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Stochastic Parameters in Lunar Laser Ranging

by

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The Problem

- Approximately 8000 lunar laser ranges are used to estimate > 80 parameters
- **Most** parameters are constant throughout the span of data [**Global** parameters: GM_{Bary}, reflector locations, tidal dissipation, orbit initial conditions, etc.]
- **Some** parameters change throughout the data span [station latitudes and longitudes, caused by stochastic behavior of UT1, PMX, PMY]
- *The Problem:* Estimate one value for each global parameter over entire span concurrently with a separate value for each of UT1, PMX, PMY (or UT0 and VOL) at every LLR data point

Standard Formulation

Given m observations and partial derivatives to estimate n parameters x_j

$$A \quad x = z$$

$$m \left\{ \underbrace{\begin{bmatrix} a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \end{bmatrix}}_{n} \right. \begin{bmatrix} x \\ x \\ x \\ x \\ x \\ x \end{bmatrix} = \begin{bmatrix} z \\ z \\ z \\ z \\ z \\ z \\ z \\ z \\ z \\ z \\ z \end{bmatrix}$$

Solution Method

Usual method is to form $A^T A\, x = A^T z$ (or equivalent) and solve for x. Works if $m \geq n$ and if system is non-singular.

But: for LLR stochastic parameters. $m = 8000$, $n \cong 24000$.

To make system determinate:

- Use a priori covariances for stochastic Parameters
- Assume exponential correlation between data points

Use Householder-triangularized square-root information matrices for data accumulation

Square-Root Matrices

A^TA is positive definite, so $A^TA = R^TR$, where R is upper triangular.
Householder orthogonal transformation H on A produces R.

$$H \quad A \quad = \quad R$$

$$H \begin{bmatrix} a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \\ a & a & a & a & a & a \end{bmatrix} = \left.\begin{bmatrix} r & r & r & r & r \\ r & r & r & r & r \\ & r & r & r & r \\ 0 & & r & r & r \\ & & & r & r \\ & & & & r \end{bmatrix}\right\} n$$

$$\underbrace{\qquad\qquad}_{n}$$

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Step 1. Determine Weights

t_{i-1} $\qquad$ t_i $\qquad$ t_{i+1}

$$w_i = \exp[-(t_i - t_{i-1})/k]$$

where k is input (typically 15 days for UT1 and 40 days for PMX, PMY)

Step 2. Apply Weights to *R*

Multiply left 3 columns by weight to increase a priori sigmas. Multiply top 3 rows by weight to decrease correlation. (Elements shaded in dark blue are multiplied twice.)

$$W^{\mathrm{T}}RW = \underbrace{\begin{bmatrix} r & r & r & r & r & r & r \\ & r & r & r & r & r & r \\ & & r & r & r & r & r \\ & & & r & r & r & r \\ & & & & r & r & r \\ & & & & & r & r \\ & & & & & & r \end{bmatrix}}_{n+1} \left.\vphantom{\begin{bmatrix} r \\ r \\ r \\ r \\ r \\ r \\ r \end{bmatrix}}\right\} n+1$$

