Sinusoidal Gravitational Waves from Mass-varying Binary Systems

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Abstract

It is believed that most quasars and galaxies present two common features: the presence in their core of a supermassive object, and the experience of one or more encounters with other galaxies. In this scenario, it is likely that a substantial fraction of active galactic nuclei harbour a supermassive binary, fueled by an accretion disk. These binaries would certainly be among the strongest sources of sinusoidal gravitational waves. We investigate their evolution considering, simultaneously, the accretion of the black hole's masses from the disk, and the gravitational waves emitted during the orbital motion. We also consider other astrophysical scenarios involving a coalescing binary with non constant, masses.

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1 Introduction

Soon after the discovery of the astrophysical phenomena subsequently indicated with the name of Active *Galactic Nuclei* (AGN), it was argued that their power supply was ultimately gravitational in origin. If this is the common feature of the wide range of models included in the AGN category, then the natural conclusion is, as first pointed out by Zeldovich & Novikov (1964) and Salpeter (1964), that an AGN 'prime mover' is a supermassive black hole (SBH). Among other compelling arguments in favour of the black hole hypothesis are their efficiency and stability, together with various observational discoveries, like rapid X-ray variability, small scale radio jets, broad emission lines, etc. (Blandford 1990; Osterbrock 1993).

For these reasons, most of the theoretical work about the AGN phenomenon has been focused on the SBH hypothesis, namely that essentially all active galaxies contain $\sim 10^6 - 10^9 M_\odot$ black holes in their nuclei, and that these objects, together with their orbiting accretion disks, are the prime movers for most of the powerful activity.

On the other hand, there are also compelling reasons to believe that a great number of galaxies have undergone at least one merger since the epoch of their formation (see Rees 1990, and references therein). Indeed, many current models assume that the central object is activate, or simply refueled, as a result of these interactions with another galaxy (Osterbrock 1993), This new evidence, along with the central SBH hypothesis, suggests that, due to the dynamical friction exerted by the surrounding environment, a certain number of active galaxies could harbour a binary black hole system (111111 S), with separation of the order of parsecs. This conclusion is supported by the observed bending and apparent precession of radio jets emerging from AGN (Begelman, Blandford, & Rees 1980), In fact, the S-symmetry observed in many radio sources and in a considerable fraction of quasars at $z < 1$ (Hutchings, Price, & Gower 1988) might be due to the presence of such binaries. However, in many quasars the jets are strongly curved on the milliarcsec scale as well. If this curvature is also due to precession, much shorter precession periods, and thus smaller binary separations, are required. This indicates that mass flow into the galactic nucleus through the accretion disk dominates not only the activity, but also the evolution of the central system (Roos 1989).

Moving from these considerations, recent works have investigated the evolution of a BBHS in the violently relaxed *core* of a merged galaxy, taking into account the flow of gas and stars into the newly formed nucleus (Begelman et al. 1980; Roos 1989; Ebisuzaki, Makino, & Okumura 1991; Fukushige, Ebisuzaki, & Makino 1992).

These AGN models are interesting per se, and also in conjunction with the issue of gravitational waves (GW) detection. In fact, close binary systems of supermassive compact objects are currently considered as the most certain observable sources for detectors like LIGO (Abramovici et al. 1992) and VIRGO (Bradaschia *et al.* 1990), and also for those experiments based on the Doppler tracking of an interplanetary spacecraft (see, e.g., Thorne & Braginsky 1976; Park & Vishniac 1991; Bertotti et al. 1992). In the standard model for the emission of GW from a binary system, the energy loss is proportional to the square of the third derivative of the quadruple moment. 'l'his description, however, does not take into account other energy loss mechanism, due for example to the interaction with an accretion disk.

In this work, we analyze the evolution of a BBHS when the effects of mass accretion and GW emission are considered simultaneously. In §2 we will describe in detail a single BBHS, and find, under some reasonable assumptions, the behavior of the separation between the two components. in §3 we show how the detection of these waves could provide some useful information on the physical characteristic of the AGN, namely its mass and accretion rate. in §4 we extend our analysis to a simple population of BBHS systems, and find the evolution law for the distribution function, We conclude, in §5, pointing out other astrophysical situations in which a relevant mass change can affect tile evolution of the binary system, and thus also the waveform of the emitted GW.

2 Evolution of a binary system in presence of mass accretion

The evolution of a close binary system in presence of mass loss or gain has been thoroughly analyzed in the past, for example as a tool to explain the irregularities and secular changes in some spectroscopic binary stars (Kruszewski 1966). Only later a similar analysis has been applied to relativistic objects, like a neutron star binary (Clark & Eardley 1977; Jaranowsky & Krolak 1992).

