pasadena, CA 91109

ABSTRACT

An algorithm is presented for identifying state space models from frequency domain data. The main advantage of this approach is that it avoids windowing distortions associated with other frequency domain methods. Other advantages are that an arbitrary frequency weighting can be introduced to shape the estimation error, and the system order can be overspecified. The numerical properties are demonstrated on real physical data taken on a complex flexible structure, leading to the successful identification of a multivariable (4-input/3-output) 100 state model over a bandwidth of 100 Hertz. The results indicate that the algorithm would be useful in applications requiring the accurate identification of high - order systems over wide bandwidths.

1. INTRODUCTION

Recently, it has been found that realization algorithms based on Markov parameters can be effectively applied to the problem of state-s~mcc system identification. Such algorithms include the original minimal realization approach of Ho and Kalman [9], and more recent improvements based on the singular value decomposition, i.e., Kung's algorithm [14], the ERA algorithm of Juang and Pappa [1 I], and the ERADC algorithm of Juang, Cooper and Wright, [12]. In such methods, the Markov parameters are used to determine a balanced state-slmcc model, and the model order is determined by the Hankel singular values. This leads to a clear trade-off between model order and identification quality in terms of singular value plot. This approach also takes advantage of the excellent numerical properties of balanced state-space realizations, for application to systems with lligller-order dynamics.

One difficulty that arises when applying realization algorithms for system identification is that the Markov parameters are difficult to measure directly. This problem has led researchers to develop state-space realization algorithms from input/output data. Such approaches include the method of Moonen ct. al. [18], the CVA method of Larimore [15][16], as well as the observer-lxiscd approaches of Juang and co-workers [13][21]. To date, these algorithms have been developed primarily in the time-domain.

often in practice, frequency rather than time domain data is available. With the new generation of sophisticated microprocessor based spectrum analyzers and automatic test equipment, this is becoming more often the case. In order to apply realization algorithms for identification in this case, one must compute the Markov parameters from frequency data. It is at this point that windowing distortion are often introduced. For example, an Inverse Discrete Fourier Transform (IDFT) of the frequency data provides an estimate of the Markov parameter sequence which is distorted by time-aliasing effects [19]. Alternatively, the ERA-FD algorithm of Juang and Suzuki [1 O] has been developed to construct a state-s])acc realization without taking an IDFT. Unfortunately, however, the ERA-FD algorithm uses a recursive formula whit.]1 is only exact if the plant impulse response dies out, completely within a time window equal to the number of frequency grid points. This is exactly the same assumption required to prevent time-aliasillg distortions when determining the Markov parameters from an ID FT.

In this paper, the State-Space Frequency Domain (**SSFD**) algorithm, is introduced for identifying state-space models from frequency response data. The SSFD algorithm utilizes frequency data as a startling point, but is free from the windowing distortions mentioned above. The key idea is to curve-fit the frequency domain data by minimizing a 2-norm error criterion, as outlined in [4], and then to determine the system Markov parameters through a linear (always invertible) algebraic relation. This two-step approach avoids windowing distorti ons by eliminating implicit time-aliasing effects. Other advantages of the SSFD approach arc that the system order can be overspecified in the curve-fit step, and an arbitrary frequency weighting can be introduced to weight the curve fit error. Frequency weighting is useful for optimizing the estimate with respect to noise properties, or for shaping the estimation error in control design applications [6].

2. BACKGROUND AND NOTATION

The frequency domain identification problem can be formulated as the problem of finding a n_u-input/n_y-output rational transfer function matrix $G(z^{\text{“}})$ which minimizes the 2-norm of the error between itself and specified frequency domain data $\mathcal{G}$, i.e.,

$$\min_{G} \sum_{i=1}^{m} \mathrm{w}^2(\omega_i) \left\| \mathcal{G}(\omega_i) - G(e^{-j\omega_i T}) \right\|_f^2 \tag{1}$$

Here, the data $\mathcal{G}(\omega_i), i = 1,\ldots, m$ is given by noisy values of the transfer function matrix evaluated over a grid of m frequency points; $\mathrm{w}(\omega_i)$ is a specified weighting function of frequency; and the Frobenious norm is defined as,

$$||X||_f^2 = Tr\{X^* X\} \tag{2}$$

with "* " denoting the complex conjugate transpose. For optimization purposes, the transfer function matrix $G(z^{-1})$ is considered to be in the form of the ratio of a matrix numerator polynomial $B(z^{-1})$ and an nth-order monic scalar denominator polynomial $a(z^{-1})$, i.e.,

$$G(z^{-1}) = \frac{B(z^{-1})}{a(z^{-1})} \tag{3}$$

$$B(z^{-1}) = B_0 + B_1 z^{-1} + \ldots + B_n z^{-n} \tag{4a}$$

$$a(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_n z^{-n} \tag{4.b}$$

where $B_k \in \Re^{n_y \times n_u}, k = o, \ldots, n$.