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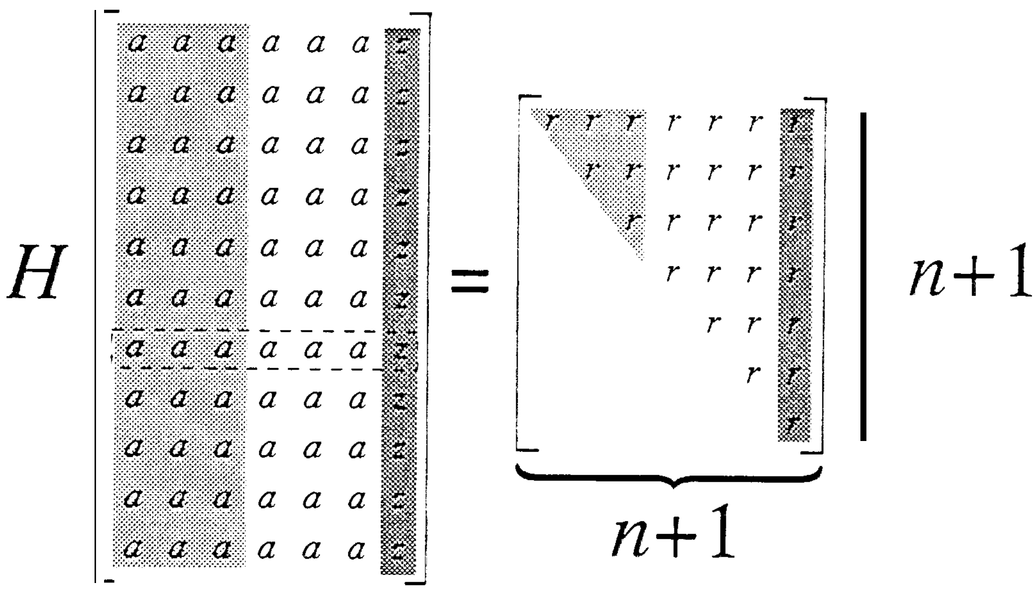
Step 3. Add point to *R*

Put stochastic partials in left columns of A.
Augument A with observation vector z.
Tranform to R, one point (row) at a time.



Step 4. Add *a priori* to *R*

Invert *a priori* covariance matrix for UT1, PMX, PMY to get information matrix. Multiply by $(1-w_i)$. Take Cholesky square root. Accumulate in *R*.

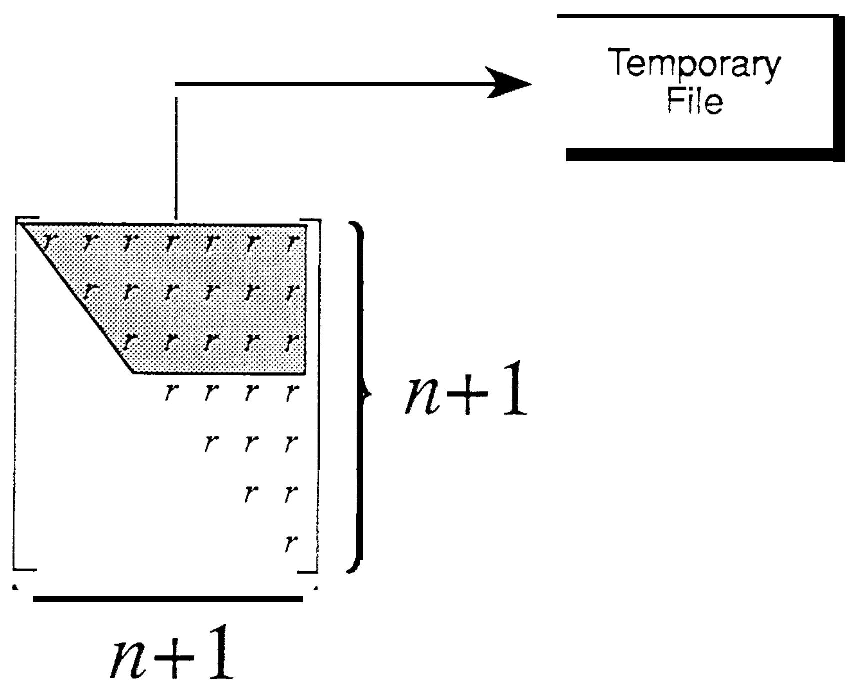
$$P^{-1}=\Lambda.$$

$$[(I-W)^{T}\Lambda(I-W)]^{1/2}=R_{ap}=\begin{bmatrix} a & a & a \\ & a & a \\ & & a \end{bmatrix}$$

$$H\begin{bmatrix} a & a & a \\ & a & a \\ & & a \end{bmatrix} = \underbrace{\begin{bmatrix} r & r & r & r & r & r & r \\ & r & r & r & r & r & r \\ & & r & r & r & r & r \\ & & & r & r & r & r \\ & & & & r & r & r \\ & & & & & r & r \\ & & & & & & r \end{bmatrix}}_{n+1} \Bigg| \; n+1$$

