

Effects of Shape and Spin on the Tidal Disruption of P/Shoemaker-Levy 9

A.W. Harris (JPL)

One can express a generalized Roche density, $\rho_c = C\rho_p (R_p/r)^3$, where ρ_c is the critical density below which a body passing a distance r from a planet will be disrupted. ρ_p is the density of the planet, R_p is the radius of the planet and C is a constant. For the classical Roche limit, which defines the limit of hydrostatic equilibrium of a fluid satellite of a planet, $C = 14.8$. It has long been recognized that such factors as internal strength, viscosity, or even just the strength from hydrostatic compression of loose solid particles (the effect that keeps a sand pile at its "angle of repose" rather than flowing out into a thin sheet) contribute to making real objects more resistant to disruption than given by the classical limit. Indeed, the satellite Phobos is well within its classical Roche limit of Mars. Various studies including material property effects suggest that values of C in the range 2-5 may be more appropriate for real bodies. For comparison, one simple text-book derivation of the Roche limit considers the point at which two equal-sized spheres would pull apart. This yields $C = 16$ for a non-rotating pair, or 24 for a synchronously rotating pair. Another simple model considers the point at which a small test particle (regolith grain) would lift off the surface of a larger sphere, which yields $C = 2$ or 3 for the non-rotating or rotating cases, respectively. Clearly the former model yields about the right number for the fluid (classical) Roche limit, and the latter one about the right value for real materials. However, the studies including material properties have all considered initially spherical bodies. Taking the simplified criterion of balancing forces on a test particle at the surface of a larger spheroid, I have derived a generalized criterion for disruption including shape and rotation of the body:

$$\rho_c \approx 2\rho_p \left(\frac{R_p}{r} \right)^3 \left(\frac{\omega}{\omega_0} \right)^2 \left(\frac{a}{b} \right),$$

where ω is the rotation rate of the body, $\omega_0 = \sqrt{4\pi G/\rho_p}$ is the surface orbit frequency about a body of unit density, and a/b is the axis ratio of the body, modeled as a prolate spheroid. For P/Shoemaker Levy 9, passing $r \approx 1.4 R_p$ from Jupiter:

$$\rho_c \approx 0.98 + \left(\frac{3.3^2}{P_{rot}} \right)^2 \left(\frac{a}{b} \right).$$

Thus for a non-rotating sphere, $\rho_c \approx 1$, but for a 2:1 elongate nucleus, $\rho_c \approx 2$ for a non-rotating body, and even more for a rotating one. Thus for bodies of plausible shapes and spins, one cannot reliably infer that the original nucleus of P/Shoemaker Levy 9 must have had an ice-like density rather than a rock-like one.