Interferometric SAR Processing;

with emphasis on:

Methods to Determine the Absolute Phase

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Søren N. Madsen

Jet Propulsion Laboratory
California Institute of Technology

OVERVIEW

- TOPSAR processing system
- Motion compensation
- Absolute phase determination
- Height/backscatter resampling/regridding

Basic Across-Track Interferometer Equations

(1) **Doppler equation:**

$$f_D = \frac{2}{\lambda}\hat{n}_{los}\cdot\vec{v} \quad \Leftrightarrow \quad \hat{n}_{los}\cdot\vec{v} = \frac{\lambda f_D}{2}$$

(2) **Interferometer equation:**

$$\phi = \frac{2\pi}{\lambda}\hat{n}_{los}\cdot\vec{B} \quad \Leftrightarrow \quad \hat{n}_{los}\cdot\vec{B} = \frac{\lambda\phi}{2\pi}$$

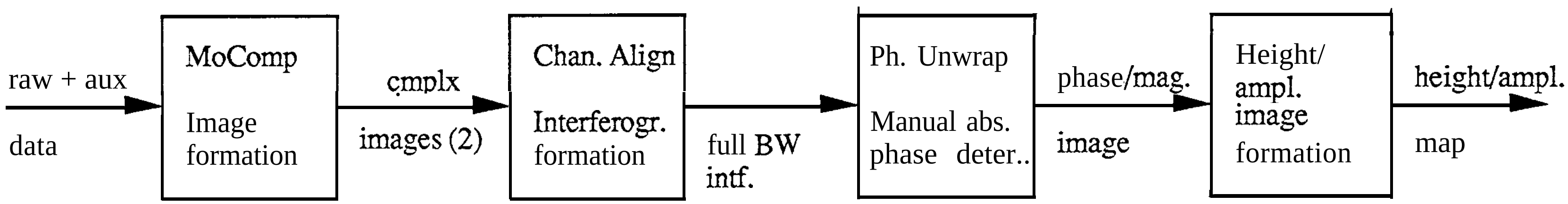
(3) **Range equation:**

$$\vec{r}_{\text{target}} = \vec{r}_{\text{radar}} + r_{sl}\,\hat{n}_{los}$$

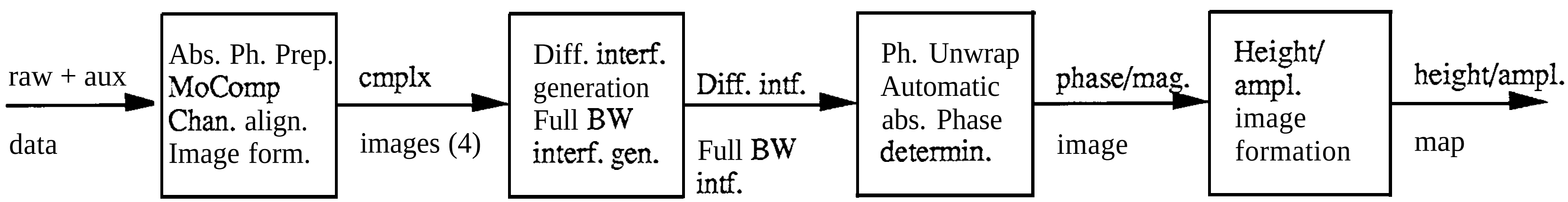
- **The target position is thus determined in 3–dimensional space.**