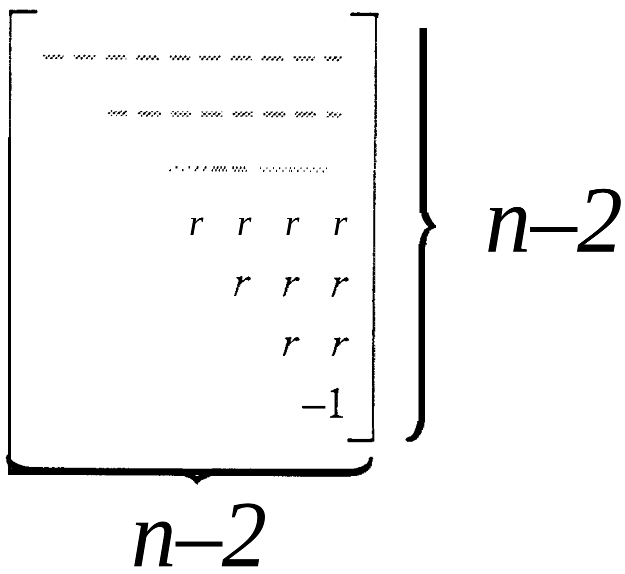
Step 5. Save Stochastic *R*

Write top three rows of R onto intermediate file. Repeat the above steps for all observations and partials. Process all data twice. once in forward direction, once in reverse.



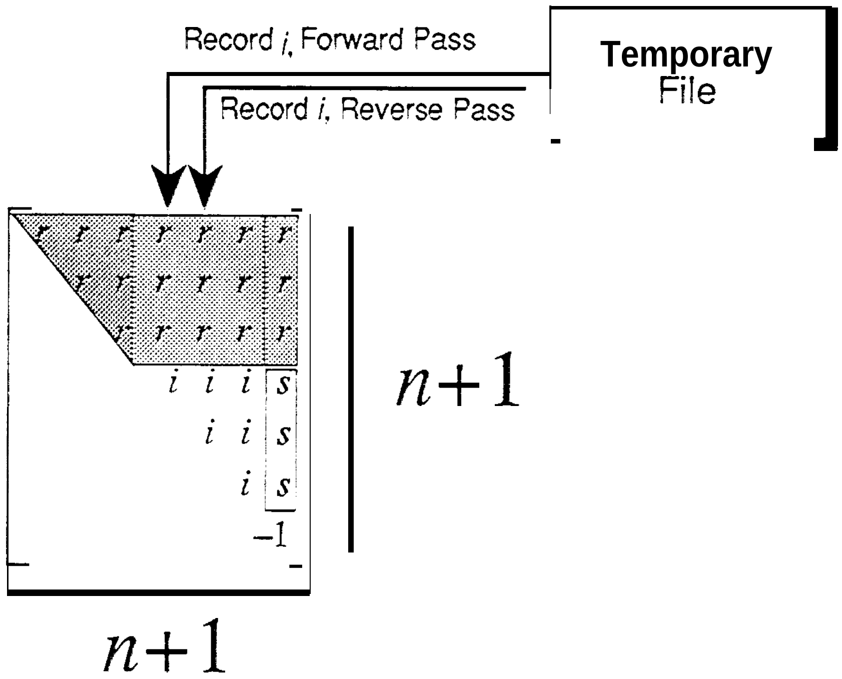
Global-Parameter Solution

Replace the last element by –1 and invert the full matrix, rows 4 through $n+1$. Right-hand column in solution, remainder is square root of covariance matrix of global parameters.



Stochastic-Parameter Solution

Read temporary file, one pair of records at a time. Pack into *R*. *Invert* only the right-hand column of the top three rows. Result is stochastic-parameter solution at that point. Invert upper left 3 x 3 to get stochastic covariance matrix.



