THE EARTH VIEWED AS A DEFORMING POLYHEDRON: METHOD AND RESULTS

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As geodesists, it is natural for us to think of the Earth's surface as approximated by a network of points, We extend this concept to one of a rotating, braced polyhedron whose origin is at the Earth's center of mass, and whose vertices are defined by GPS stations. I-he realization of a terrestrial reference frame with estimated site velocities requires the specification of 3 Euler angles and their first time derivatives to align site coordinates with convention. We illustrate practical aspects of such reference frame alignment with examples from global GPS data acquired from June 1992 to March 1993. Finally, we describe the JPL GPS coordinate solution, JGC9301, which has also been submitted to the IERS Annual Report.

INTRODUCTION

GPS *is* quite unlike. any other geodetic technique., because we. can use it to look at the Earth with high spatial and temporal resolution. For example, the. GPS global network provides us with a daily snapshot of the Earth, allowing us to look with high temporal resolution at the motion of sites before, during, and after a large earthquake. At the. other extreme of the spatial and temporal scale, GPS has great potential for mapping post-glacial rebound of the Earth's crust.

Currently, the GPS global network has over 30 simultaneously operating receivers. Given that the current "core" network will double within the next few years, and that the total number of permanent receivers will possibly reach 200 within 5 years (most of them in regional arrays), we are faced with the rather daunting and exciting task of reducing all these data into a consistent picture of the Earth.

This paper does not address the. technical issues of communication, storage, and data processing for such a vast data rate, suffice it to say that regional data reduction, least-squares partitioning, and collaborative. exchange of subnetwork solutions, will all play a role. This requires international collaboration, and the IGS already provides the cohesiveness, organization, standards, and goodwill that is necessary to make this work.

The main focus of this paper is to view the Earth an evolving polyhedron, whose vertices are defined by the GPS sites. We review the prime estimable parameters of the free-net work approach [1, 2], and then go on to describe how a time-series of coordinates can be derived without imposing external constraints on any particular site coordinate or velocity. We show examples of time-historic.s of site latitude, longitude, and height, taken from a 13-week time period in 1992, including the effects of the Landers earthquake of 28

June, 1992 in California. Finally, we present cartesian coordinates for 38 stations at epoch 1992,5, with 3 rotation angles applied so that the polyhedron is oriented to ITRF91 [3,4]. We compare the scale, geocenter, and individual station coordinates of our solution with ITRF91.

ESTIMABLE PARAMETERS OF THE POLYHEDRON

Figure 1 illustrates parameters which are well-constrained by the GPS data, even when all station coordinates and satellite orbits are freely estimated without a priori constraints. Certain functions of these parameters may be even better determined (for example, the angle between a long baseline and the spin axis, or the differential geocentric distance between two nearby stations).

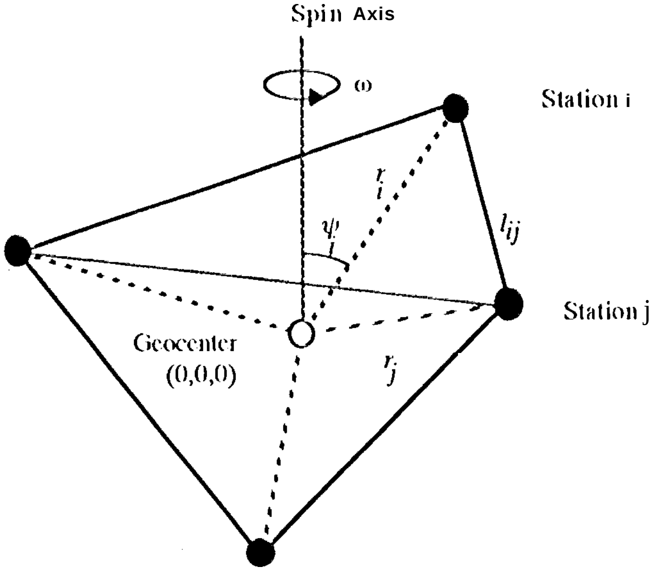

Fig 1. This figure illustrates estimable parameters, that is, those parameters which are well-constrained by the GPS data, and do not require external constraints. The parameters include baseline length between station i and station *j*, l_{ij}, geocentric distance of station *i*, r_i, colatitude of station *i* to the instantaneous spin axis, ψ_i, and the rate of rotation, ω.

