Analysis and Simulation for a Spotlight-Mode Aircraft SAR in Circular Flight Path

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Abstract - A spotlight aircraft SAR in a circular flight path can efficiently obtain an ima ge with very high azimuth resolution 01 a wider azimuth viewing angle. Ai i analysis 011 the spotlight SAR is made regarding the required PRF, the predicted resolution, and the computation complexity as a function of the aircraft altitude and the distance between a target and the center of the flight path projection. An efficient processing algorithm based 011 the exact wide beam spectrum is p resented. The results of simulation indicate that the impulse responses meet the predicted resolution performance.

Keywords: SAR, spotlight-mode, resolution, PRF, time-bandwidth product, reference function, processing algorithm.

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A wide azimuth 1-main SAR can offer higher resolution or wider azimuth viewing angle; two factors that help better characterize the backscattering property of targets for various science applications. One disadvantage of wide beam SAR is that a much higher pulse repetition frequency (PR 1') is usually required since PRF is proportional to the radar beam angle. T'llis j)lc}l~lclllc alll)cl c.s()l\'c{l[l sil]g,as])()tligl~t-11)[)(1(! concept: steering a na rrow beam SAR to a fixed spot on the ground. The drawback of a spotlight-mode SAR is its limited coverage. A conventional spotlight-mode SAR operates along a straight line path as shown in Figure la, It, can be shown that spotlight-mode SAR that follows a straight line path has difficulty in achieving the ulti nate resolution of $\lambda/4$. It also cannot utilize the full 180 degree of azimuth viewing angle that can be attained only when the synthetic aperture lengt i approaches infinity.

A spotlight-mod e SAR can also be operated from a circular flight path as shown in Figure 1.b. This type of spotlight-mod e SAR offers several advantages: (1) relatively easy to achieve the ulti mate azimuth resolution, (2) it allows a full 360 degree of viewing angle, (3) there is no need to steer the radar, and **(4)** the required PRF can be scaled down according to the ratio between the radius of the radar spot to the radius of the flight path.

SPOTLIGHT SAR ANA LYSIS

For an aircraft SAR in a circular path as shown in Figure 2, the slant range history of a]Joi]lt-target is given by

$$R(\theta_1) = \sqrt{A + B\cos\theta_1},$$

where $A = R_0^2 + 2R_b^2 - 2R_0R_b\cos\theta_c$

and $B = 2R_b(R_0\cos\theta_c - R_b)$.

In the above equations, θ_c is the radar elevation angle, R_0 is Slant! range between the radar and the Lal'get> at the beam center, R_b is the radius of the flight path, and θ_1 is the angle between the radar at the minimum range to the target, the center of the flight circle, and the radar at time of interest. Since time is directly proportional to θ_1, this function can also be expressed using time as the variable, i.e.

$$R(t) = \sqrt{A + B\cos(\frac{v}{R_b}t)}$$

where v is the speed of the aircraft. Let Z denotes the altitude of the aircraft, it is obvious that R_0 and Z are related by $Z = R_0\sin\theta_c$. Below, we shall denote the distance between the target and the center of the path projection on the ground as R_T, which is equal to $R_b - R_0\cos\theta_c$.

To provide better insight into the spot-light SAR characteristics, it is assumed that the radar beam width is unlimited such that any target within the path projection is illuminated all the time during mapping. To determine the azimuth resolution, it is necessary to determine the maximum Doppler bandwidth. This can be accomplished by evaluating the zero crossing time of the Doppler frequency rate. The Doppler history is the derivative of the slant range, i.e.

$$f_d(t) = \frac{-d(2R(t)/\lambda)}{dt} = \frac{k_0Bv}{R_b}\sin(\frac{vt}{R_b})R^{-1}(t)$$

Plots of both the slant range history and Doppler history are given in Figure 3. The history of the Doppler frequency begins with 0Hz at the minimum slant range point, decreases to the minimum Doppler (negative value), increases back to 0Hz at the maximum slant range point, keeps increasing up to the maximum Doppler, and then decreases back to 0Hz after a complete cycle. Based on this, one can divide a complete circle into two apertures with equivalent bandwidth. Later, the aperture with a Doppler ranging from its maximum to its minimum and consisting of the minimum slant range point is referred to as the **principle aperture**. The other aperture is referred to as the **complement aperture**.