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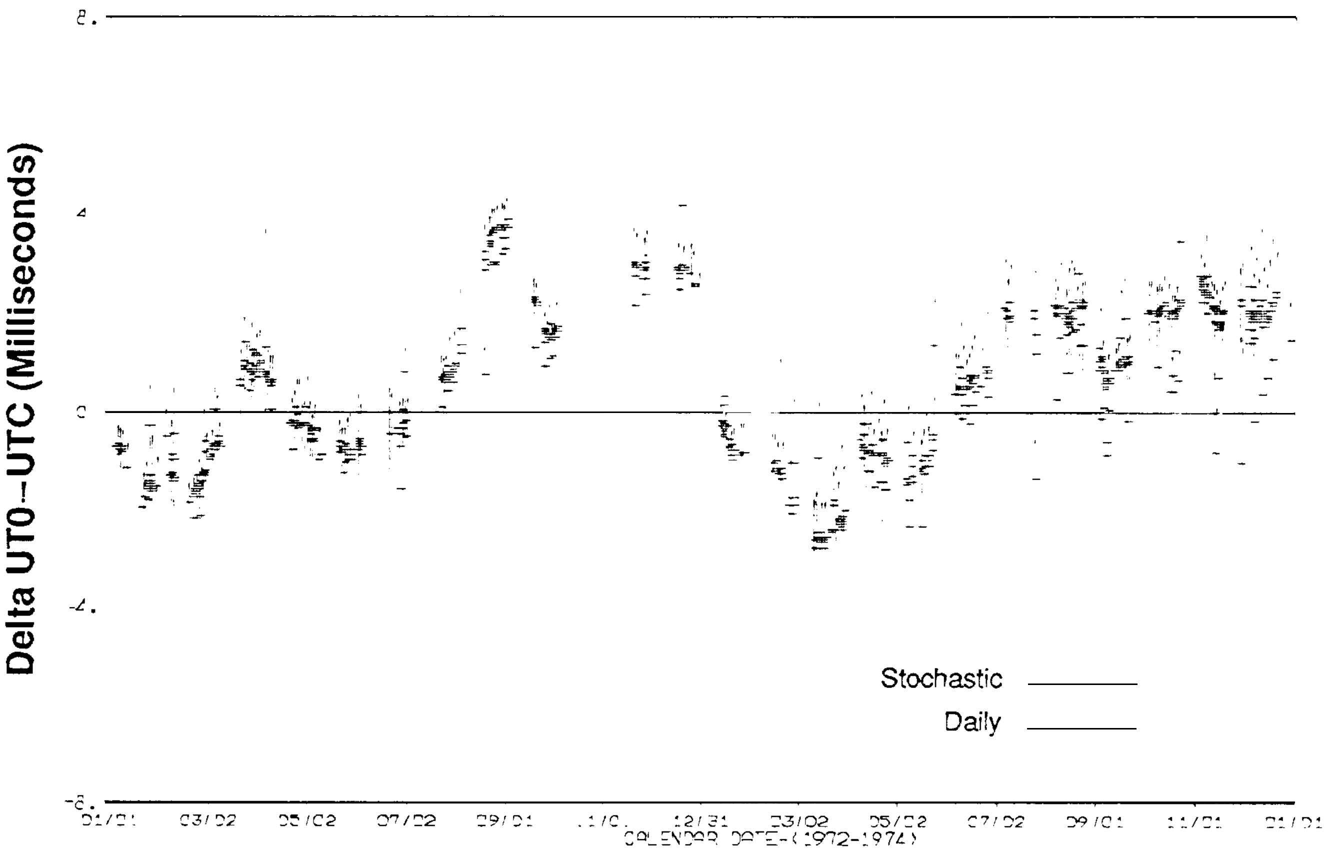
Stochastic vs. Daily UTO–UTC



Stochastic vs. Daily McDonald V.O. L.