LONG-TERM PERSISTENCE OF SOLAR ACTIVITY

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Abstract. We examine the question of whether or not the non-periodic variations in solar activity are caused by a white-noise random process. The Hurst exponent, which characterizes the persistence of a time series, is evaluated for the series of ^{14}C data for the time interval from about 6000 BC to 1950 AD. We find a constant Hurst exponent, suggesting that solar activity in the frequency range of from 100 to 3000 years includes an important continuum component in addition to the well known periodic variations. The value we calculate, $H \approx 0.8$, is significantly larger than the value of 0.5 that would correspond to variations produced by white noise process. This value is in good agreement with the results for the monthly sunspot data reported elsewhere, indicating that the physics that produces the continuum is a correlated random process and that it is the same type of process over a wide range of time interval lengths.

A Introduction

Early studies of solar activity focused on the search for periodicities. The fundamental discovery of the 1 l-year cycle by Schwabe followed by the discovery of the secular Gleissberg modulation and by the modern spectral findings in studies of the radiocarbon data (Damon and Sonett, 1989), showed that the spectrum of solar activity is composed of a number of basic frequencies superposed on a background continuum. This background continuum was at first assumed to be determined by short term variations with a random distribution (Vitinskii, 1976). However the rediscovery of the Grand Minima of solar activity (Eddy, 197S) led to a re-examination of the question of the nature of the non- periodic part of the variations of the sun's activity. Here wc address the question of whether or not the non-periodic continuum variations arc produced by a white noise random process. In a time series, white noise is a process in which the amplitudes of the variations at different times arc independent of onc another. The test for independence and search for correlations in a time series can be carried out by usc of a simple analytical tool, the calculation of the Hurst exponent. This exponent describes the coherence or persistence in a long time series. It was developed in the context of some problems in hydrology and has been widely used in that field (see Mandelbrot and Wall is, 1969; Feder, 1988 for reviews). It was first used in solar physics when it was applied to a series of monthly sunspot numbers by Mandelbrot and Wallis (1969). They found that continuum variation of the sunspot numbers in the time interval range of from 1 to 200 years was not random in that it had a Hurst exponent significantly larger than 1/2. In this paper we investigate solar activity persistence for time intervals from 100 to 3000 years. We find the Hurst exponent for solar activity using the series of ^{14}C data (Stuiver and Pearson, 19S6) as a proxy.

A Definition of the Hurst exponent

The concept of the Hurst exponent is most easily understood by first considering one- dimensional Brownian motion. We can ask what the position of a particle will be after many steps if, at each step, the displacement is independent of the displacement of the particle" at any other step. In Brownian motion the step lengths arc given by a Gaussian probability distribution. Then, as is well known, the particles mean-squared distance from its starting point will increase as the square root of the time, If we plotted the "mean-squared distance from the starting point in time t " versus the "time interval t " we would get a well defined curve showing that relationship. The relationship is easier to visualize if wc plot the logarithms of the quantities since then we would find that the slope of the line, i.e. the exponent in the relation of distance versus time, is 1/2. Brownian motion is a random process with Hurst exponent equal to 1 /2.

The concept of Brownian motion may be generalized by assuming that the step lengths arc dependent on one another, i.e. correlated. This was done in a series of papers by Mandelbrot and co-workers (see Feder, 1988), in which they discussed the case in which the value of a random function at time t depended on all previous increments at earlier times of an ordinary Gaussian random process with zero average and unit variance. Under these circumstances they showed that, after a large number of steps, the "mean-squared distance from the stating point in time t," would increase as some power H of the time. The power would depend on the time correlations, i.e., how each new step depended on all previous steps. Again if the logarithms of the quantities where plotted, the power in the relation between distance. and time would be given by the slope, that is, by the Hurst exponent H. A Hurst exponent greater than 1/2 indicates that the process is proceeding in such a way that, if in the past we have had a positive increment, then in the future we can expect on average an increase. A Hurst exponent less than 1/2 conversely means that if we have had an increase in the past, then on average we can expect a decease in the future.

The situation is easily imagined by considering gamblers in Las Vegas. There are three kinds of gamblers. The first kind is an optimist and thinks if he has had a run of good luck it will continue because he is on a roll. He is betting that the game is set up with a Hurst exponent greater than 1/2. The second gambler is also an optimist and thinks if he has been losing it will turn around because it has to come out to zero eventually. He is betting the Hurst exponent is less than 1/2. The third type believes to himself to be rational and figures the game is 'set up like Brownian motion (i.e. it is a "even game") and he had better not gamble at all because he is as likely to lose as to win on the next step.