The Doppler frequency rate variation is given by

$$f_r(t) = \frac{df_d}{dt} = \frac{v}{R_b}\cos(\frac{vt}{R_b})R^{-1}(t) + \frac{Bv}{2R_b}\sin^2(\frac{vt}{R_b})R^{-3}(t)$$

The solution of the time at which f_r is equal to zero is given by

$$t\,|_{f_r=0} = \cos^{-1}\left(\frac{-A + (A^2 - B^2)^{1/2}}{B}\right)$$

Therefore, the Doppler bandwidth, denoted as F, is given by two times the absolute value of the Doppler at one of the solution given above. Since F varies as a function of both the aircraft altitude Z and target radius R_T, it is given by

$$F(Z, R_T) = \frac{2\sqrt{2}v}{\lambda R_b}(A^2 - B^2)^{-1/4}\sqrt{|A^2 - B^2 - A\sqrt{A^2 - B^2}|}$$

The azimuth resolution is thus given by the reciprocal of the bandwidth multiplied with the effective velocity $\frac{R_T}{R_b}v$, or

$$\Delta X(Z, R_T) = \left(\frac{R_T}{R_b}v\right)\frac{1}{F(Z, R_T)}$$

The variations of the resolution and the Doppler bandwidth as a function of the target location are plotted in Figure 4.a and Figure 4.b. It should be noted that these plots also include the targets located outside of the vertical cylinder containing the orbit because they can also be imaged as long as range ambiguity can be avoided. It is interesting to see that the ultimate resolution of $\lambda/4$ can be achieved for targets falling on the orbit plane and bounded by the circular orbit. The resolution width increases linearly as targets shift radially outside of the orbit. The Doppler bandwidth decreases as the targets approach to the center of the orbit, This implies that for targets located on the orbit plane and bounded by the orbit, the required PRF is proportional to the distance from target to orbit center.

Consider a special case where $Z = 0$ and R_T approaches R_b. According to the bandwidth equation given above, we may find that the required PRF is equal to $4v/\lambda$. This indicates that the required sampling spacing is exactly $\lambda/4$ which equals to the ultimate azimuth resolution.

The time interval of the principle aperture is given by

$$T_p(Z, R_T) = 2 \cdot \cos^{-1}\left(\frac{-A + (A^2 - B^2)^{1/2}}{B}\right)$$

It will be more convenient to express the aperture time interval as a value normalized by the period of a complete flight circle. The complexity of SAR processing is usually determined by the number of samples within the aperture or the value of the time-bandwidth product TBP given by

$$\mathrm{TBP}(Z, R_T) = T_p(Z, R_T)F(Z, R_T)$$

Both normalized time interval of the aperture and the time-bandwidth product are shown in Figure 5.a and Figure 5.b).

In summary, the analysis in this section leads to the determination of the PRF for operating the spotlight radar, the predicted resolution, the time interval of the aperture and the time-bandwidth product to be selected for signal processing.

SPOTLIGHT SAR PROCESSING ALGORITHM

Several algorithms were presented for processing spotlight-mode SAR data or the rotating object data. These algorithms include the well known range-Doppler algorithm applied to subapertures, the backprojection processing method (Munson, et. al., [1]) commonly used in computer-aided tomography (CAT), and the polar format processing algorithm (Ausherman, et. al [2]). One essential assumption required for both the CAT and polar algorithms is that the dimension of the imaged area is much less than the radar to target distance. Other processing algorithms devised for imaging rotating object include a tomographic extension of Doppler processing algorithm (Mensa, et,. al. [3]) and a range-Doppler processing algorithm (Walker, J. [4]). The first method is suitable only for imaging sparse arrays of objects due to its higher integrated sidelobe ratio (ISLR); and both algorithms also rely on the assumption of large radar to target distance. An exact solution for a circular aperture acoustic imaging system was presented by Norton [5]. This algorithm requires the implementation of a quasi-fast Hankel transform which is not very efficient.