This type of parameterization is inconvenient for least squares estimation and for reporting results. Quite simply, the polyhedron is overspecified. (For example, we could actually compute r_j given all other parameters.)

It is much more convenient to represent the station coordinates as cartesian coodinates. However, cartesian coordinates themselves are not estimable! Even if we define the spin axis lie on the z-axis at a certain time, the azimuthal angle of the polyhedron is not defined. If we also choose to explicitly estimate the spin axis direction, then a total of 3 Euler angles are undefined. Note that, if we estimate station velocities, these 3 Euler angles are also free to drift at a constant rate, hence we would need to specify 3 Euler angles and their 3 first time derivatives (or, equivalently, 3 Euler angles at two epochs).

We must keep in mind that we are choosing the cartesian coordinate representation (or the equivalent representation of latitude, longitude and height for a specified ellipsoid) for convenience only, and that the coordinates themselves are not necessary for interpretation (a notion crucial to the development of relativity theory).

SITE COORDINATES

We have chosen to estimate all cartesian coordinates and a daily pole direction, all with very loose constraints. As a final step, the free-network GPS solution is oriented to ITRF91 [3, 4] at epoch 1992.5. When deriving the rotational angles between two reference frames, it is essential to simultaneously estimate the 3 angles, and also 3 translational components and a scale parameter. The reason for this is that the angles are correlated with the translations, so if the GPS solution's location of the "geocenter" (Earth's center of mass) disagrees with the reference solution, the estimate of 3 angles alone will absorb some of the translational offset, thus giving an erroneous orientation: Having estimated all 7 parameters, only the 3 angles are then used to transform the GPS solution.

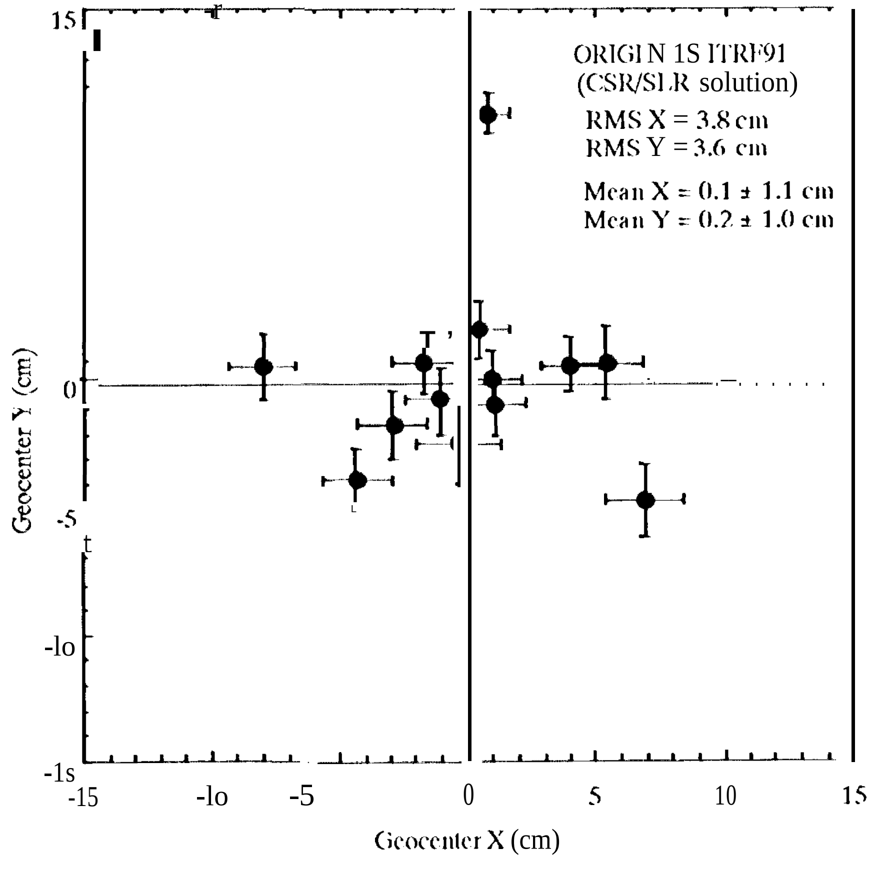

Fig 2. Weekly GPS solution for the geocenter (Earth center of mass), as compared with the origin of 1"1'1<191, which is based on a satellite laser ranging solution by CSR.

in the following examples, we use data from a 13 week period from June-August, 1992. For this purpose, we simply assumed a zero-velocity model for station coordinates, and formal a fully weighted average solution for the free-network polyhedron, which was then oriented to ITRF91 using the above procedure. We. then took each weekly solution, and estimated a 7-parameter transformation into the 13-week combined solution.