The Hurst exponent is linearly related to the exponent of the power spectrum, $\alpha = 2H + 1$ (Mandelbrot, 1977). For example, the value $H = 1/2$ corresponds to a time series having a -2 power spectrum; the Kolmogorov spectrum for the homogeneous turbulence appears from data with $H = 1/3$ corresponding to $\alpha = 5/3$, Thus, in principle the Hurst exponent can be found by standard Fourier analysis, However in practice (for most geophysical, space physics, and astronomical data) we deal with short and sparse records. This makes the application of Fourier transform difficult. It is very popular to use a substitution for the Fourier analysis; the so called the method of maximal entropy (M F, M). MEM is good for finding discrete lines in the spectrum, but it was not designed for handling the continuous part of spectrum. The Hurst method was designed to be a good tool for characterizing the continuous part of the spectra. In contrast to the power spectral exponent, the Hurst exponent is confined to the unit interval, $0 \leq H \leq 1$, and has clear statistical meaning: it defines a value and sign of the correlation between events in the sequence presented by an observed time series.

A The Hurst Exponent for ^{14}C Data

The original tree-ring high-precision ^{14}C data for dendrohronologically dated 20 year tree-rings intervals were taken from the compilation given in Stuiver and Pearson (1986). The period covered in our study is 7720 years, i.e. 386 data points. The cosmic rays responsible for the production of ^{14}C arc modulated by the solar wind and Earth's rnagnctic fields. The geomagnetic modulation can be approximated by a sinusoidal curve with a period of about 11, 000 years (Damon and Linick, 1986; Damon, Cheng and Linick, 1989). This modulation was removed from the data by subtracting the sinusoidal curve from the original data. The remaining variations have been modulated by the solar wind throughout the heliosphere and reflect solar activity.

The method for the determination of the Hurst exponent from the observation of a time series has been developed by Mandelbrot and Wallis (1969). The method relies on the fact that, compared to a Brownian particle, the excursion of a particles position from its starting point will be large (small) if the Hurst exponent is larger (smaller) than 1/2. This is illustrated in Figure 1 (from Fedex-, 1988) in which we see that the range of values of a parameter (maximum value minus minimum value) increases as the Hurst exponent in the generating function increases. Hence, if wc study the behavior of the range of values in an observed time series, we can estimate the Hurst exponent for the process that produced that time series. Here wc apply this method to a time series of the ^{14}C data which is used as a proxy for solar activity. Following Mandelbrot and Wallis (1 969) wc set the following definitions for the ^{14}C record from t = O to $t = T$, denoting the data time series as $C(t)$: 'I'he accumulated mean value over a time interval τ is,

$$< C >= \frac{1}{\tau}\sum_{t=0}^{\tau} C(t),$$

The accumulated departure from the mean over a time interval τ is

$$X(t,\tau) = \sum_{u=1}^{t}[C(u)- < C >].$$

The difference between the maximum and minimum values of the accumulated departure in this time interval, i.e. the range, is given by

$$R(\tau) = \max_{0 \le t \le \tau} X(t,\tau) - \min_{0 \le t \le \tau} X(t,\tau).$$

It is convenient to usc the normalized range R/S, where $S(\tau)$ is the empirical variance over the same time interval. For each τ, the time t can take $T - \tau + 1$ values, and the average over them will be used to construct the dependence of the normalized range on τ. The slope of the log-log plot of these quantities will give the Hurst exponent.

Figure 2 shows our results for the ^{14}C record. The values of τ plotted were chosen to be equally spaced on the log-log plot. For τ = 1,1?(1) = S(1) = O, for $\tau = 2$, $R/S = 2$ in every case. So the starting point for the analysis is $\tau = 3$. We will normally assume that $\tau \geq 5$. The plotted points arc the averages of the values of log R/S taken for each interval of length τ. The slope of the line along which the points lie is very different from 1/2; which is the slope of the dashed line given in Figure 2. The slope of the line which wc fit to the data using the least squares method is

$$H = 0.84 \pm 02,$$

This indicates a high degree of persistence in the variations of solar activity. The departure of the data from a straight line at high values of τ may be duc to sampling effects and should not be considered indicative of a change in Hurst exponent for these longer term variations. The question of whether the exponent changes for τ larger than 3,000 years needs to be looked into using a data set that covers a longer period of time.