We consider a binary system consisting of two supermassive black holes, following circular newtonian orbits around the common center of mass, This system, according to General Relativity, radiates gravitational waves, which subtract energy and orbital angular momentum from the system itself. The effect of this radiation is also to circularize the orbit. For this reason we have assumed the orbits to be circular. Also the frictional drag exerted by the surrounding gas and stars during the formation and early evolutionary stage of the BBHS should circularize the orbits, even if this point is still controversial (Begelman *et al.* 1980; Fukushige *et al.* 1992).

The angular momentum carried away by the GW in the unit of time is given by the quadruple formula (Peters 1964), which for point masses reads (units c = G = 1 hereafter)

$$\frac{dJ_{GW}}{dt} = \frac{32}{5}\frac{\mu^2 M_T^{5/2}}{a^{7/2}}, \qquad (1)$$

where a is the separation between the BHs, $M_T = m_1 + m_2$ is the total mass, and $\mu = m_1 m_2 / M_T$ is the reduced mass of the system.

When we consider such a system in the core of an AGN, we must also take into account the effect of the accretion disk. This material, made up of gas, stars or even more exotic objects, like ordinary black holes, is able to transfer or subtract energy and orbital angular momentum to/from tile BBHS (Kandrup & Mahon 1992).

We will assume that the disk lies on the orbital plane, and that the mass falls radially with respect to the center of mass of the binary (i.e. without any velocity component tangential to the BHs orbits). Moreover, we neglect any frictional or gravitational effect exerted by the disk on the BBHS, as well as the intrinsic angular momentum (spin) of the BHs.

Forgetting for a moment the GW effect, according to Newton's law we have, for each body ($i = 1, 2$),

$$\frac{d}{dt}(m_i \vec{v}_i) = \vec{F}_i^N + \dot{m}_i \vec{\xi}_i \,, \tag{2}$$

where the dot denotes derivative with respect to time, $\vec{F}_i^N$ is the gravitational force, and $\vec{\xi}_i$ is the velocity of the accreting mass. The vector product of equation (2) with $\vec{r}_i$ gives the conservation equation for the total angular momentum

$$\frac{d\vec{J}}{dt} = \sum_{i=1}^{2} \dot{m}_i \left(\vec{r}_i \times \vec{\xi}_i \right) \,. \tag{3}$$

Thus, since we are assuming that $\vec{r}_i$ and $\vec{\xi}_i$ are (anti) parallel, the orbital angular momentum is conserved during accretion. Taking now into account the angular momentum carried away by the GW (see eq. (1)), we can write

$$\frac{d}{dt}\left(\mu\sqrt{M_T a}\right) = -\frac{dJ_{GW}}{dt} \,. \tag{4}$$

For simplicity, we will assume, from now on, that the two BHs have the same mass, m. Then, substituting equation (1) in equation (4), one gets the following equation governing the separation a

$$\dot{a} = -\frac{128}{5}\left(\frac{m}{a}\right)^3 - 3\left(\frac{a}{m}\right)\dot{m} \tag{5}$$

in order to integrate equation (5) we should have to know the time dependence of $\dot{m}$. Unfortunately, while we can make some reasonable hypothesis on the magnitude of $\dot{m}$, based on the Eddington assumption (see later), very little earl be said about the real time dependence of $\dot{m}$, as well as the duration of the active phases. Due to our ignorance on this subject, we will make the very simple assumption $\dot{m} = \nu = const.$ Then, integrating equation (5) and taking $t = 0$ as the initial time, one gets

$$a(t) = a_0 \left(1 + \frac{t}{\tau_m}\right) \left[\left(1 + \frac{\tau_m}{16\tau_g}\right) \left(1 + \frac{t}{\tau_m}\right)^{-16} - \frac{\tau_m}{16\tau_g} \right]^{1/4} , \quad (6)$$

where we have introduced the two fundamental time scales

$$\tau_m \equiv \frac{m_0}{\nu} \quad (7)$$

and

$$\tau_g \equiv \frac{5}{512} \frac{a_0^4}{m_0^3} . \quad (8)$$

'The meaning of these two quantities is as follows: τ_m gives the mass e-folding time for accretion at the rate ν, while τ_g gives the time required by the binary to coalesce as a consequence of the energy and angular moment urn carried away by the G W, assuming circular orbits and $\nu = 0$. For a very slow accretion rate, i.e. for $\tau_m >> \tau_g$, the time evolution of a is dominated primarily by GW emission, and equation (6) reduces in this limit to the well known formula (Misner, 'J'borne, & Wheeler 1973)

$$a(t) = a_0 \left(1 - \frac{t}{\tau_g}\right)^{1/4} . \quad (9)$$

From equation (6), we can easily find the coalescing time, i.e. the time at whit]] a vanishes,

$$\tau_c = \tau_m \left[\left(1 + \frac{16\tau_g}{\tau_m}\right)^{1/16} - 1 \right] . \quad (10)$$

In the limit $\tau_m \gg \tau_g$, by expanding equation (10) in powers of τ_g/τ_m, to first order we get

$$\tau_c = \tau_g \left[1 - \frac{15}{2} \frac{\tau_g}{\tau_m} + O\left(\frac{\tau_g}{\tau_m}\right)^2 \right] . \tag{11}$$

Figure 1 shows the value of τ_c as a function of τ_m, in units $\tau_g = 1$. As expected, the mass accretion speeds up the evolution fo the system. In particular, if the two time scales are almost equal, then the coalescing time is approximately 20% of τ_g.