Fig. 2 shows the translational offset of each weeks solution. These translations can be interpreted as the discrepancy between the GPS determination of the geocenter and the origin of ITRF91. Since we know of no mechanism which can induce few-cm level variations in the Earth's center of mass (relative to the crust) over such a short period, we must interpret Fig. 2 as a measure of the stability of the GPS origin, which is implicitly defined through orbital dynamics. Hence, Fig. 2 illustrates one aspect of orbital mismodeling. There is no evidence of a bias between GPS and ITRF. The z-component is not as precisely constrained by the GPS data, but nevertheless agrees on average to better than 10 cm with ITRF. (Our most recent solution, described below, agrees with the ITRF origin to within 2 cm in all 3 components).

After removing each week's geocenter, scale, and orientation so that it is transformed into the 13-week reference frame, we obtain weekly estimates of station coordinates. Fig. 3 shows a representative examples of time-series of coordinates for Wettzell, Germany. Wettzell is typical of all northern hemisphere sites. The average RMS for geocentric coordinates are summarized in Table 1 below.

Fig. 4 shows the motion of Pinyon Flat Observatory (PIN1), California, due to the Landers earthquake of 28 June, 1992. It is important to realize that this plot is showing, the latitude of the station (not baseline estimates, such as those shown in [5] and [6]). This illustrates the power of this technique to observe absolute co-seismic displacement, without reference to any particular fixed station. In fact, the geocentric coordinates are generally better determined than baseline coordinates for long baselines. Baseline precision is at the level of 2 parts per billion, which exceeds 4 mm for baselines longer than 2000 km.

Table 1

RMS OF WEEKLY GPS GEOCENTRIC COORDINATES

Coordinate	Northern Hem. (mm)	Southern Hem. (mm)
Latitude	4.0	11
Longitude	4,4	14
Height	7.5	23