The difficulty in processing spotlight-mode SAR data collected from a circular path is that the Doppler history of a point target involves many higher order terms and that the depth of focus is very shallow. To overcome the first problem, the algorithm proposed here make use of an exact 2-D spectrum of a point target (Jin, 1992 [6]) in a range Doppler like processing approach. To overcome the second problem, this algorithm efficiently updates its reference function as frequently as required.

According to [6], the magnitude and the phase of a reference spectrum are given by

$$A(\omega_r, \omega) = \sqrt{2\pi} \left| \frac{d^2(-(\omega_r + \omega_0) \cdot 2R(t_s)/c)}{dt^2} \right|^{-1/2} \quad (1.a)$$

$$\psi(\omega_r, \omega) = \exp\left\{ j\left(\frac{\cos^{-1}(\alpha(\omega_r,\omega))\omega}{v/R_b} - 2k_r\sqrt{A + B\alpha(\omega)} - \frac{\pi}{4} \right)\right\} \quad (1.b)$$

where ω is the azimuth angular frequency, ω_r is the range angular frequency, ω_0 is the angular frequency of the carrier, and k_r is equal to ω_r /c, where c is the speed of light.

$$\alpha(\omega_r,\omega) = \frac{-\frac{\omega^2}{k_r^2} \pm \sqrt{\frac{\omega^4}{k_r^4} + 4\frac{\omega^2 v^2}{k_r^2 R_b^2}(B - R_0^2) + 4\frac{v^4}{R_b^4}B^2}}{2\frac{v^2}{R_b^2}B}$$

The basic correlation steps for the spot-light mode processing are summarized 1) C1OW.

(1) Perform subsampling for the echo pulses according to the radius of the spot area. This step is not required if the PRF of the sensor is tuned to that radius such that there is no redundant data. Transform the SAR data into its spectrum by a 2-1) 1“1”’1’ process.

(2) Perform a 2-D SAR correlation. For each azimuth line with a constant range position, a reference spectrum is generated according to equation (1). Correlation is performed by multiplying the data spectrum and the conjugated reference, averaging in range, and performing an inverse azimuth 1“1”’1’. This is repeated for each azimuth line.

(3) Perform geometric resampling to correct for the geometry and grid spacing. The mapping between image pixels generated from step (2) and points on the ground for a spot-light mode SAR is given in Figure 6.

SIMULATION

A simulation was performed to test the proposed algorithm and to verify the analysis on the azimuth resolution. Plotted in Figure 7.a and 7.b. are the impulse responses of two aircraft spotlight systems, both with zero altitude. The first one has a range bandwidth being twice the carrier frequency such that the range resolution is comparable to the azimuth resolution. The second system has a range bandwidth being .15 the carrier frequency. The 3-db resolutions in range and azimuth of the first system are very close to $\lambda/4$. The 3-db resolutions of the second system are also close to $\lambda/4$. However, it has a much worse integrated sidelobe ratio.

CONCLUSION

The spotlight mode aircraft SAR in a circular flight path has been analyzed. The proposed processing algorithm is both efficient and accurate. This algorithm will be tested using the data acquired JPL/NASA aircraft SAR (AIR SAR) in the near future.