The crucial quantity which appears in equations (6) and (10) is thus the ratio

$$\frac{\tau_g}{\tau_m} \simeq 3 \times 10^6 \, m_8^{-4} \left(\frac{a_0}{1 \; pc}\right)^4 \left(\frac{\nu}{M_\odot / \; yr}\right) , \tag{12}$$

where m_8 is the initial mass in units of $10^8 M_\odot$. For values of m and m typical of AGN models, this ratio is usually bigger than one, unless one is willing to adopt extremely small separations. This fact is of considerable importance, because we are forced to abandon the usual idealized description, based on equations (8) and (9), in favour of the more general equations (6) and (10). On the other hand, the quantity given in equation (12) can not be arbitrarily large, In the standard accreting model, the total radiated power is assumed to be limited by the Eddington luminosity

$$L_E \simeq 2.6 \times 10^{46} 71\}6 \; erg \; S^{-1} . \tag{13}$$

In principle, this limit only applies for spherical accretion, when most of the models actually gives $L << L_E$ (Chang & Ostriker 1985; Park & Ostriker 1989), but valid arguments suggest that this limit remains valid in every realistic (i.e. without an unnatural segregation between radiation and fuel, see Turner 1991) anisotropic model, which generally gives $L \lesssim L_E$ (Rem 1984).

Associated with L_E is an Eddington accretion rate, that would be able to sustain an Eddington luminosity with efficiency ϵ for conversion of mass into radiant energy

$$\dot{m}_E \approx \frac{0.2}{\epsilon} m_8 \; M_\odot / yr \tag{14}$$

Equation (14) gives a lower limit for the e-folding time

$$\tau_E \simeq 4.4 \times 10^8 \epsilon \, yr, \tag{15}$$

which is independent of the mass. Note that equation (14) implies an exponential growth of the mass and its derivative, in contradiction with our hypothesis $\dot{m} = const.$ Nonetheless, since we are implicitly assuming that we observe the AGN for a time $T \ll \tau_m$, during which the r.h.s. of equation (14) can be assumed constant, we will consider the Eddington values (14) and (15) as upper (lower) limits for $\dot{m}$ and τ_m, respectively. Thus, the ratio in equation (12) is bounded from above by the quantity

$$\frac{\tau_g}{\tau_E} \sim \frac{7 \times 10^5}{\epsilon} \left(\frac{a_0}{1 \, pc}\right)^4 m_8^{-3}. \tag{16}$$

Some comments should refer to the unknown physical parameter ϵ, which describes the radiative efficiency. A detailed review of this aspect has been given by Turner (1991), in the attempt of accounting for the masses and luminosities of the oldest quasars ($z > 4$). From his considerations, one can adopt the reasonable value $\epsilon = 0.1$, independently of the details which perturb the fuel reservoir quite far from the inner giant object.

Finally, another useful quantity is the critical separation, a_c, defined as the initial separation which gives coalescence after a certain time t. From equation (10), this is given by

$$a_c(t) = \left\{ \frac{32 m_0^3 \tau_m}{5} \left[\left(1 + \frac{t}{\tau_m} \right)^{16} - 1 \right] \right\}^{1/4}. \tag{17}$$

In using equation (17), we must remember that the origin of time was taken at the instant of formation of the binary. This quantity is of particular importance when one is looking for the GW bursts resulting form the final coalescence, since the probability of these events depends on the percentage of systems which formed with separation around $a_c(t_{obs})$.

3 The Braking Index

In the previous section we have calculated the decreasing of the separation a as a consequence of the emission of GW and the accretion of mass. Since the frequency of the waves is twice the orbital angular frequency, the net effect of their emission is an increase of their own frequency. If the masses are constant, then a is governed by equation (9), and from Kepler's third law one finds

$$f(t) = f_0\left(1 - \frac{t}{\tau_g}\right)^{-3/8}. \tag{18}$$

however, in our case the situation is not so simple, since now the total mass is increasing, and $a(t)$ is given by the more complicate equation (6), The overall effect is, thus, a more rapid increase of $f(t)$. This fact can have considerable importance in relation to GW detection experiments. In fact, a gravitational train emitted by a binary system is detectable as a pure sinusoid only if the frequency remains in the same resolution bin during the observations, i.e. as long as $\dot{f}T \lesssim$ Af $\equiv 1/T$, where T is the duration of the experiment. 'l'his constraint has been sometimes overlooked, in the past, since it implies that the spectral region where the signal can be scare.hed for must be consistently restricted (Giampieri & Tinto 1993).