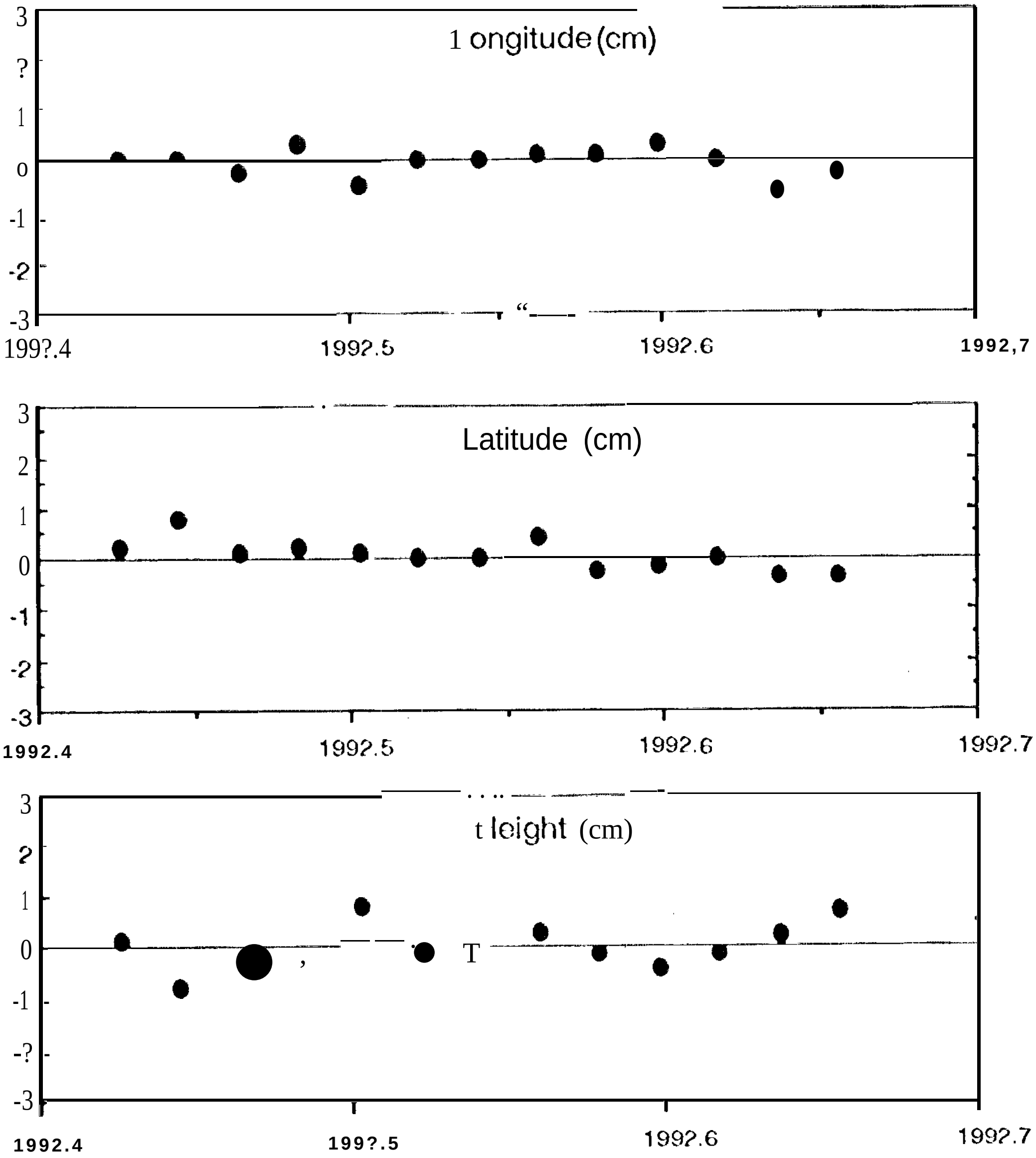

Fig 3. Weekly GPS solution for the geocentric coordinates of Wettzell, Germany. The 13-week average solution has been subtracted out. RMS in lat. and long. is 3 mm, and 7 mm for height.