Several algorithms arc presently avail able for solving (1). These methods tend to be of two types, fixed-~mi]lt iterations [1][7][17][22][26][27], or fixed-point iterations combined with modified gradient methods [4] [24] [27] [29]. For example, a simple but approximate algorithm found in the work of Sanathanan and Koerner [22] is given by the following fixed-l)c)int iteration, (denoted here as the *SK* iteration),

SK Iteration:

$$a^{k+1}, B^{k+1} = \arg\min_{a,B} \sum_{i=1}^{m} \mathrm{w}^2(\omega_i) \left\| \frac{1}{a^k(e^{-j\omega_i T})} \left(\mathcal{G}(\omega_i) a(e^{-j\omega_i T}) - B(e^{-j\omega_i T}) \right) \right\|_f^2 \tag{5}$$

with initial condition $a^0 = 1, B^0 = 0$ With a^k fixed at each iteration, the cost function in (5) is quadratic. Hence, the SK iteration is implemented as a sequence of linear least squares problems.

Remark 1 The original work of Sanathanan and Koerner [22] is formulated in the Laplace s domain. Details of the formulation in the z-domain with some practical improvements can be found in [5] for single- ilq)ut/single-out put systems and in [4] for multivariable systems. ∎

With no noise and correct choice model order, the SK iteration converges to the optimal solution of (1) in a *single* iteration since the error in (5) vanishes identically. In the presence of noise and/or incorrect model order, the SK iteration converges to a value which is not generally optimal in any sense. However, it has been shown by Whitfield [27] that the algebraic condition satisfied by the fixed point of the SK iteration differs from a vanishing gradient only by a second-order term in the residual. Hence, while the SK iteration is not generally optimal, it is often close, Hence, a useful approach taken in Bayard [4] is to use the SK iteration to initialize a Gauss-Newton (GN) iteration which forces the gradient to vanish upon convergence. A multivariable formulation of the SK and GN iterations as well as sparse matrix methods for speeding up computation and reducing memory requirements can be found in Bayard [4].

Remark 2 One common difficulty in minimizing cost (1) by complex curve fitting, is that an estimate of plant order n is required to be known a-priori. Since this is generally not the case in practice, a tedious cut and try process is usually required to find the best model order. This difficulty is overcome in the SSFD algorithm introduced in the next section by allowing the polynomials B and a to be over.~pccijid, and then determining model order based on Hankel singular values. ∎

Remark 3 At this point, it is noted that state-space models can be constructed directly from polynomials B and a using standard canonical forms (i.e., companions form, controllable/observable canonical form, Jordon canonical form, etc.). However, aside from the problems of model order mentioned in Remark 2, the problem with this simplistic approach is that *extreme numerical* sensitivities are known to result from using state-s~)zicc realizations based directly on the polynomial coefficients (see for example, Laub and Little [20] for a discussion of numerical sensitivities of companion forms, cent rollable/observable canonical forms, etc.). The Jordon canonical form offers no relief, since this requires the factorization of polynomials a and B. The factorization of polynomials is inherently poorly conditioned, particularly when the polynomials are higll-order, or if any roots are in close proximity of cinch other (cf., Wilkinson [28]). The SSFD algorithm introduced in the next section avoids numerically sensitive polynomial manipulations and canonical forms by constructing a balanced stat,c-space realization directly from Markov parameter estimates. ∎

3. STATE-SPACE 1UWQUENC% DOMAIN IDENTIFICATION

In this section, a new algorithm is introduced for state-sl)acc model identification from frequency response data. The key idea is to overparametrize the polynomials B and a when minimizing (1), and then to determine the system Markov parameters through a linear algebraic relation.

Given G', one can divide $a(z^{-1})$ into 11(2'l) o give the Markov parameter sequence $\{H_i\}$,

$$\frac{B(z^{-1})}{a(z^{-1})} = \sum_{i=0}^{\infty} H_i z^{-i} \tag{7}$$

which gives upon cross-multiplying,

$$B_0 + B_1 z^{-1} + \ldots + B_n z^{-n} = (1 + a_1 z^{-1} + \ldots + a_n z^{-n}) \sum_{i=0}^{\infty} H_i z^{-i} \tag{8}$$

Equating coefficients of the first N powers of z^{-1} in (8) gives the following system of linear

equations,

$$\begin{vmatrix} I & 0 & \cdots & \cdots & \cdots & \cdots & o \\ a_1 I & I & & & & & \\ & a_1 I & I & \ddots & & & \\ a_n I & \ddots & a_1 I & I & \ddots & & \\ 0 & \ddots & \ddots & \ddots & \ddots & & \\ & \ddots & \ddots & \ddots & \ddots & & 0 \\ o & \cdots & 0 & a_n I & \cdots & a_1 & I \end{vmatrix} \begin{vmatrix} H_0 \\ H_1 \\ \vdots \\ H_n \\ H_{n+1} \\ \vdots \\ \vdots \\ H_N \end{vmatrix} = \begin{vmatrix} B_0 \\ B_1 \\ \vdots \\ B_n \\ 0 \\ \vdots \\ \vdots \\ 0 \end{vmatrix} \tag{9}$$

Given the estimated polynomial $a(z^{-1})$ anti polynomial matrix Ii'(.z^{-1}), the multivariable Markov parameters $H_i \in \Re$ ""x"", $i =$ O, ..., N can be calculated by solving the above system of equations. Sine.c the matrix to be inverted is lower triangular with ones on the diagonal, it is always invertible and a solution always mists. Furthermore, since this system of equations is block triangular it can be solved recursively by backsubstitution.