ELEMENTS OF AN ACROSS-TRACK INTERFEROMETRIC PROCESSOR

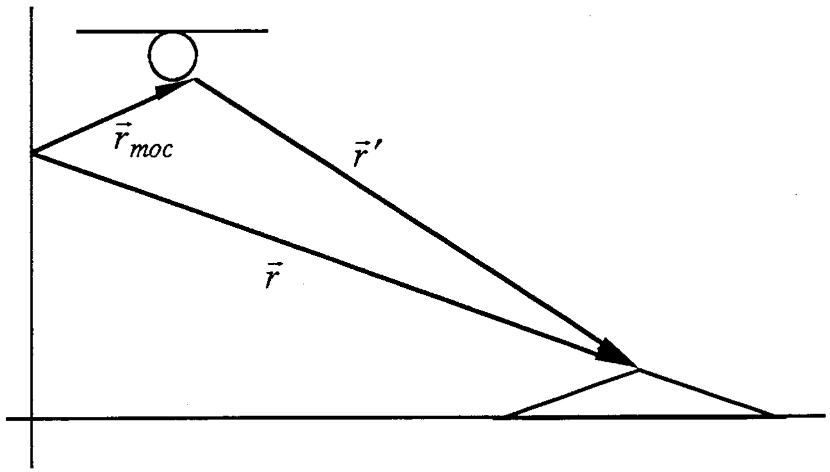
- **"Traditional Processing Scheme"**



- **"Hybrid Processing Scheme" (TOPSAR)**



WHY THE MOTION COMPENSATION PROBLEM IS NONTRIVIAL?



$$\vec{r} = \vec{r}_{moc} + \vec{r}'$$

$$r^2 = r'^2 + r_{moc}^2 + 2\vec{r}_{moc} \cdot \vec{r}'$$

$$r - r' = \frac{r_{moc}^2 + 2\vec{r}_{moc} \cdot \vec{r}'}{r + r'} \approx \vec{r}_{moc} \cdot \hat{r}'$$

- Platform position needs to be accurately known ❑> Accurate INS/GPS required!

- Target location needs to be known!! Fundamental problem as:
  – range dispersion exists ($\rho_r = c/2B$)
  – target height is not known a priori => defocus + (!) systematic error when tracks are non parallel (roll rate $\neq 0$)

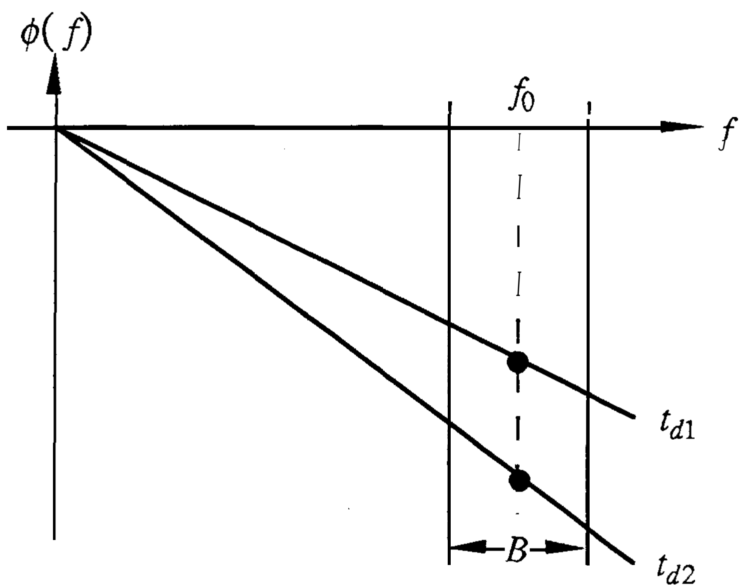
- Motion Compensation corrections must be implemented carefully in processor => this is particularly important for absolute phase determination

ABSOLUTE PHASE - WHAT IS IT?

- **Interferometric phase relation**

$$\phi = \frac{2\pi f_0}{c}(r_1 - r_2) = 2\pi f_0 (t_{d1} - t_{d2})$$
$$= \tilde{\phi} + 2\pi p \quad ; \quad \tilde{\phi} \in]-\pi, +\pi[$$

ϕ, the absolute phase is directly proportional to differential range and time!



$$F\{h(t - t_d)\} = e^{-2\pi f t_d} F\{h(t)\}$$

The absolute phase relates to both the phase difference and the slope difference!

ABSOLUTE PHASE – WHAT IS IT? (2)

- **Full bandwidth interferogram gives fractional time in units of** $1/f_0$

$$t_{d1} - t_{d2} = \frac{\tilde{\phi}}{2\pi f_0} + \frac{p}{f_0}$$

- **Phase slope/differential phase must give** p **with accuracy better than ±0.5!**

- **Implications for motion compensation:**

$$e^{j2\pi f t_{moc}} = e^{j2\pi f (t_{shift} + \varepsilon / f_0)}$$ **; 4S<<0.5**