ACKNOWLEDGMENTS

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Reference:

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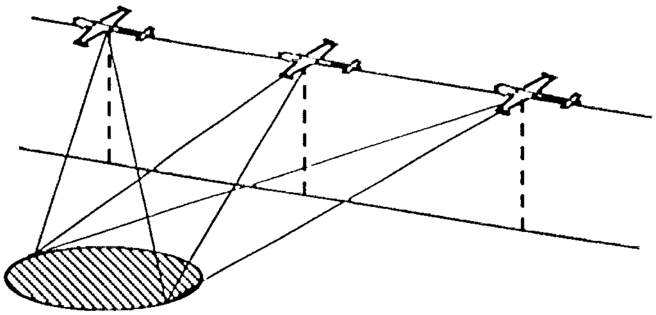

Figure 1.a. A straight line path Spotlight-mode SAR platform

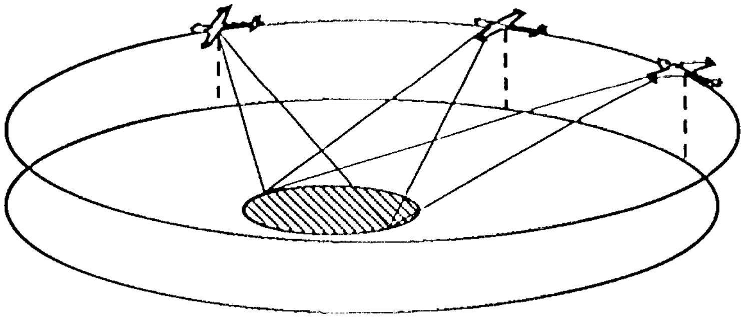

Figure 1.b. A circular path Spotlight-mode SAR platform

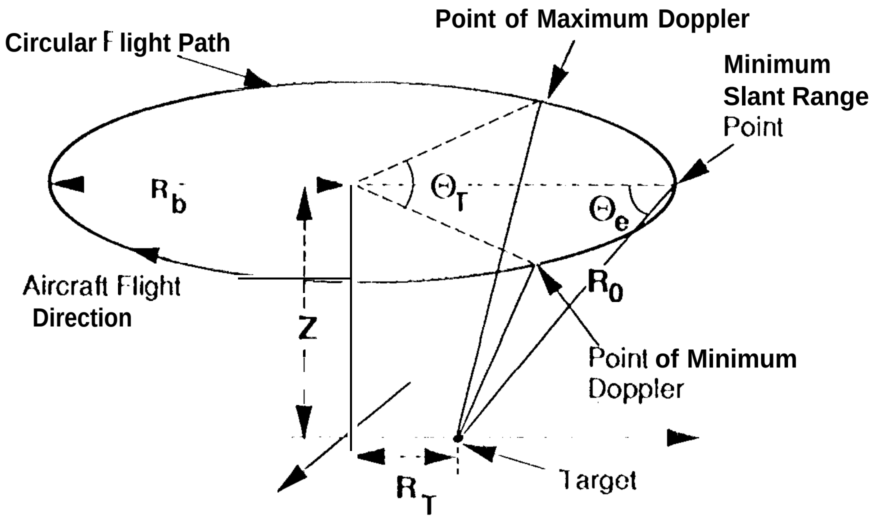

Figure 2. imaging geometry from a circular orbit

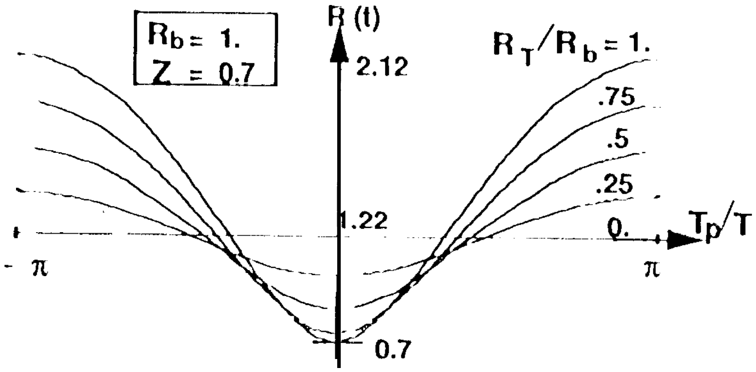