In fitting the data to a straight line to determine the Hurst exponent, not all the points should be considered as being of equal accuracy. When τ is small compared with the total length of the data record, then the number of independent determinations of log R/S is relatively large and the average plotted is a statistically good estimate. However, when τ is comparable to the record length the plotted value of log R/S is not as reliable an estimate. Thus for large values of τ the value determined from our almost 8000 year data set will depend somewhat on the particular 8000 years chosen for analysis. We investigated this effect by using two different 386 point times series generated by a random number generator. We found that the values of log R/S for a given log τ for these two samples were identical except that, at high τ the two curves began to gradually differ from one another. For this reason the three or four points for the largest values of τ should not be considered in fitting the graphed points to a straight line.

The data that we used in this analysis permits us to examine the persistence of solar activity variations of duration of about 100 to 3,000 years. The persistence of variations on shorter time scales was investigated by Mandelbrot and Wallis (1 969) using monthly sunspot number values. We have repeated their analysis as shown in figure 3. In this frequency range the points no longer lie on a straight line. Instead the line is distorted because of the 11 year sunspot cycle, as discussed by Mandelbrot and Wallis. The power in the 11 year cycle is large compared to the power in the continuum at these frequencies. The important point for this study is that the behavior of the continuum is still apparent. We have superposed a line with slope 0.86 to compare the persistence of the continuum in the 1 to 100 year range with that found for the 100 to 3000 year range, From the Figure it is clear that the persistence of the variations in the higher frequency range of the continuum shown in Figure 3 is consistent with the persistence in the lower frequency range.

A Discussion

We have shown that the variations of a well recognized proxy for solar activity arise from correlated random process. Wc conclude that the variations of the continuum component of solar activity itself are also distributed differently from simple white noise. Since the line spectrum at these frequencies does not contain much power (Damon and Sonett, 1989) the largest variations can be expected to arise from the continuum. In particular, the amplitude of the variations of solar activity that arc expected to occur in a time interval are large compared to the variations expected from a white noise process. We have also given evidence that the continuum variations are distributed in the same way for periods from 1 to 3000 years.

Actually, for a physical system with random behavior the range is expected to behave differently in three sets of time intervals. Asymptotically, the range for very long time intervals may be independent so that R/S will vary as $\tau^{1/2}$, i.e. like a white noise process. For very small intervals R/S may have some complicated behavior with no statistical meaning. The fact that

$\log(R/S)$ is linearly dependent on τ for a large number of intermediate T's has a non-trivial meaning. It says that the solar activity in this intermediatc regime is self-similar. The tilnc-size (the largest interval) in this regime defines some characteristic time (a memory) in activity behavior. With the present data wc could not reach this limit, and can only state that the solar activity is strongly persistent throughout the period covered by the radiocarbon data set we used in this study. We plan to usc a longer time series of ^{10}Be in an attempt to find the white noise limit; if it exists in solar activity,

The physical process by which the sun produces persistent variations in its activity is as yet unknown; however we can expect the persistence of the variations to be produced by a persistence in the random variations of the sol ar dynamo. It was conjunctured that the solar dynamic system has the nature of a stochastic strange attractor (Ruzmaikin, 1981; Weiss *et al.*, *1984).* The observed regularities of solar activity evidence that the attractor is not a pure random strange attractor such as the well-known Lorenz attractor, it is rather a coherent strange attractor. Speaking in the language of spectra we mean the following: The spectra of purely periodic (or quasi periodic) functions are composed of delta functions (lines). In contrast, a purely chaotic dynamic system has a continuous spectrum with no lines. The observed solar activity spectrum is characterized by some lines, for example the 1/(11 year) line, superimposed on a broad continuum, One may call this type of spectrum '<noisy periodicity", or "coherent strange attractor". An example of a coherent strange attractor is provided by dynamic systems near the accumulation point of the Feigenbaum period-doubling bifurcations leading to chaos. There is some evidence that the solar attractor might be of that type (Feynman and Gabriel, 1988; Ruzmaikin *et al.*, 1992).