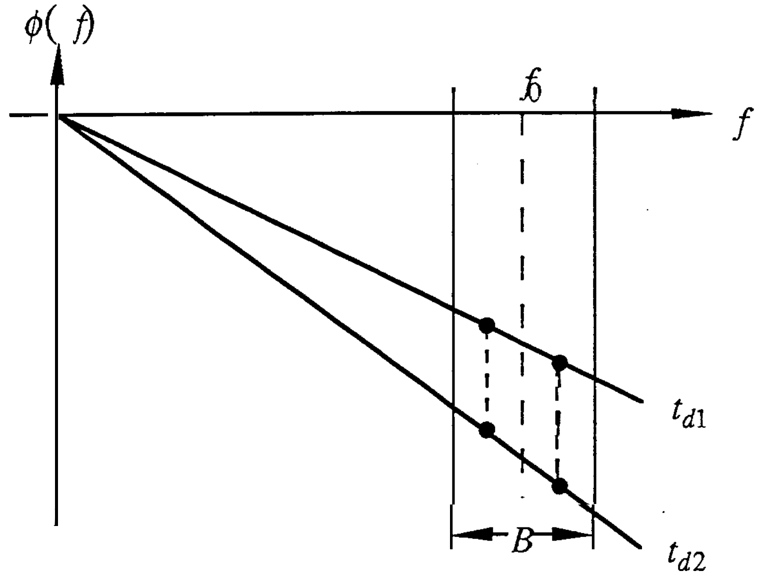
$$\Leftrightarrow \; t_{shift} - t_{moc} < \varepsilon / f_0$$

Assume complex sampling at f_s **and resampling accuracy** $1/(2Nf_s)$

$$\frac{1}{2Nf_s} << \frac{1}{2}\frac{1}{f_0} \;\Leftrightarrow\; N >> \frac{f_0}{f_s}$$

HOW TO ESTIMATE ABSOLUTE PHASE?

- Split spectrum estimation



- Most successful technique to date!
- 98% success rate for JPL TOPSAR data acquired in 1991
- Much higher error rate for 1992 data!
  Why? Is presently being studied

AMBIGUITY RESOLUTION (CONT.)

- **Note that interferometric phase is a function of carrier frequency**

$$\phi_1 = 2\pi \frac{f_1 B}{c} \sin(\alpha - \theta_i)$$
$$\phi_2 = 2\pi \frac{f_2 B}{c} \sin(\alpha - \theta_i)$$

- **Differential phase is equivalent to low frequency interferometer**

$$\Delta\phi = 2\pi \frac{(f_1 - f_2) B}{c} \sin(\alpha - \theta_i)$$

- **No ambiguity for**

$$|f_1 - f_2| < \frac{c}{2B}$$

- **Split spectrum processing!**

- **Split spectrum technique is <u>not</u> dependent on ground control points!**

MATHEMATICS OF ABSOLUTE PHASE DETERMINATION

- **RF signals received by antennae 1 and 2:**

$$s_1(t) = \exp(j2\pi f t)$$
$$s_2(t) = \exp(j2\pi f(t - t_d))$$

- **Baseband signals**

$$c_1(t) = \exp(j2\pi(f - f_0)t)$$
$$c_2(t) = \exp(j2\pi(f - f_0)(t - t_d) - j2\pi f_0 t_d)$$

- **Interferogram**

$$w(t) = c_1(t)c_2^*(t + t_{d0})$$
$$= \exp(j2\pi(f - f_0)(t_d - t_{d0}) + j2\pi f_0 t_d)$$

MATHEMATICS OF ABSOLUTE PHASE DETERMINATION (CONT.)

- **Two band processing at $f_+ \pm B_f / 2$ and $f_- \pm B_f / 2$:**

$$\phi_+(t) = 2\pi f_+(t_d - t_{d0}) + j2\pi f_0 t_d$$
$$\phi_-(t) = 2\pi f_-(t_d - t_{d0}) + j2\pi f_0 t_d$$
$$\phi_{diff}(t) = 2\pi(f_+ - f_-)(t_d - t_{d0})$$