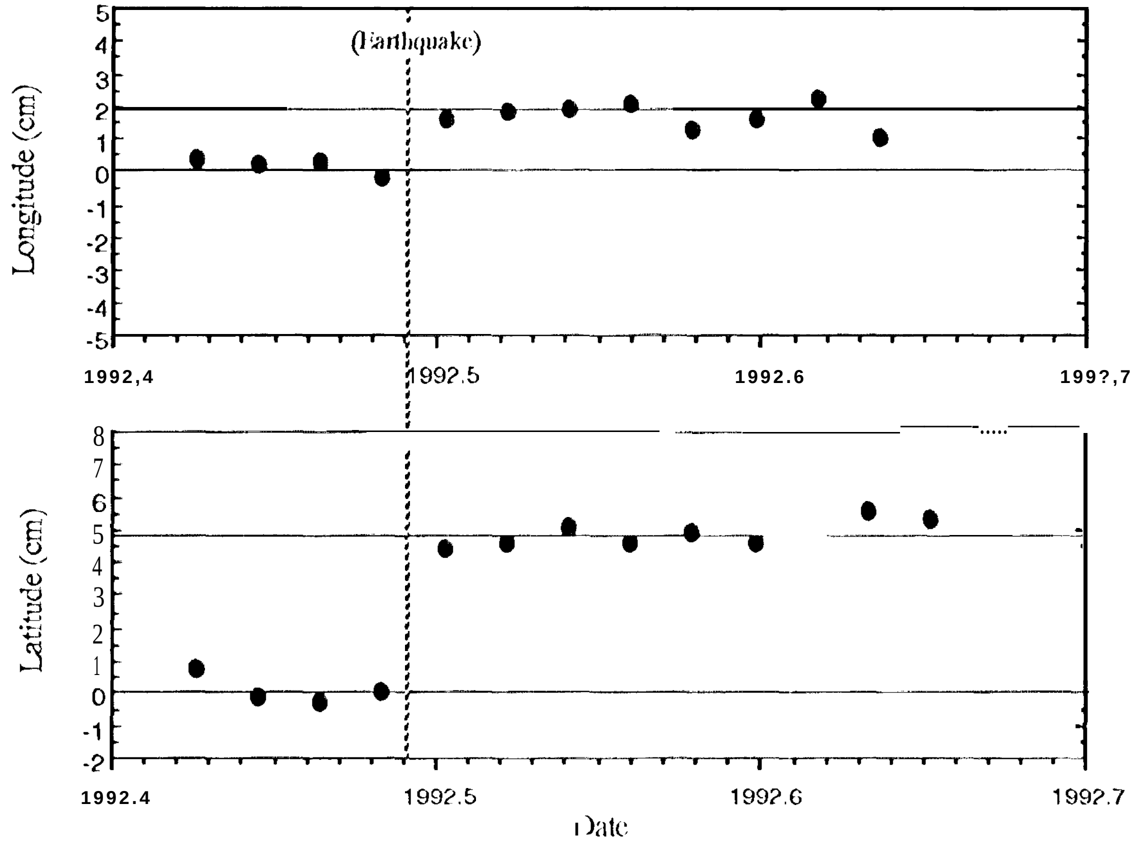

Fig 4. Weekly GPS solutions for the geocentric coordinates of Pinyon Flat Observatory, California. The step-function is due to co-seismic displacement associated with the Landers earthquake of 28 June, 1992.

SITE VELOCITIES

We mentioned above the additional complexity in reference frame definition when station coordinates are estimated as an epoch position plus a velocity: Euler angle rates must be specified, otherwise the polyhedron is free to rotate about some arbitrary pole. For example, velocities in the longitudinal direction would be perfectly correlated with the Earth spin rate.. Fixing the Euler angle rates will affect the apparent drift of the Coordinates of the Earth's spin axis ("apparent," because it does not affect the estimables parameter, which are the colatitudes of all stations with respect to the instantaneous spin axis!). Conversely, the Euler angles and their rates may be arbitrarily fixed by defining the direction of Earth's pole on 2 days, and fixing the longitude and longitudinal velocity of one station. The choice we suggest here, is to apply a rate constraint such that the station velocities are aligned in some average. sense with conventional geological plate. motion models, such as NUVEL, NNR-1 ("NNR" means "no net rotation") [7].

One way to achieve this is to expand the notion of a 7-parameter transformation into a 14-parameter transformation (the original 7-parameters plus the.ir rates). We could then solve for the. Euler angles and rates and directly apply it to our free network solution. The advantage of such a technique is that no station (or station velocity) has special treatment in the reference frame definition, and no coordinate (or velocity) has zero formal error.