Figure 3.a Slant Range History Plots

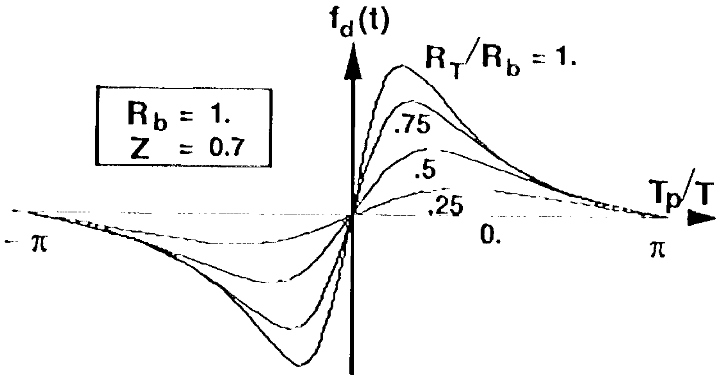

Figure 3.b Doppler Frequency History Plots

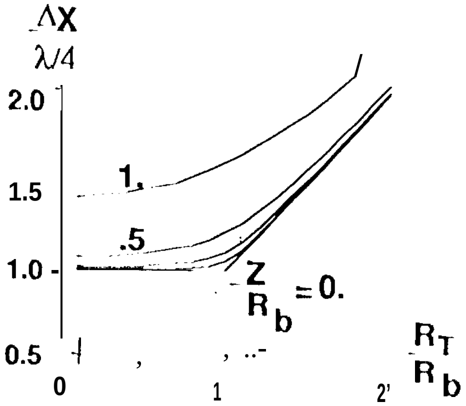

Figure 4a. Resolution vs target location

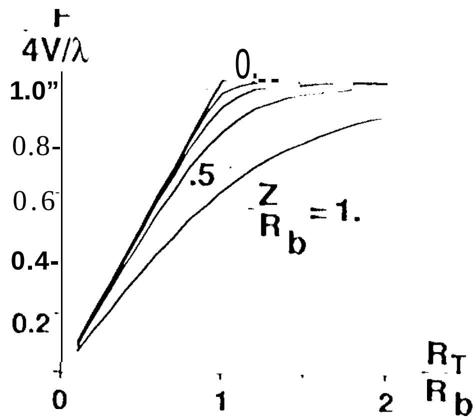

Figure 4.b. Bandwidth vs target location

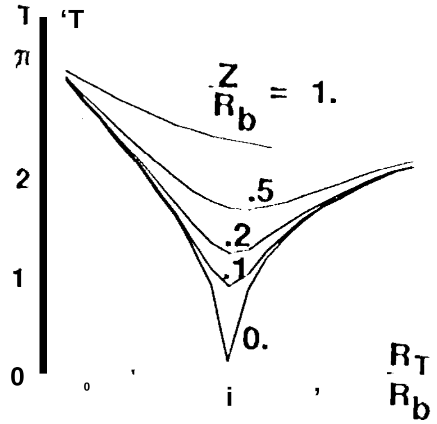

Fig. 5a, Aperture Interval vs target location

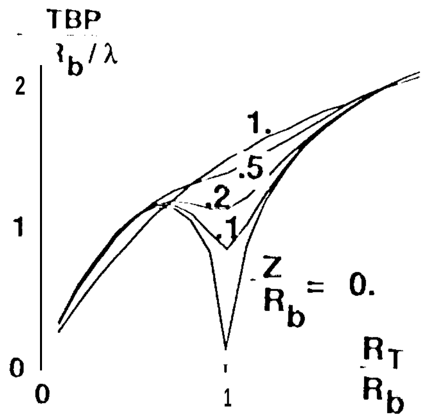

Fig. 5.b. Time-bandwidth product vs target location

