Processing Techniques for Topographic Mapping
Using Interferometric SAR

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Søren N. Madsen

Jet Propulsion Laboratory
California Institute of Technology
\[ r(x) = |\vec{r}_{tgt} - \vec{r}_{plt}(x)| \quad \leftrightarrow \quad \vec{r}_{tgt} = \vec{r}_{plt}(x) + r(x)\hat{\text{los}} \]

\[ \delta r_d = c t_d = 2\hat{\text{los}} \cdot \delta \bar{x} \quad \Rightarrow \quad f_D = \frac{2\hat{\text{los}} \cdot \bar{v}}{\lambda} \]

\( \delta r_d \) is the differential range delay from array element to array element

\( \delta \bar{x} \) is the array element spacing
SPATIAL DISCRIMINATION MECHANISMS (2)

Introduce a second parallel track!

\[ \tilde{r}_{tgt} = \tilde{r}_{plt,1}(x_1) + r_1(x_1)\hat{n}_{los,1} = \tilde{r}_{plt,2}(x_2) + r_2(x_2)\hat{n}_{los,2} \]

\[ (\hat{n}_{los,1})_x = (\hat{n}_{los,2})_x = (\hat{n}_{los})_x \quad \text{and} \quad \tilde{r}_{plt, i}(x) = (x, y_i, z_i) \Rightarrow x_2 - x_1 = (r_1 - r_2)(\hat{n}_{los})_x \quad (3) \]

\[ \tilde{r}_1 = r_1 \hat{n}_{los,1} \quad \text{and} \quad \tilde{r}_2 = r_2 \hat{n}_{los,2} \quad \text{and} \quad \tilde{r}_1 = \tilde{r}_2 + \tilde{r}_{21} \]

\[ r_1 - r_2 = \frac{\tilde{r}_1 + \tilde{r}_2}{r_1 + r_2} \cdot \tilde{r}_{21} = \hat{n}_{los,av} \cdot \tilde{r}_{21} \quad (4) \]
The fundamental location equations of SAR interferometry:

\[
\begin{align*}
\vec{r}_{tgt} &= \vec{r}_{plu,1}(x_1) + r_1(x_1)\hat{n}_{los,2}(x_2) + r_2(x_2)\hat{n}_{los,2} \\
\hat{n}_{los,x} &= \frac{\delta r_d}{2 \delta x} \quad \text{and} \quad x_2 - x_1 = (r_1 - r_2)\hat{n}_{los,x} \\
\end{align*}
\]

(5)

\[r_1, r_2 >> |\vec{r}_{21} \Rightarrow \]

\[\text{target} = \vec{r}_{\text{radar}} + r_{st}\hat{n}_{los} \]

(6)

\[f_D = \frac{2}{\lambda} \hat{n}_{los} \quad \text{and} \quad \hat{n}_{los} \cdot \vec{v} = \frac{\lambda f_D}{2} \]

(7)

\[\hat{n}_{los} \sim \hat{2} \quad r_1 - r_2 = \frac{\lambda \phi_{\text{intf}}}{2\pi} \]

(8)
Note that the processor can apply Doppler centroid or zero Doppler referencing - both will work as long as matching values for platform position, range, Doppler are applied.
PHASE UNWRAPPING

- SAR interferogram provides complex numbers. Only $\phi \mod 2\pi$ is measured directly!

- To solve for topography the differential range must be determined. The appropriate number of $2\pi$ must be added to the interferogram phase measurement.

- The problem can be broken into two parts:
  - Phase unwrapping
  - Absolute phase determination

- Satellite radar interferometry: Two-dimensional phase unwrapping, [Goldstein et al., 1988]
PHASE UNWRAPPING (2)

- The unwrapping issue, example:

Clean data:

<table>
<thead>
<tr>
<th>Absolute phase in cycles</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.0</td>
<td>19.3</td>
</tr>
<tr>
<td>0.0</td>
<td>0.3</td>
</tr>
<tr>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>0.6</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Noisy data:

<table>
<thead>
<tr>
<th>Absolute phase in cycles</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.0</td>
<td>19.3</td>
</tr>
<tr>
<td>0.0</td>
<td>0.3</td>
</tr>
<tr>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>0.6</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Unwrapped (col. 1 fret)

<table>
<thead>
<tr>
<th>Unwrapped (row 1 fret)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
</tr>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.6</td>
</tr>
</tbody>
</table>
PHASE UNWRAPPING (3)

- Residues/Changes
  Unwrap the phases of four neighboring points (2 x 2).
  Net phase increment of loop = +1 cycle => residue with charge +1.
  Net phase increment of loop = -1 cycle => residue with charge -1.

- Cut lines/Three building
  Charges are connected by cut lines to form neutral trees.
  Any closed integration path not intersecting cut lines will enclose
  an accumulated charge of zero. Phase unwrapping not crossing
cut lines will thus be consistent- but not necessarily right!

- Neutrons
  Points that by some criteria has been determined as likely cut line
  locations. Possible criteria: non monotonic phase variation with
  range, large phase variance, high contrast edges in amplitude
  image. Neutrons are only used for “guiding” cut lines.