Our last remark concerns the role of solar activity persistence in solar-terrestrial relations, specifically in climate variability. The solar irradiance has been found to change by a factor of 0.1 % during the last solar cycle. A change of irradiance of about 0.5% is needed to produce an effect the Earth's climate. We have shown here that the time period over which such a change in solar activity can be expected to occur is significantly shorter than that which would be expected for variations produced by a white noise random (Brownian type) process.

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Figure Captions

Fig. 1, The Brownian motion displacements $B_H(t)$ evaluated with N = 700 and $B_H(0)$ = O: (a) The ordinary Brownian motion H = 1/2, (b) H = 0.7, (c) H = 0.9 (Feder, 1988). The range of values of a parameter (maximum value minus minimum value) increases as the Hurst exponent in the generating function increases.

Fig. 2. This Figure shows the almost linear dcpcndcnce of the resealed range for ^{14}C data, 'l'he Hurst exponent is found to bc H = 0.84 which evidences in favor of quite strong coherence in a long-term behavior of the solar activity, The dashed line plotted for comparison has the slope 1 /2. The time intervals τ are measured in units of 20 years.

Fig. 3. The Hurst exponent for the monthly averaged sunspot numbers in a period 1748- 1990 is found to be in agreement with the exponent for ^{14}C

$$H = 0.86* 0.05$$

The time intervals here are measured in units of one month,

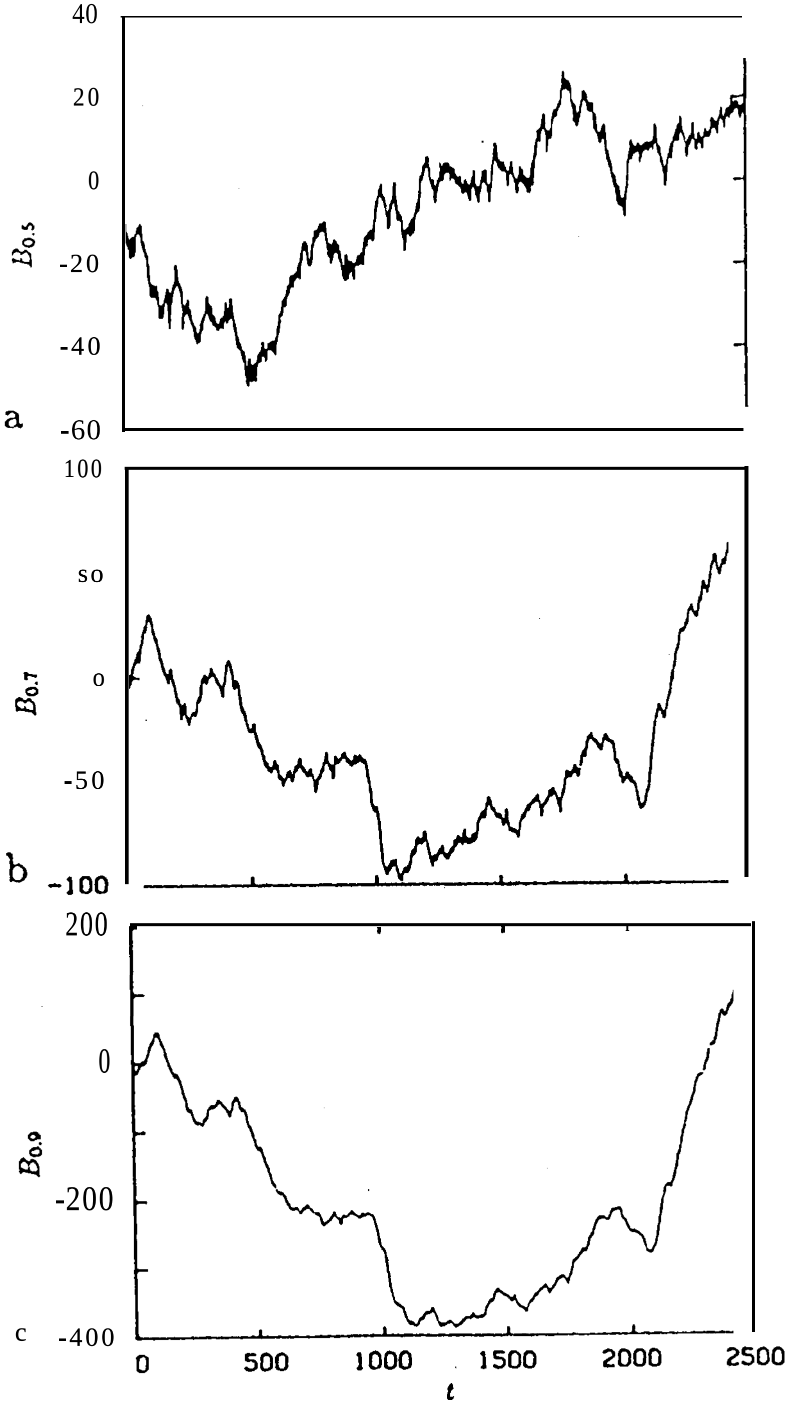
40
20
0
$B_{0.5}$
-20
-40
a
-60
100
so
0
$B_{0.7}$
-50
b
-100
200
0
$B_{0.9}$
-200
c
-400
0
500
1000
1500
2000
2500
t