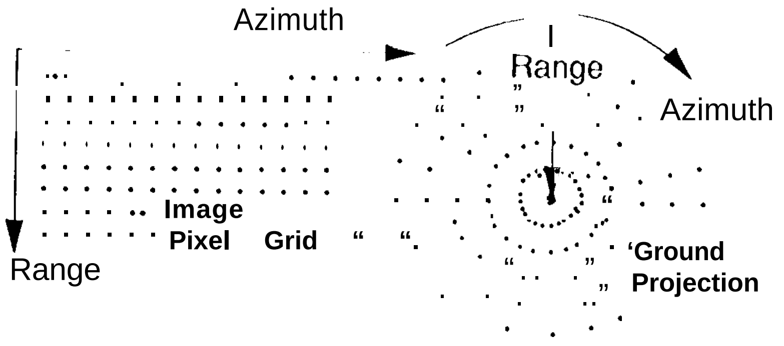

Figure 6 Spot-Light image pixel to projection mapping

Range Bandwidth : 2 x Radar Carrier Frequency
Full Resolution Processing over the Principle Aperture

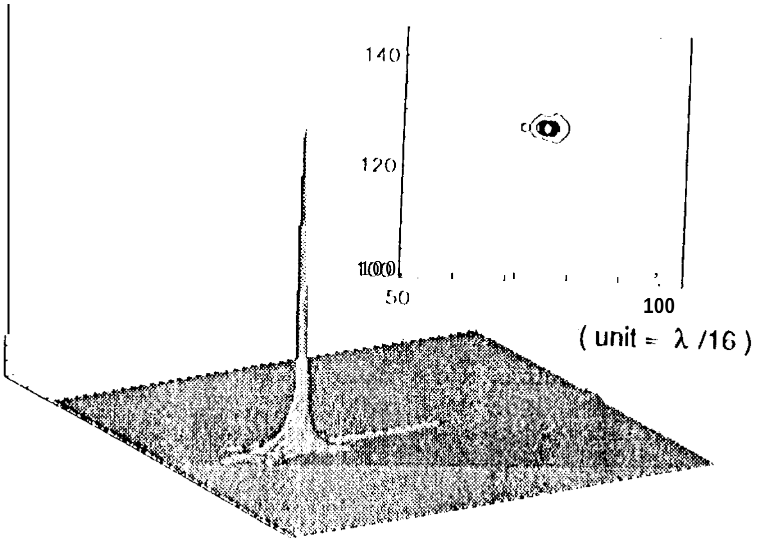

Figure 7.a Spotlight Simulated Impulse Response

Range Bandwidth .0 .15 x Radar Carrier Frequency
Full Resolution Processing over the Principle Aperture

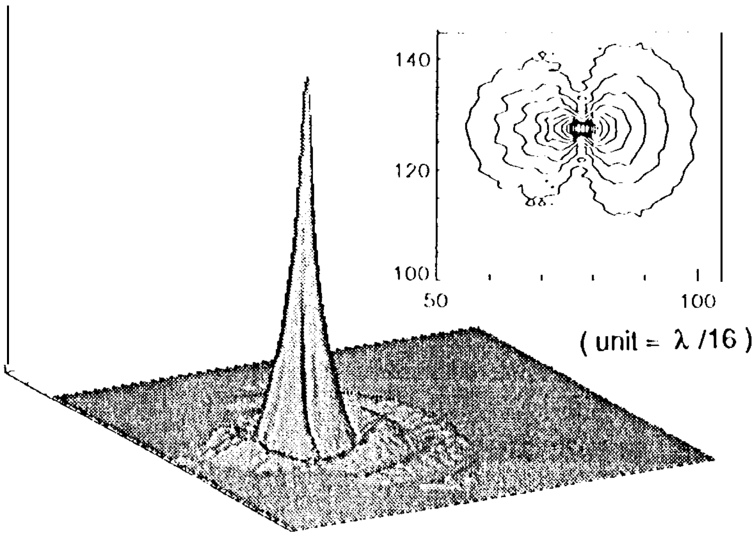

Figure 7.b Spotlight Simulated Impulse Response