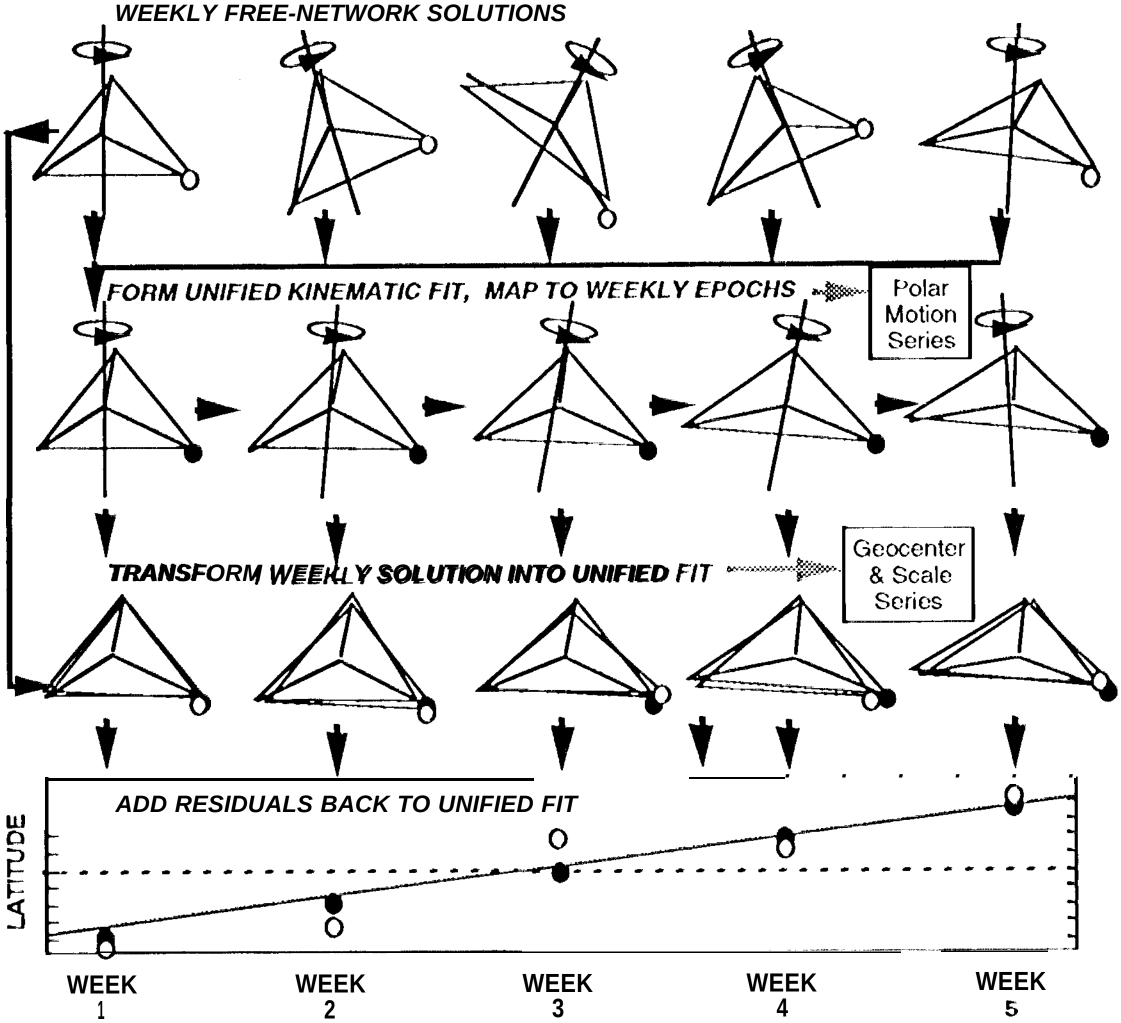

Fig. 5. A schematic description of how to derive a time-series of geocentric station coordinates. In this example, we obtain weekly station coordinates, pole positions, geocenter location, and scale parameter. In practice, the pole position is estimated every day.

Figure 5 illustrates how station coordinates at, for example, weekly epochs can be derived by mapping weekly solutions into a unified kinematic solution (with station velocities estimated). To avoid complication, the alignment to ITRF is not explicitly shown in this figure. Note that this is similar to method used to derive Figs. 3 and 4, (except that station velocities were not estimated, and pole positions were actually estimated daily).

JPL GLOBAL COORDINATE SOLUTION JGC9301

JPL solution JGC9301 has been submitted to IERS for inclusion in the Annual Report. The solution is listed in the Appendix. We describe it here to illustrate how the above techniques can be applied.

The inputs to JGC9301 are daily free-network solutions from (approximately) June, July, August, 1992, plus January and February 1993. (We do not yet have lrm-network solutions for the missing months). These daily solutions were first combined into weekly solutions, and a few (about 5 %) suspected problem days were removed by checking baseline length repeatability. Using these weekly solutions, station velocities and coordinates were estimated at a specified epoch (in this case, 1992.5). At this point, the solution was very ill-determined for the reasons given above. Since the period spanned by the data is a fraction of a year, this solution's was constrained to the ITRF91 velocity field [3, 4]. We solved for a 7-parameter transformation into ITRF91[3] at epoch 1992.5, and then applied only the 3 rotational angles to the GPS solution. The solution JGC9301, augmented with the NUVEL NNR-1 velocity field (at designated primary sites on stable plate interiors) can now be used to define the orientation for all future GPS solutions.

The geocenter and scale for JGC9301 were. not fixed to ITRF91. The differences in geocenter and scale are given in Table 2. Removing the geocenter and scale, the RMS coordinate difference is 16 mm (for 66 coordinates, 59 degrees of freedom).