Finally, a balanced state-sl)acc realization is determined from the Markov parameters $\{H_i\}$ using the ERA algorithm [1 1]. With this approach, the model reduction is performed systematically in terms of the Hankel singular values, and leads to a desired reduced-order balanced state-sl)acc realization.

The entire procedure for determining a state-s~mc.c model from frequency domain data is summarized below,

State-Space Frequency Domain (SSFD) Identification A lgorithm

Step *1.* Given frequency response data $\mathcal{G}(\omega_i)$ i = 1,..., m, solve for the transfer function G which minimizes the following 2-norm criteria,

$$\min_G \sum_{i=1}^{m} \mathrm{w}^2(\omega_i) \left\| \mathcal{G}(\omega_i) - G(e^{-j\omega_i T}) \right\|_f^2 \tag{lo}$$

where,

$$G(z^{-1}) = \frac{B(z^{-1})}{a(z^{-1})} \tag{11}$$

$$B(z^{-1}) = B_0 + B_1 z^{-1} + \ldots + B_n z^{-n} \tag{12}$$

$$a(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_n z^{-n} \tag{13}$$

Here, the model order n should be chosen as an upper bound on the true plant order so that the optimization problem is *overparametrized.* The optimization of (10) can be performed approximately using the SK iteration (5), or more precisely using a combination of the SK iteration and the Gauss-Newton algorithm as outlined in [4].

Step 2. Choose any $N \geq 2n$, and solve for Markov parameters H_i, i = O, ... N by inverting the system of equations (9). This is most rosily done by backsubstitution, giving rise to the following recursive formula,

$$H_0 = B_0 \tag{14a}$$

$$H_k = B_k - \sum_{j=1}^{k} a_j H_{k-j}; \quad k = 1, ..., n \tag{14b}$$

$$H_k = -\sum_{j=1}^{n} a_j H_{k-j}; \qquad k = n+1, ..., N \tag{14c}$$

Step 3. Choose any r and s such that $r + s \leq N$ and $min(r, s) \geq n$, and form the Hankel type matrices $H(0), H(1) \in \Re^{(r \cdot n_y) \times (s \cdot n_u)}$ where,

$$H(0) = \begin{bmatrix} H_1 & H_2 & \cdots & H_s \\ H_2 & H_3 & \cdots & H_{s+1} \\ \vdots & \vdots & \ddots & \vdots \\ H_r & H_{r+1} & \cdots & H_{r+s-1} \end{bmatrix} \tag{15}$$

$$H(1) = \begin{bmatrix} H_2 & H_3 & \cdots & H_{s+1} \\ H_3 & H_4 & \cdots & H_{s+2} \\ \vdots & \vdots & \ddots & \vdots \\ H_{r+1} & H_{r+2} & \cdots & H_{r+s} \end{bmatrix} \tag{16}$$

Step 4. Compute a balanced state-slmcc realization using the ERA algorithm, i.e.,

4.a Compute the SVD of $H(0)$ to give,

$$H(o) = U \Sigma V^T \tag{17}$$

where $\mu = min(r \cdot n_y s \cdot n_u)$, $U \in \Re^{(r \cdot n_y) \times \mu}$, $V \in \Re^{(s \cdot n_u) \times \mu}$, $\Sigma = diag\{\sigma_1, ..., \sigma_\mu\}$, and the singular values are ordered by size, $\sigma_i \geq \sigma_{i+1}$, $i = 1, ... \mu - 1$

4.b Plot the Hankel singular values σ_i to visualize trade-off between model order and identification accuracy, and truncate to keep only q singular values.

4.c Form q-th order state-space realization as,

$$x_{k+1} = A_q x_k + B_p u_k \tag{18}$$

$$y_k = C_q x_k + D u_k \tag{19}$$

where,

$$A_q = \Sigma_q^{-1/2} U_q^T H(1) V_q \Sigma_q^{-1/2} \tag{20.(1)}$$

$$B_q = \Sigma_q^{1/2} V_q^T E_u \tag{20.b}$$

$$C_q = E_y^T U_q \Sigma_q^{1/2} \tag{20.c}$$

$$D = H(0) \tag{20.d}$$

$$E_y^T = [I_{n_y \times n_y} \quad O] \in \Re^{n_y \times (r \cdot n_y)} \tag{21.a}$$

$$E_u^T = [I_{n_u \times n_u} \quad O] \in \Re^{n_u \times (s \cdot n_u)} \tag{21.b}$$

$$\Sigma_q = diag\{\sigma_1, ..., \sigma_q\}$$

U_q - submatrix formed from first q columns of U

V_q - submatrix formed from first q columns of V

■

Remark 4 The use of the ERA algorithm in Step 4 is only to simplify the presentation. Once the Markov parameters are obtained in Step 3, any state-space realization algorithm can be used in Step 4. For example, the ERADC algorithm of Juang, Cooper and Wright [12] can be used by forming the covariance matrix $R(i) = H(i)H(0)^T$, $i = 0, 1$, taking the SVD as $R(0) = V\Sigma V^T$ and replacing equations (20.a)-(20.d) with,