- Phase flooding
  Phases are unwrapped in a consistent fashion without intersecting
cut lines.
PHASE UNWRAPPING (4)

- Well behaved phase history with monotonic phase
- Discontinuous phase history (lay-over) with non-monotonic phase
PHASE UNWRAPPING RESIDUES ONLY

UNWRAPPED PHASE

RESIDUE/CUT MAP
ABSOLUTE PHASE DETERMINATION

- Note that interferometric phase is a function of carrier frequency

\[ \phi_1 = 2\pi \frac{f_1 r_{21}}{c} \sin(\alpha - \theta_i) \]
\[ \phi_2 = 2\pi \frac{f_2 r_{21}}{c} \sin(\alpha - \theta_i) \]

- Differential phase is equivalent to low frequency interferometer

\[ \Delta \phi = \phi_1 - \phi_2 = 2\pi \frac{(f_1 - f_2)}{c} r_{21} \sin(\alpha - \theta_i) \]

- No ambiguity for

\[ f_1 - f_2 < \frac{c}{2r_{21}} \]

- Split spectrum technique is independent on ground control points!
MEASURED DIFFERENTIAL PHASE AS A FUNCTION OF ABSOLUTE PHASE
MOTION COMPENSATION

- Aircraft accelerations measured by Inertial Navigation Unit (INU) located close to radar antennae

- Accelerations are processed and combined with Global Position System (GPS) data to generate a location time-line with high update rate and small drift

- Attitude data is measured by INU

- Antenna positions are derived from INU motion and lever arm corrections

- Data are processed in small overlapped patches with locally adapted reference line
MOTION COMPENSATION (2)

- Straight reference track selected to minimize antenna 1 deviations

- Antenna 2 reference off-set from antenna 1 reference by a constant baseline displacement vector; effectively making tracks parallel.

- Antenna 1,2 signals are shifted in range and phase to compensate for any measured, $\Delta r^i(x)$, deviation from straight line flight

\[
\Delta r_{sl}^i(r_{sl}, x) = \Delta \tilde{r}^i(x) \cdot \hat{n}_{los}(r_{sl})
\]

\[
\Delta \phi_{sl}^i(r_{sl}, x) = -\frac{2\pi f}{c} \Delta r_{sl}^i(r_{sl}, x)
\]
\[ \Delta r_{21} = B \sin(\alpha + \delta \alpha - \theta - \Delta \theta) \]
\[ \equiv B \sin(\alpha - \theta - \Delta \theta) + B \cos(\alpha - \theta - \Delta \theta) \delta \alpha \]
\[ \equiv B \sin(\alpha - \theta - \Delta \theta) + B \cos(\alpha - \theta) \delta \alpha + B \sin(\alpha - \theta) \delta \alpha \Delta \theta \]

\[
\alpha = 63^\circ, \delta \alpha = 1^\circ, \theta = 30^\circ, \: \Delta \theta = 5^\circ
\]
\[ \Rightarrow \delta \theta_{\text{error}} = 0.046^\circ = 0.0008 \text{ rad} = 0.8 \text{ m/km} \]

- Perfect navigation data not sufficient!
  Solutions: Iterative processing, or, inverse mocomp
### RADAR PARAMETERS, TOPSAR

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>wavelength</td>
<td>0.0567 m</td>
</tr>
<tr>
<td>bandwidth</td>
<td>40 MHz (20 MHz)</td>
</tr>
<tr>
<td>max. squint</td>
<td>15°</td>
</tr>
<tr>
<td>antenna length</td>
<td>1.5m</td>
</tr>
<tr>
<td>baseline length</td>
<td>2.5 m</td>
</tr>
<tr>
<td>baseline angle (rel. her.)</td>
<td>63°</td>
</tr>
<tr>
<td>altitude</td>
<td>9 km (approx.)</td>
</tr>
<tr>
<td>look angles</td>
<td>30-55°</td>
</tr>
<tr>
<td>slant range</td>
<td>10-15 km</td>
</tr>
<tr>
<td>output size</td>
<td>6.4 x 30 km (12.8 x 30 km)</td>
</tr>
<tr>
<td>pixel spacing/resolution</td>
<td>10 m / 15 m</td>
</tr>
</tbody>
</table>
JPL PROCESSOR, PHYSICAL CONFIGURATION

- Presently running on Alliant FX-800,
  - 12 proc. Intel i860
  - 512 Mbyte RAM
  - 10 Gbyte disk

- UNIX operating system

- Processor coded in Fortran 77

- Throughput:
  - input: 2 files each 81920 lines of 3072 bytes off-set video
  - output: -3000 lines of 640 samples (1 O* I O m pixels)= 192 km²
  2 hours 20 minutes
JPL PROCESSOR, CHARACTERISTICS

- Range-Doppler type processor
- Patch processor with local reference tracks adapted to actual track
- Doppler centroid is a function of range and is updated for each patch
- Data is motion compensated as a function as slant range
- Phase unwrapping using algorithm based on residues and neutrons
- Absolute phase determined using split spectrum technique
PROCESSOR BLOCK DIAGRAM

MoComp data

INM, D, GPS data integration

Antenna #1, Antenna #2 data

Decoding, range comp., & bandpass filtering

Motion compensation, phase shift & resampling

Doppler estimation & range fit

Azimuth compression

High band interferogram

Low band interferogram

Full resolution interferogram

Differential interferogram

Phase unwrapping

Residue calculation

Absolute phase image

3-dimensional location

ground map output
ON-GOING PROCESSOR RESEARCH RELATING TO INTERFEROMETRY

- Phase unwrapping
- Absolute phase determination
- Map resampling $\sigma_{xyz}(\text{range, azimuth}) \Rightarrow z(x, y) \& \sigma(x, y)$