Table 2
TRANSFORMATION JGC9301 -ITRF91
Standard errors are given for JGC9301 only

Parameter	**JGC9301-ITRF91**
Geocenter X	8.9 ± 11 mm
Geocenter Y	0.6 ± 10 mm
Geocenter Z	17.7 ± 15 mm
Scale	$-2.4 \pm 0.2 \times 10^{-9}$

CONCLUSIONS

In conclusion, we suggest that GPS can provide. a very strong reference frame, capable of providing geocentric coordinates with subcentimeter accuracy. The terrestrial reference frame will be deficient in 6 quantities which must be specified in order for station coordinates and their velocities to be consistent with convention. These quantities are 3 Euler angles and their first time derivatives, We suggest orienting free-network GPS solution at a specific epoch with ITRF (for example, by solving for a 7-parameter transformation, then applying the. solution for the 3 rotation angles). The 3 Euler angle rates can be fixed by applying a global rotation rate to minimize the. RMS difference in station velocity with NUVEL NNR-1 for sites on stable plate interiors.

ACKNOWLEDGEMENTS

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APPENDIX

JPL GPS Coordinate Solution: JGC9301

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** EPOCH OF ADJUSTMENT IS 1992.5; ALL VELOCITIES CONSTRAINED TO ITRF91
** GPS DATA SPAN: 1-JUN-1992 to 30-AUG-1992, and 1-JAN-1993 to 1-MAR-1993.
(1) With the following exceptions, antenna heights are as reported in
    IGS Mail#90. Note that PN1Q, GOLQ, and JP1Q refer to post-seismic positions.
(2) All "S"-type points are to the top of the choke ring, hence they should
    more properly be designated a new DOMES number, located 7 cm above the
    current S point.
(3) Unknown or unassigned DOMES sites are given the number 99999.
(4) Unknown or unassigned DOMES points are assigned the number 999.
(5) Station character ID's follow IGS Mail#90, except for the following:
    (a) CASA is an uncatalogued point near Mammoth Lakes, California.
    (h) HARV is an off-shore oil platform near Vandenberg, Calif.
    (c) KOUR is the new global tracking site at Kourou, S. America.
    (d) NYA* is "post explosives" (refering to the accident of late 1992), but
        should in principle be equivalent to NYAL. A separate solution was obtained
        to assess the new antenna height provided by Statens Kartwerk. Assumed
        antenna heights to top of ring were: NYAL=5.286 m, NYA*=5.273 m
    (e) PAM* is to the top of the choke ring (for June-August., 1992).
    (f) PIE1 is a new point at Pie Town, New Mexico. According the M. Bryant,
        GSFC, the tie from the ref. point of CDP 7234 to JPL 4009 S is
          DX= 36.9162 m, DY= 34.8267 m, DZ= 35.2550 m
    (g) USU2 (until Aug 9, 1992) and USU3 (since Aug 9, 1992) are both
        different points than USUD (which was valid only for Jan'91 expt.)
    (h) VNDP is a new Rogue monument at Vandenberg, Calif.
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JPL GPS Coordinate Solution: JGC9301 (continued)