$$A_q = \Sigma_q^{-1/2} V_q^T R(1) V_q \Sigma_q^{-1/2} \tag{22.a}$$

$$B_q = \Sigma_q^{1/2} V_q^T H(0) E_u \tag{22.b}$$

$$C_q = E_y^T V_q \Sigma_q^{1/2} \tag{22.c}$$

$$D = H(0) \tag{22.d}$$

■

Remark 5 A powerful technique is to use the SSFD algorithm in combination with the frequency data acquired using the multisinusoidal input designs outlined in [3][25] [29]. Such input designs consist of a sum-of-sinusoids at harmonically related frequencies (usually a 2^n FFT grid). Windowing distortions are avoided if data is collected after the system has reached periodic steady-state [3]. The sinusoids can be conveniently phased to minimize peaking in time using a formula given by Schroeder [23], or using more recent improvements based on nonlinear programming [8]. In combination with the SSFD algorithm, this gives a complete "end-to-end" frequency domain identification methodology free from windowing distortions in the noiseless data case. The effectiveness on real data is verified experimentally in the next section, and apparently represents a significant advance in the state of practice. ■

Remark 6 Instead of truncating Hankel singular values for model reduction, it may be more appropriate to retain a high order State-Space model from the SSFD algorithm, and then use more sophisticated frequency weighted methods for model reduction. This approach is particularly useful for control design purposes where one wishes to maintain an accurate reduced-order model over a specified control bandwidth. In this approach, the model order from the SSFD algorithm never needs to exceed $n * min(n_y, n_u)$ since this gives an exact state-sl)acc realization of the polynomial model D/a. ∎

Remark 7 The use of overparametrization in Step 1 is key to obtaining good results with the SSFD algorithm. In the noiseless case, overparametrized curve fits have been characterized in [2]. Using a minimum-norm solution to the overparametrized problem, it is shown in [2] that the extraneous dynamics come in as stable pole-zcm cancellations and hence do not bias the Markov parameter estimates. Furthermore, in the presence of small noise levels, the extraneous dynamics are perturbed to become stable "near" pole-zero cancellations. These are subsequently realized as "nearly unobservable/uncontrollable" subspaces in the state- sl)ac.c model and are removed systematically by truncating small Hankel singular values. As shown in [2], these overparametrization results are only true for the shift z operator, and do not carry over to the Laplace s or δ-rule operators. •

4. NUMERICAL EXAMPLES

Example 1

The first example will study the effect of windowing distortion in various frequency domain algorithms. Consider the sampled-data plant obtained by a zero-ordm hold (at sampling period $\tau = .03$ seconds), on the following 3-mode continuous-time plant,

$$G(s) = \sum_{i=1}^{3} \frac{b_i}{s^2 + 2\zeta\omega_i s + \omega_i^2}$$

where $b_i = 10, 20, 25$, $\omega_i/(2\pi) = .41, 2\ .3, 9$, for $i = 1, 2, 3$ respectively, and $\zeta = .01$. Noiseless frequency domain data on a uniform 256 point grid on the unit circle, between O and $1/(2T) = 16.66$ Hertz is shown as the solid line in Fig. 1 (magnitude) and Fig. 2 (phase). Noise has not been added so that the estimation error can be studied with respect to windowing distortion.

The first algorithm to be studied uses the Markov parameter estimates obtained by the inverse DFT (IDFT) of the frequency data (suitably symmetrized to give a real time-domain signal). These Markov parameter estimates are used in an ERADC algorithm [12] with $r = 12$, $s = 24$ in (15) and (16). The singular values fall off sharply after 6 states and the resulting 6th order state-s~)acc realization in shown as the dashed line marked as IDFT/ERADC in Fig. 1 (mag) and Fig. 2 (phase). The distortion due to windowing effects is clearly seen in both the magnitude and phase plots.

The ERA-FD algorithm [1 O] is studied next. 12 rows are used in the "shifted transfer function matrix" described in [10]. A 6th order state-spa.cc realization is constructed from the data and shown as the dashed line marked ERA-FD in Fig. 1 (mag) and Fig. 2. (phase). As in the previous method, the distortion due to windowing effects is clearly seen in both plots.

The SSFD algorithm is studied next, where $n = 12$ in Step 1; equation (5) is iterated 5 times; N = 100 in Step 2, and r = 12, s = 24 in Step 3. The ERADC realization is used in Step 4. (see Remark 4). The singular values fall off sharply after 6 states so that we choose $q = 6$. The resulting 6th order state-s~mcc model is shown as the dash-dot line marked SSFD in Figs. 1 and 2. The result is so ac.curate that it is hardly discernable from the actual data. However, there is still a small discrepancy (seen in the first valley to the right of the first mode) which may be attributable to computer round-off error. overall, the distortionless character of the SSFD estimate is readily apparent compared to the IDFT/ERADC and ERA-FD approaches.