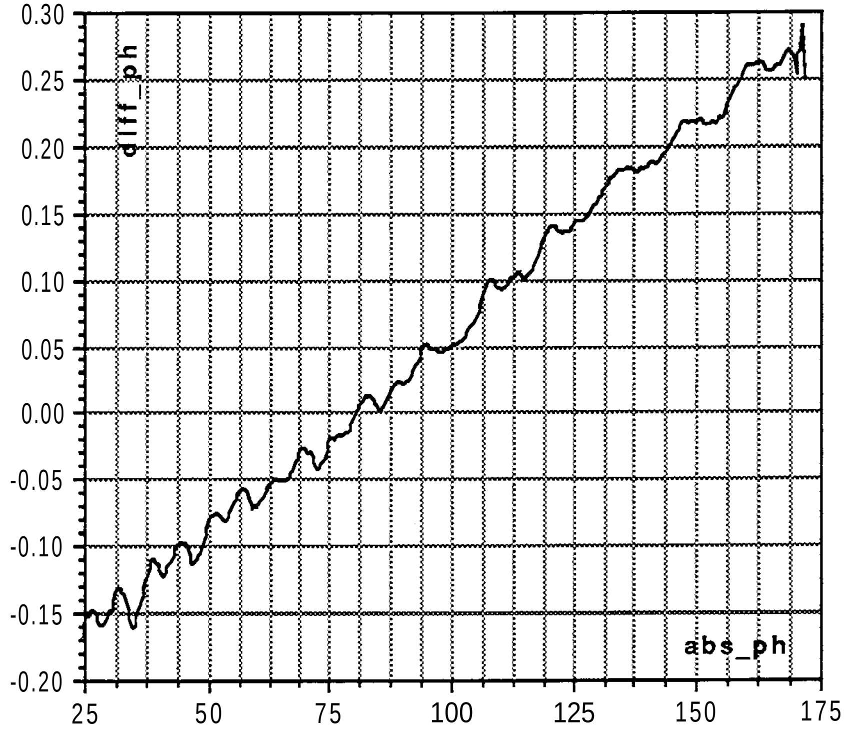
- **Relation between differential and absolute phase**

$$\phi_{abs} = \frac{f_0}{f_+ - f_-}\phi_{diff} + 2\pi f_0 t_{d0}$$

- **Implementation**

$$\phi_{abs} = \frac{f_0}{f_+ - f_-}\phi_{diff} + 2\pi f_0 t_{d0} = \phi_{unwrap} + 2\pi n$$
$$\tilde{n} = \mathrm{nint}\left(\frac{1}{2\pi}E\left\{\frac{f_0}{f_+ - f_-}\phi_{diff} + 2\pi f_0 t_{d0} - \phi_{unwrap}\right\}\right)$$
$$\phi_{abs} = \frac{f_0}{f_+ - f_-}\phi_{diff} + 2\pi f_0 t_{d0}$$

MEASURED DIFFERENTIAL PHASE AS A FUNCTION OF ABSOLUTE PHASE



HOW TO ESTIMATE ABSOLUTE PHASE(2)

- **Correlation Method**
  – after phase unwrapping we have:

$$t_{d1} - t_{d2} = \frac{\tilde{\phi}_{\text{unw}}}{2\pi f_0} + \frac{p}{f_0}.$$