In order to describe the time evolution of $f(t)$, wc take into account also its derivatives $\dot{f}$ and j. In particular, the quantity of interest is the *braking index*

$$k \equiv \frac{f\ddot{f}}{\dot{f}^2}. \tag{19}$$

Now, Kepler's third law and equation (6) give

$$\frac{\dot{f}}{f} = -\frac{3}{2}\frac{\dot{a}}{a} + \frac{1}{2}\frac{\dot{m}}{m} = \frac{3 \cdot 2^{14/3}}{5} m^{5/3}(\pi f)^{8/3} + \frac{5\nu}{m}. \tag{20}$$

Deriving again equation (20) with respect to t, and neglecting the second order terms in the nondimensional quantity $\nu(\pi m f)^{-8/3}$, we eventually find

$$k \simeq \frac{11}{3} - \beta\nu(\pi m f)^{-8/3}, \tag{21}$$

where β is a numerical coefficient given by $\beta \simeq 0.77$. The value $k = 11/3$, corresponding to the limit case $\nu = 0$, is the constant value of the braking index when one consider GW alone, as one can easily check from equation (1 8). If we were able to detect a sinusoidal GW at the frequency f emitted by a BBHS, then measuring k one could determine the interesting quantity $\nu m^{-8/3}$. Figure 2 shows this result when ν is assumed to be equal to the Eddington limit (see eq. (14)), with $\epsilon = 0.1$. Given the GW frequency f, or, equivalently, the orbital period $P = 2/f$, and the deviation Ak from the general relativistic value 11/3, one can determine the mass of the system. Alternatively, if the mass is known, one can get relevant information on v, as shown in Figure 3.

To be more useful, this method ought to be extended to more realistic situations, where, for example, mass transfer between the two components occurs. in other words, we would like to drop the simplifying assumptions of equal masses and constant accretion rates, and find the expression of k which generalizes equation (21). This request can be easily fulfilled, since to obtain equation (21) we have not made any use of the solution (6) for $a(t)$; all we need is an expression for $\dot{a}$ in terms of a itself, μ, M_T and their derivatives $\dot{\mu}, \dot{M}_T$. From the adiabatic invariant one finds

$$\left(\frac{\dot{a}}{a}\right)_m = -2\frac{\dot{\mu}}{\mu} - \frac{\dot{M}_T}{M_T}, \tag{22}$$

while equation (1) gives

$$\left(\frac{\dot{a}}{a}\right)_g = -\frac{64}{5}\frac{\mu M_T^2}{a^4} \tag{23}$$

Substituting equations (22) and (23) in equation (20), we obtain, neglecting terms of order $O(\dot{m}/m)^2$ and $O(\ddot{m}/m)$,

$$k \simeq \frac{11}{3} - \frac{35}{96}\left(\frac{\dot{\mu}}{\mu} + \frac{2}{3}\frac{\dot{M}_T}{M_T}\right)\mu^{-1} M_T^{-2/3}(\pi f)^{-8/3}, \tag{24}$$

which reduces to equation (21) when $m_1 = m_2$ and $\dot{m} = const.$ From equation (24) we see that k, in the general case, can assume values above and below the reference value 11 /3, according to the sign of the term between parentheses in the last equation. For example, if some material is flowing from one BH (call it component '1') to the other one (component '2'), then

$$\dot{\mu} = \dot{m}_2\left(\frac{m_1 - m_2}{M_T}\right), \tag{25}$$

$$\dot{M}_T = 0. \tag{26}$$

Thus, k is bigger (lower) than $11/3$ if and only if m_1 is less (more) massive than m_2. The mass transfer has, in practical, no effect on k as long as $m_1 \simeq m_2$. Moreover, we stress the possibility that a single BBHS could exploit different behaviors of μ and M_T during its evolution. From the observational point of view, this fact gives rise to a dispersion of the measured values of k, depending on which of the various possible effects, namely mass loss- transfer-accretion, is dominant on each particular system at that particular time. Thus, measuring k in a wide population of BBHS, one can get very interesting information on the variability of these systems, which seems to be an important step toward the understanding of the fuel mechanisms.