Example 2

This example demonstrates the SSFD algorithm on experimental data taken from the J 1'1, Advanced Reconfigurable Control Testbed shown in Fig. 3. A 4-input, 3-output transfer function is considered, where each actuator is an active strut, and each sensor is an accelerometer. The frequency response data is obtained using a 512 Schroeder phased sum-of-sinusoids input design at a sampling rate of 200 Hertz (background on the design of Schroeder-phased inputs, and their use in unbiased estimation can be found in [3]). The magnitude response is shown as the dashed line in Fig. 5 (phase is available but not shown).

Let the overparametrized model order be $n = 60$ in Step 1 of the SSFD algorithm, and let $w(\omega_i) \equiv 1$, (i. e., a uniform weighting). Since there are 12 numerator polynomials and 1 denominator polynomial, this requires the simultaneous estimation of 780 parameters. The SK algorithm is iterated 12 times, using the sparse matrix SVD algorithm developed in [4]. The sparse matrix SVD algorithm was indispensable for this problem, reducing RAM requirements by better than an order of magnitude (from approximately 60 Megabytes to 6 Megabytes) and reducing computation time two orders of magnitude (from approximately 30 hours to 20 minutes).

Steps 2, 3, and 4 (the realization portion) of the SSFD algorithm are computed using 7' = 61, s = 61. The singular values are plotted in Fig. 4. It is seen that there is a sharp drop off at 180 states since there is an *exact* state-space realization of this size (i. e., $180 = \min(n_y, n_u) * 60$). However, for demonstration purposes, a model order of 100 is chosen (i. e., g = 100). This is also reasonable since the error from the singular value plot is seen to have dropped approximately 3 orders of magnitude at this point.

A magnitude plot of the state-space model realized from Step 4 of the algorithm is shown in Fig. 5 (solid line) superimposed on the response data (dashed line). The state space model is stable, and is seen to match the data well over the full 100 Hertz bandwidth.

5. CONCLUSIONS

The SSFD algorithm has been introduced for identifying state-space models from frequency response data, A key feature is that in the noiseless case the SSFD algorithm avoids windowing distortions inherent to other frequency domain algorithms. This important property was verified on a 3 mode numerical example. In the presence of noisy data, the estimate can be frequency weighted to shape the error, as is desired in control applications. The model order in the SSFD algorithm can be overspecified since the final system order is determined based on Hankel singular values, and the state-space model is constructed without polynomial factorization or numerically sensitive canonical forms.

The SSFD algorithm was tested on a flexible structure experimental data set, demonstrating the successful identification of a multivariable (4-input/3-output) 100 state model over a bandwidth of 100 Hertz. The general results are encouraging, and indicate that the approach would be useful in such areas as adaptive optics, flexible structures, helicopter/rotocraft testing, high performance tracking, or any other applications requiring the accurate identification of Iligll-order systems over wide bandwidths.

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FIGURES

Fig. 1 Magnitude plots for 3-mode example: Noiseless data (solid); IDFT/ERADC (dashed); ERA-FD (dashed); SSFD ((lash-dot,).

Fig. 2 Phase plots for 3-mode example: Noiseless data (solid); IDFT/ERADC (dashed); ERA-FD (dashed); SSFD (dash-dot).

Fig. 3 J 1 'L Advanced Reconfigurable Control (ARC) Testbed

Fig. 4 Hankel singular values

Fig. 5 Multivariable experimental identification results using SSFD algorithm: Raw experimental data (dashed); Identified state-space model, 100 states, 4-input/3-output (solid).