Fig 1

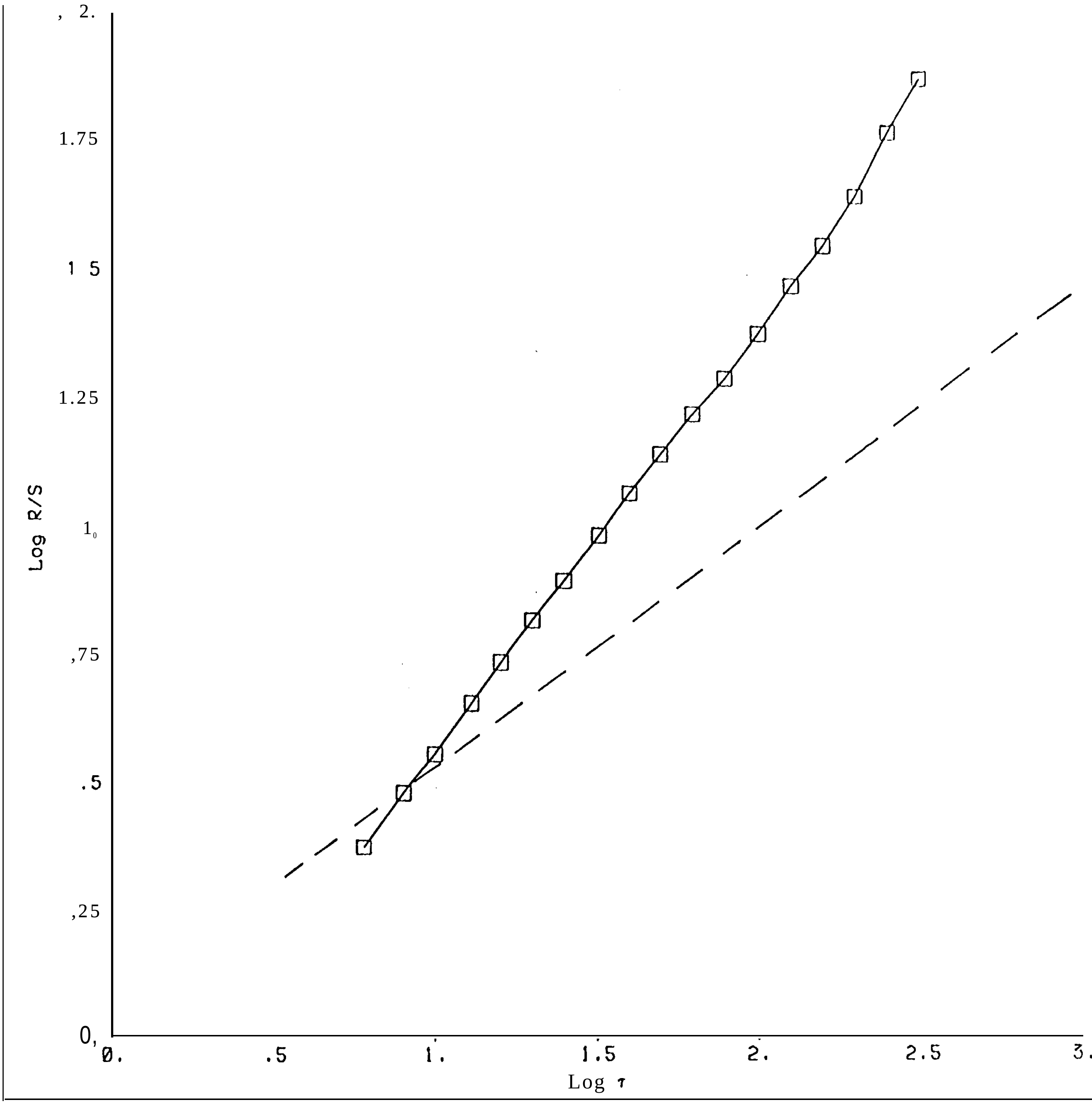

Fig. 2 Ruzmaikin