4 Evolutionary effects

Following the discussion concluding the previous section, we will now consider a population of BBHS. Independently of their formation epoch and mechanism, we will start considering them at a given 'initial' time t_0, when each of them is characterized by a separation a_0. For simplicity, we will assume that m_0 and ν are equal in all these systems, and focus our attention on the evolution of a. in particular, we want to find the distribution function, i.e. the function which gives the number of systems with a given separation a, at an observation time t, given the same distribution at the initial time t_0. To be more specific, we define the number of BBHS with initial separation between a_0 and $a_0 + da_0$ as $g_0(a_0)da_0/a_0$, with a_0 belonging to the interval $[a_{min}, a_{max}]$. The lower limit a_{min} can be considered as the minimum separation compatible with our newtonian assumption (for Post- Newtonian corrections to the binary orbit see Lincoln & Will 1990), while the upper limit a_{max} is determined by the sensitivity and the bandwidth of the receiver.

We want to find how go evolves with time. At a given time t, the binary systems with $a_0 < a_c(t)$ have disappeared, while those with $a > a_c(t)$ are distributed in accordance with the number conservation law, i.e.

$$g(a,t)\frac{da}{a} = g_0(a_0)\frac{da_0}{a_0}\,, \tag{27}$$

where a is related to a_0 by equation (6). Differentiating equation (6) we thus find

$$g(a,t) = g_0(a_0)\underbrace{\left(\frac{a}{a_0}\right)^4}_{GW}\underbrace{\left(1+\frac{t}{\tau_m}\right)^{12}}_{DISK}\,. \tag{28}$$

In equation (28), we have made explicit the origin of the multiplicative factors. in fact, the term $(a/a_0)^4$ is typically due to the emission of GW alone, as can be seen differentiating equation (9) (Bond & Carr 1984). Note, however, that in our case the relationship between a and a_0 is not the same as in the 'unperturbed' equation (9). in other words, the factorization in equation (28) is only apparent, since τ_m appears also, through equation (6), in the term indicated with 'G W'. Equation (28) can be rewritten as

$$g(a,t) = g_0(a_0)F(a_0,t)\,, \tag{29}$$

with the introduction of the transfer function $F(a_0, t)$, given explicitly by

$$F(a_0, t) = 1 + \frac{\tau_m}{16\tau_g}\left[1 - \left(1 + \frac{t}{\tau_m}\right)^{16}\right] . \tag{30}$$

Note that this quantity is always less than one, and depends on a_0 only through the GW time scale τ_g. Since $\tau_g \propto a_0^4$, this means that the evolution is much faster near a_{min} than near a_{max}. Figure 4 gives an example of this behavior. Note also that, as expected, $F(a_c(t), t) = 0$.

5 Conclusions

In this work we have considered the effect of an accretion disk on the GW emission from a BBHS. The main result is equation (6), which gives the evolution of the separation a as due to the GW emission together with a (constant) mass accretion. We have found that the coalescing time can be considerably shorter when the mass increases at the Eddington rate. Finally, we gave an approximate expression for the braking index, in terms of the reduced and total masses of the system.

We stress that most of the results presented here have been obtained under quite general assumptions. Therefore, they can be applied also in other interesting astrophysical objects, like a binary pulsar, etc.. For example, we can consider a stellar binary system, loosing its mass adiabatically. Then, assuming again equal masses and $\dot{m} = const.$ ($\tau_m < 0$ in this case), we find that a evolves according to the same equation (6).

In some circumstances, however, the adiabatic hypothesis could appear inadequate. In this case, equation (6) is no longer valid. Anyway, we are able to replace it, as long as the fractional change in the binding energy due to the stationary mass loss is proportional to the fractional change of the mass itself, i.e.

$$\frac{\delta E}{E} = \alpha \frac{\delta m}{m} . \tag{31}$$

This case includes the previous one, since the adiabatic law corresponds to $\alpha = 5$, as well as other important cases; for example, Jeans's mode of mass ejection implies $\alpha = 3$ (Huang 1963).

Then equation (5) must be replaced by

$$\dot{a} = -\frac{128}{5}\left(\frac{m\mu}{a}\right)^3 - (\alpha - 2)\left(\frac{a}{m}\right)\dot{m}\ , \tag{32}$$

which can be easily integrated, to give

$$a = a_0 \left(1 + \frac{t}{\tau_m}\right)^{2-\alpha} F_\alpha(t)^{1/4}, \tag{33}$$

where

$$F_\alpha(t) = \begin{cases} 1 + \dfrac{\tau_m}{4(\alpha - 1)\tau_g}\left[1 - \left(1 + \dfrac{t}{\tau_m}\right)^{4(\alpha-1)}\right] & \alpha \neq 1 \\ 1 - \dfrac{\tau_m}{\tau_g} \ln\left(1 + \dfrac{t}{\tau_m}\right) & \alpha = 1 \end{cases} \tag{34}$$