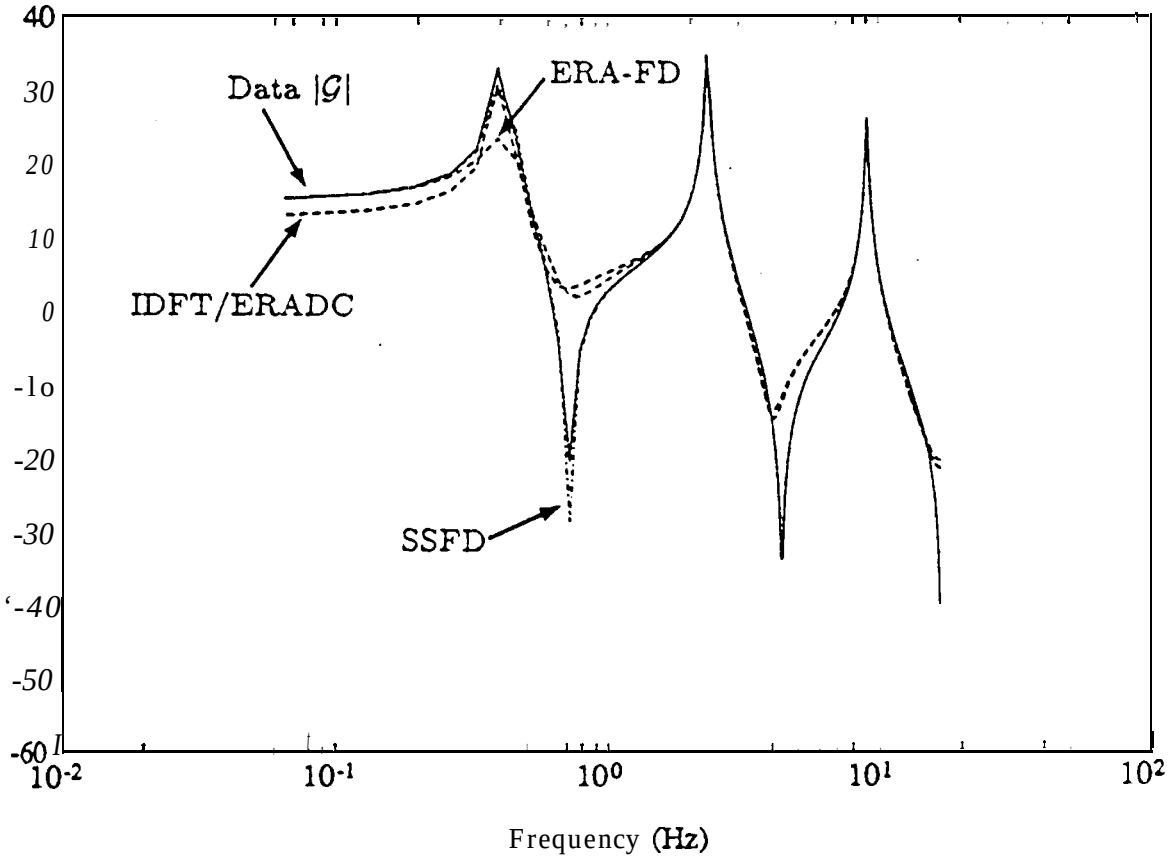

Fig. 1 Magnitude plots for 3-mode example: Noiseless data (solid); IDFT/ERADC (dashed); ERA-FD (dashed); SSFD (dash-dot).

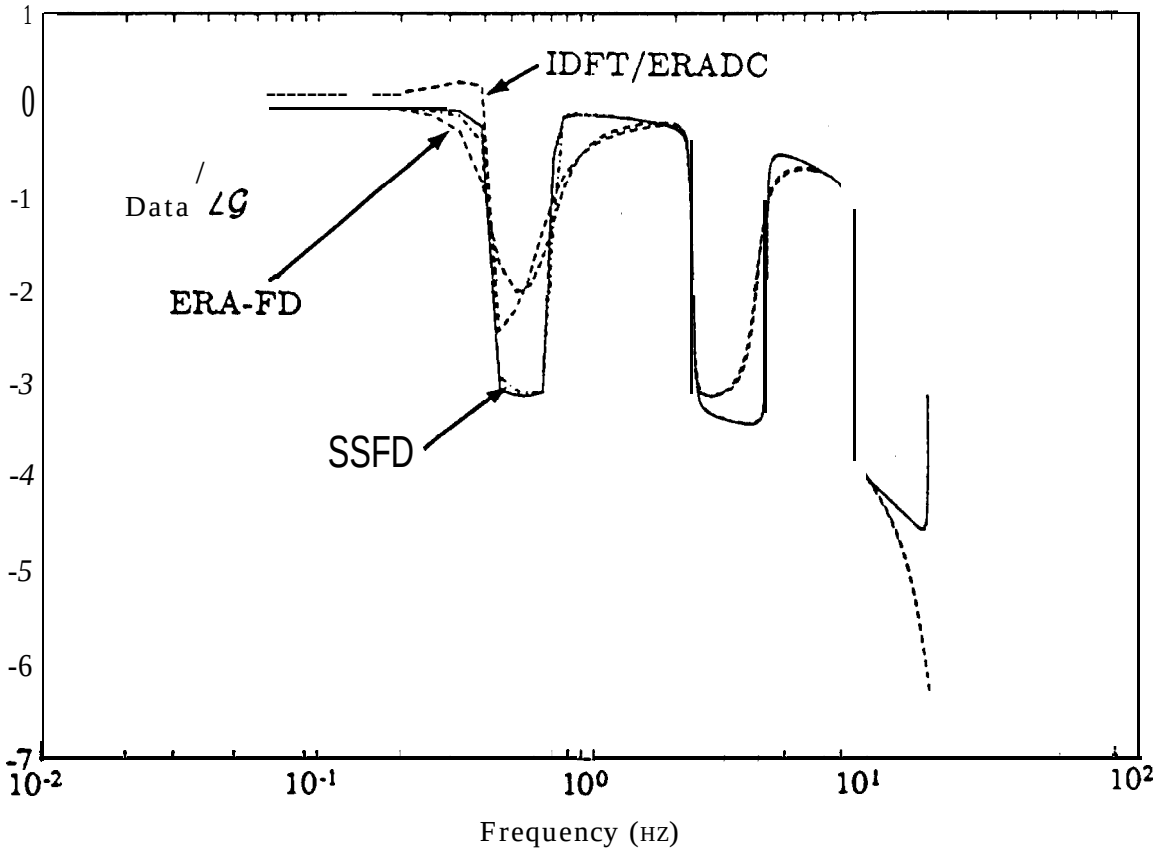

Fig. 2 Phase plots for 3-mode example: Noiseless data (solid); IDFT/ERADC (dashed); ERA-FD (dashed); SSFD (dash-dot).