et al

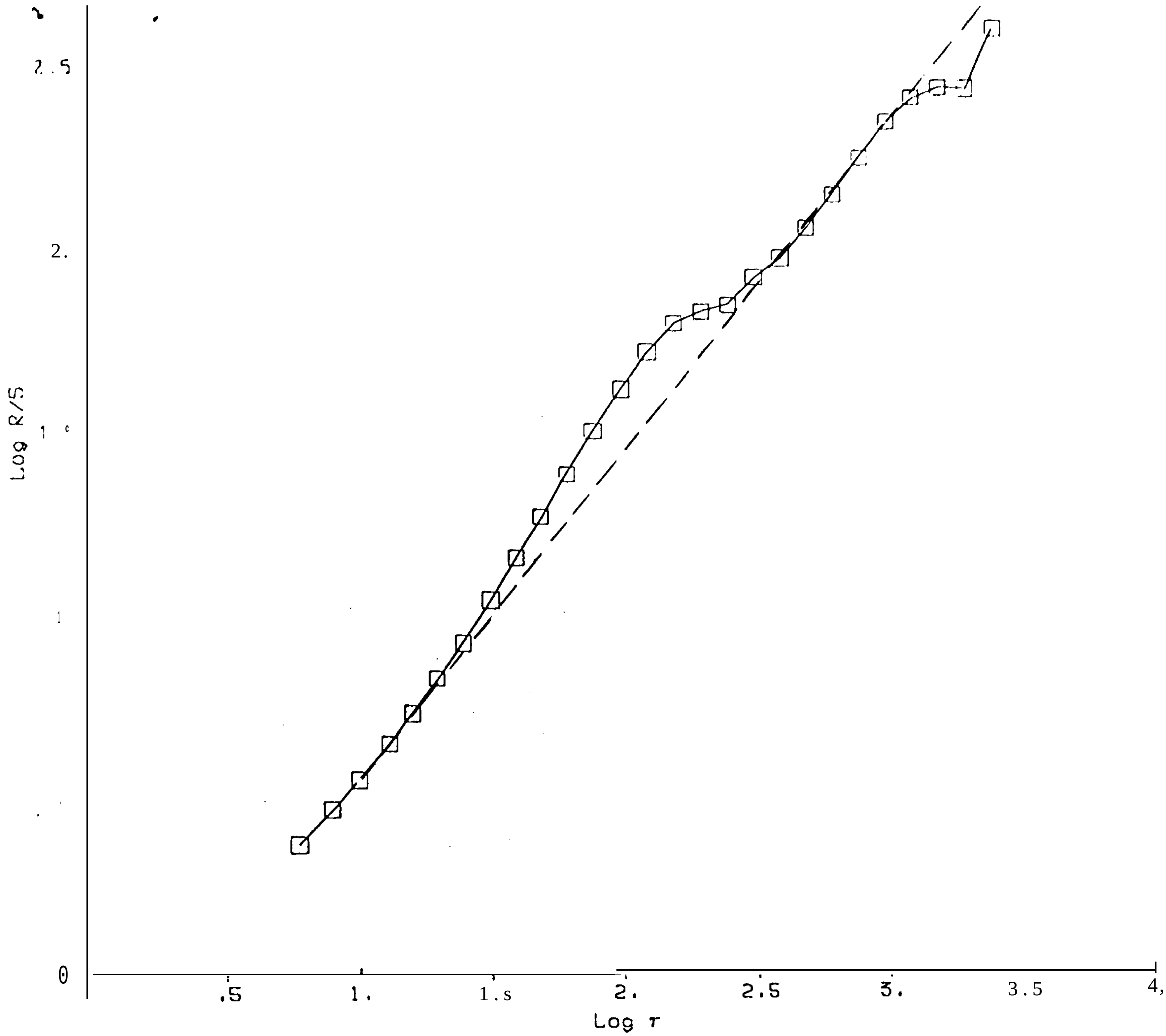

Fig 3 Ra...