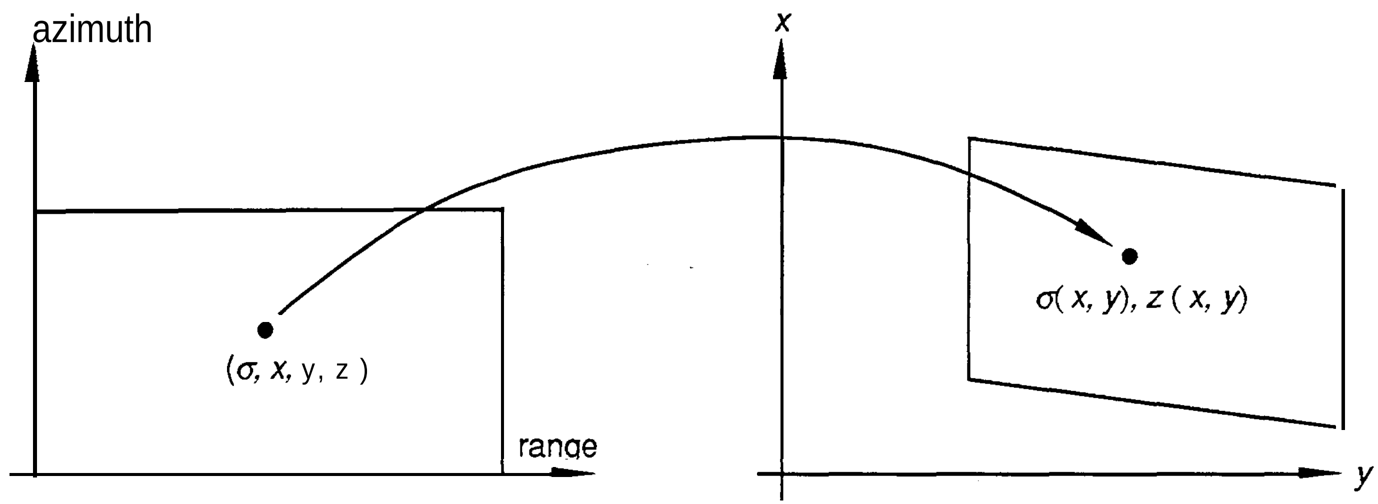
with p constant for entire patch

– Use the unwrapped phase, ϕ_{unw}, to interpolate and phase shift one complex image relative to the other. The two images should now be identical except for the residual range shift caused by the constant bias of the unwrapped phase relative to the absolute phase.

- Determine the off-set, p, relative to the absolute phase by cross-correlating the two complex images

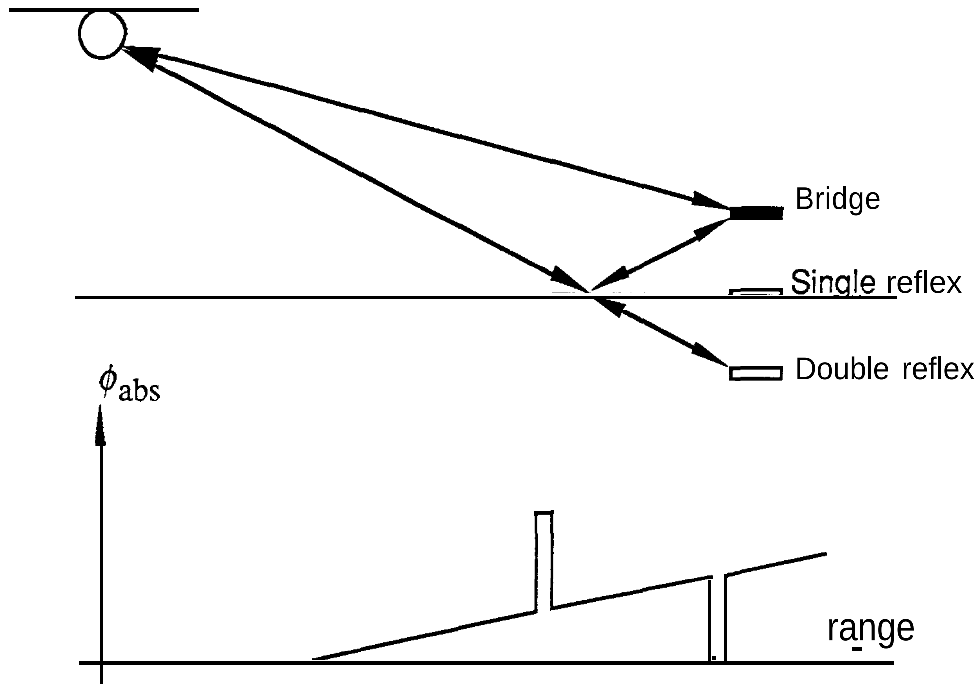
- **Preliminary conclusion: Method often in error by ±1 or ±2 ambiguities, not very robust. More studies needed.**

The Output "MAP" Resampling Problem



- Unevenly spaced points in (x, y) space
  - possible solutions 2-D bilinear interpolation or facet interpolation
- Problem! Location noise is comparable to the point spacing
- Problem! Lay-over and multipath

THE OUTPUT "MAP" RESAMPLING PROBLEM (2)



- **Preliminary result: Nearest neighbor interpolation + pixel fill seems to be most accurate and robust! (bilinear has been tried)**

- **Image patch filled from far range to near range, overwriting multipath echoes**

- **More work required!**

CONCLUSIONS

- Fully automatic processing has been proven feasible

- Key elements in across-track interferometric processing:

  - Absolute phase determination
  - Motion Compensation
  - Phase unwrapping
  - Output "map" resampling

- Still seems room for improvements!