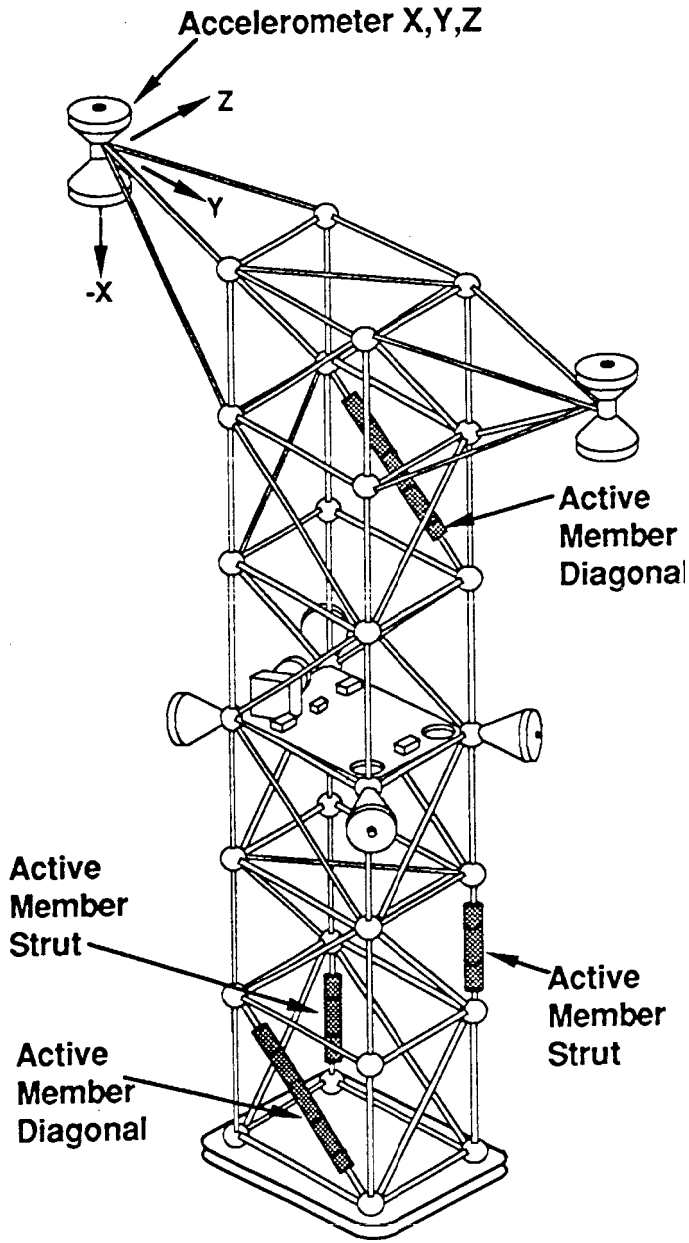
Accelerometer X,Y,Z
Z
Y
-X
Active
Member
Diagonal
Active
Member
Strut
Active
Member
Strut
Active
Member
Diagonal

Fig 3

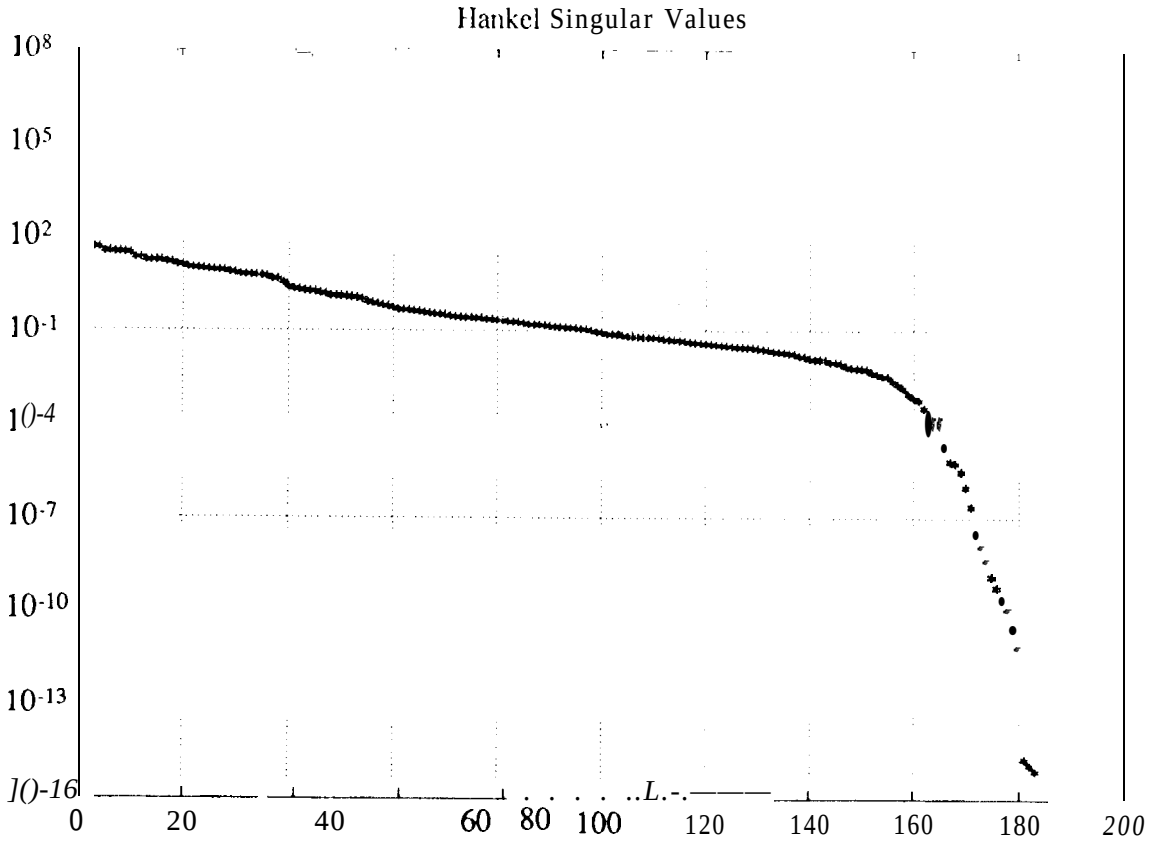
Hankel Singular Values
10^8
10^5
10^2
10^-1
10^-4
10^-7
10^-10
10^-13
10^-16
0
20
40
60
80
100
120
140
160
180
200