DOMES #	Station	X(m)	Y(m)	Z(m)	SX(m)	SY(m)	SZ(m)
40129M003	ALBH	-2341332.8188	-3539049.5063	4745791.3986	0.0027	0.0036	0.0032
401 04M002	ALGO	918129.6062	-4346071.2190	4561977.7926	0.0032	0.0036	0.0036
50103S017	CANB	-4460996.1118	2682557.1741	-3674443.9985	0.0063	0.0063	0.0050
40437M999	CASA	-2444430.1146	-4428687.6998	3875747.4434	0.0239	0.0338	0.0270
40105M002	DRAO	-2059164.5868	-3621108.3908	4814432.4129	0.0027	0.0036	0.0032
40408M001	FAIR	-2281621.3153	-1453595.7717	5756961.9615	0.0027	0.0032	0.0040
40405S028	GOLD	-2353614.1045	-4641385.4774	3676976.5198	0.0036	0.0045	0.0036
40405S031	GOLQ	-2353614.0916	-4641385.4647	3676976.5243	0.0054	0.0076	0.0058
11001M002	GRAZ	4194424.0635	1162702.4962	4647245.2583	0.0045	0.0036	0.0045
40400M006	JPL1	-2493304.0622	-4655215.5740	3565497.3586	0.0036	0.0040	0.0036
40400M007	JP1Q	-2493304.0487	-4655215.5673	3565497.3406	0.0050	0.0072	0.0054
99999S001	HARV	-2686069.1359	-4527084.4727	3589502.2322	0.0040	0.0054	0.0040
30302M002	HART	5084625.4517	2670366.5648	-2768494.0472	0.0104	0.0090	0.0054
13212M007	HERS	4033470.3093	23672.7011	4924301.1537	0.0045	0.0036	0.0045
40424M004	KOKB	-5543838.0765	-2054587.5465	2387809.5811	0.0054	0.0050	0.0036
13504M003	KOSG	3899225.3394	396731.7611	5015078.2819	0.0032	0.0027	0.0032
99999S999	KOUR	3839591.5927	-5059567.6757	579956.8479	0.0076	0.0086	0.0036
13407S012	MADR	4849202.5739	-360329.1847	4114913.0528	0.0036	0.0032	0.0032
31303M001	MASP	5439189.2326	-1522054.8584	2953464.2000	0.0054	0.0040	0.0036
12734M008	MATE	4641949.8225	1393045.2204	4133287.2514	0.0040	0.003?	0.0032
66001M001	MCMU	-1310695.2319	310468.8975	-6213363.4752	0.0054	0.0063	0.0081
10503S011	METS	2892571.0552	1311843.3063	5512634.0591	0.0027	0.0027	0.0036
10317M001	NYAL	1202430.7419	252626.6293	6237767.4903	0.0027	0.0027	0.0063
10317M001	NYA*	1202430.7483	252626.6281	6237767.5077	0.0036	0.0032	0.0094
10402M004	ONSA	3370658.7584	711876.9849	5349786.8156	0.0027	0.0027	0.0032
92201S999	PAM*	-5245202.1159	-3080476.4838	-1912828.0770	0.0099	0.0086	0.0045
92201M003	PAMA	-5245195.1164	-3080472.3882	-1912825.5272	0.0121	0.0108	0.0050
40129M002	PGC1	-2327188.0475	-3522529.0014	4764832.3874	0.0040	0.0050	0.0050
40456M999	PIE1	-1640916.6978	-5014781 .1876	3575447.1450	0.0040	0.0054	0.0045
40407M002	PIN]	-2369510.3526	-4761207.2139	3511396.1471	0.0040	0.0054	0.0040
40407M003	PNIQ	-2369510.3751	-4761207.2145	3511396.0951	0.0050	0.0072	0.0054
40433M004	QUIN	-2517230.9574	-4198595.2959	4076531.3450	0.0036	0.0050	0.0040
40499M002	RCM2	961318.9938	-5674090.9670	2740489.5737	0.0045	0.0068	0.0040
41705M003	SANT	1769693.2841	-5044574.1095	-3468321.1600	0.0068	0.0086	0.0058
40460M001	S101	-2455521.6655	-4767213.4340	3441654.9141	0.0086	0.0139	0.0099
40101M001	STJO	26)2631.3467	-3426807.0053	4686757.7401	0.0032	0.0032	0.0032
23601M001	TAIW	-3024781.8690	4928936.9104	2681?34.5286	0.0063	0.0072	0.0050
10302M003	TROM	2102940.4466	721569.3569	5958192.0724	0.0027	0.0027	0.0036
21729S999	USU2	-3855262.6529	3427432.2180	3741020.9954	0.0076	0.0072	0.0063
21729S999	USU3	-3855263.0376	3427432.5738	3741020.4726	0.0063	0.0063	0.0050
40420M999	VNDP	-2678090.4952	-4525439.04?3	3597432.4703	0.0423	0.0625	0.0437
14201S020	WETT	4075578.7195	931852.6398	4801570.0361	0.0036	0.0032	0.0036
14201M999	WKT*	4075577.6580	931852.3942	4801568.7689	0.0040	0.0032	0.0045
50107M004	YAR1	-2389025.3445	5043316.8547	-3078530.9517	0.0063	0.0076	0.0050
40127M003	YELL	-1224452.3754	-2689216.0862	5633638.2826	0.0027	0.0032	0.0036