Correspondingly, the coalescing time (1 O) becomes

$$\tau_c = \tau_m \cdot \begin{cases} \left(1 + \dfrac{4(\alpha - 1)\tau_g}{\tau_m}\right)^{1/4(\alpha-1)} - 1 & \alpha \neq 1 \\ \exp(\tau_g/\tau_m) - 1 & \alpha = 1 \end{cases} \tag{35}$$

When $|\tau_m| \gg \tau_g$, equation (35) gives

$$\tau_c = \tau_g \left[1 + \frac{5 - 4\alpha}{2}\frac{\tau_g}{\tau_m} + O\left(\frac{\tau_g}{\tau_m}\right)^2\right] \tag{36}$$

From equation (36) we deduce that

$$\tau_c \gtrsim \tau_g \quad \text{for } \alpha \gtrsim 5/4,$$

$$\tau_c \simeq \tau_g \quad \text{for } \alpha \simeq 5/4,$$

$$\tau_c \lesssim \tau_g \quad \text{for } \alpha \lesssim 5/4.$$

These formula could be encountered, for example, in a type 1 Supernova progenitor, according to the Double Degenerate White Dwarfs (DDWD) model (Iben & Tutukov 1984). *In* this model, both WD loose their 11-rich envelope just before the final coalescence. in the usual description of the DDWD model the GW emission is considered only as a mechanism capable to get the binary close enough for the common envelope phase (or phases) to occur, This idealized mode], in which the various phenomena, namely the mass transfer and ejection, and the final coalescence, take place with different time scales, is obviously justified by the intention of giving a satisfying description of the SNI progenitor. However, from the results of our work, we can conclude that all these effects can be taken into account simultaneously, and that they can have considerable influence on the observability of this kind of sources.

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Figure Captions

Figure 1

The coalescing time, τ_c, as a function of the mass e-folding time, τ_m. Both quantities are expressed in units of the GW time scale', τ_g.

Figure 2

How to determine the mass of the binary, knowing the GW frequency f and the braking index k, expressed here in terms of the deviation from the GR value 11 /3, We have assumed an efficiency $\epsilon = 0.1$.

Figure 3

How to determine the efficiency ϵ, for a BH mass of $10^8 M_\odot$, from the knowledge of the GW frequency f and the braking index k. As in fig.2, $\Delta k = 11/3 - k$.

Figure 4

The evolution of the distribution function $g(a_0)$ at a generic time t. Binaries with $a_0 < a_c(t)$ have disappeared. Those with an initial separation larger than at(t) have evolved according to equation (4.3). The initial distribution g_0 is assumed to be flat over the interval $[0, a_{max}]$ (dot-dashed line).