FIG. 4

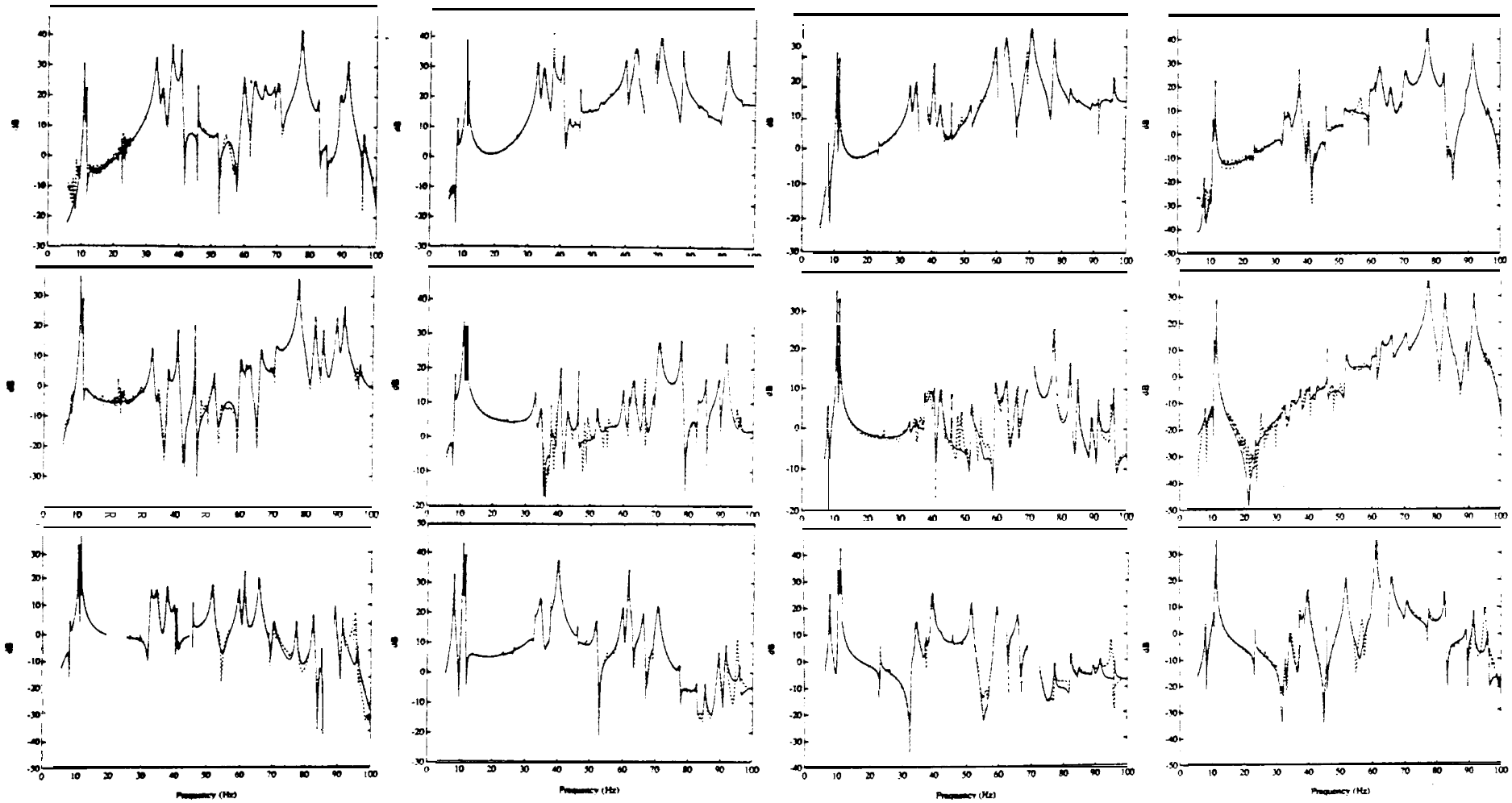

FIG. 5