Figure 1

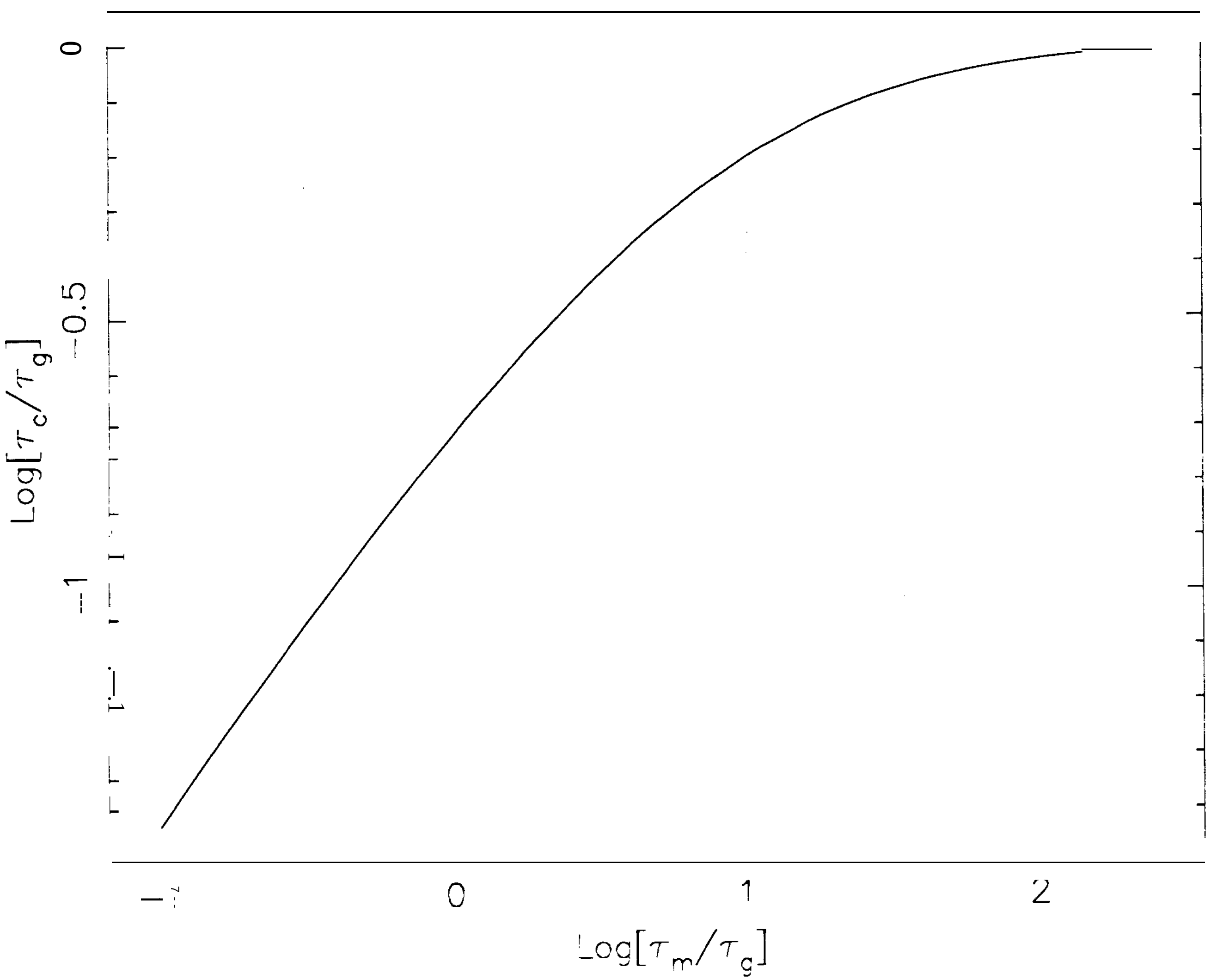

Figure 2

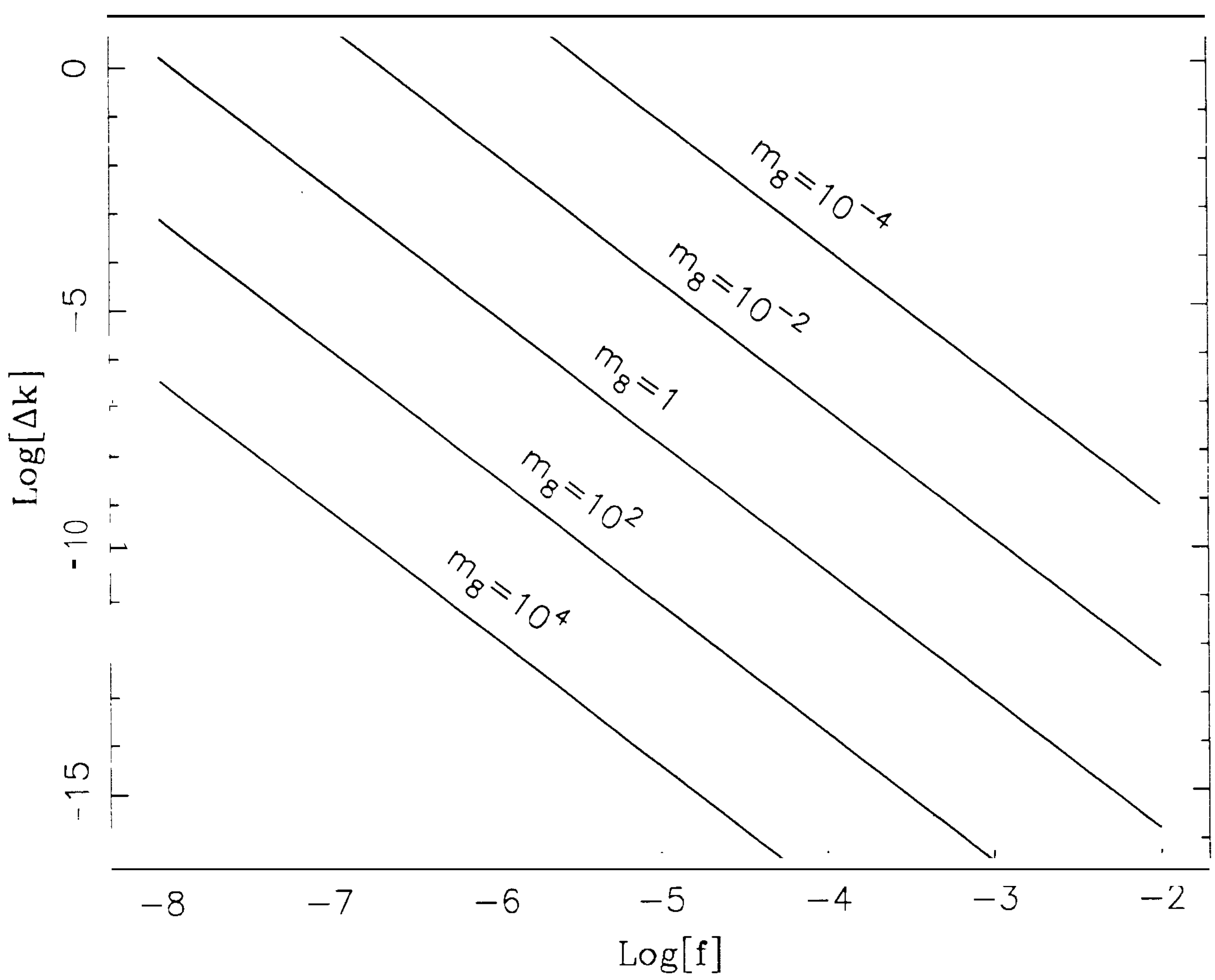

Figure 3

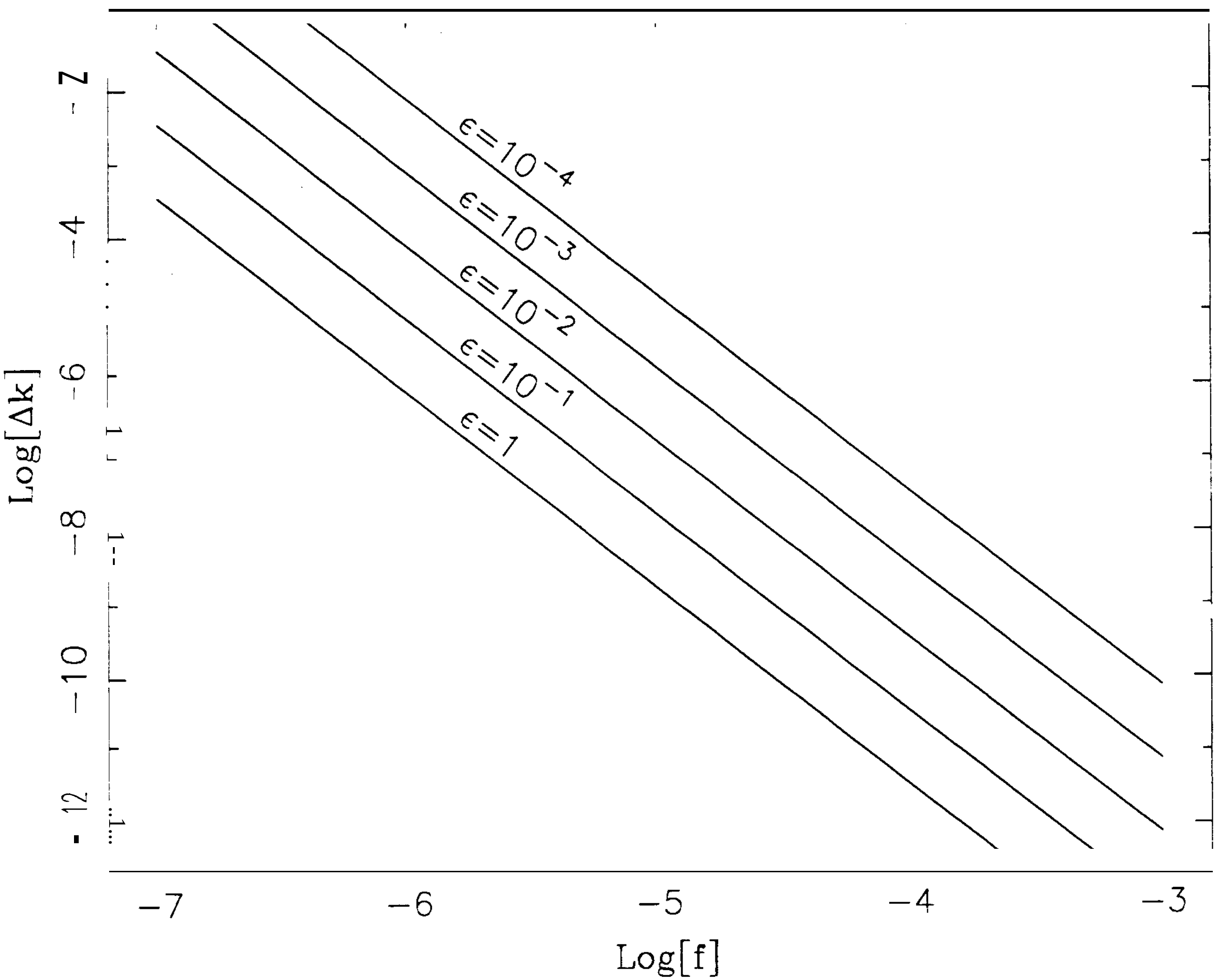

Figure 4