Bayesian Estimation of Soil Parameters from Remote Sensing Data

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ABSTRACT

We consider the problem of finding a mathematically optimal algorithm to estimate soil parameters based on radar and/or other measurements. Specifically, given measurements $m_1, m_2, .., m_J$ representing radar cross-sections of a given resolution element at different polarizations arid/rrr different frequency bands, and given an approximate model expressing the dependence of these measurements on the dielectric constant ϵ and the r.m.s. surface height h of the corresponding resolution element, we would like to make an "optimal" estimate of the actual ϵ and h that gave rise to the particular m_1, m_2, observed. By "optimal" we mean that our algorithm should produce estimates that are, on average, as close as possible to the actual values. To obtain such an algorithm, we assume that we have at our disposal a data catalogue consisting of careful measurements of the soil parameters ϵ and h, on one hand, and the corresponding remote sensing data m_1, m_2 etc, on the other. We also assume that we have used this data to write down, for each j, an approximate formula which computes an average value of m_j to associate to the corresponding values of ϵ, h. Rather than throw away the data catalogue at this stage, and use the average formulas in a deterministic fashion to solve the inverse problem, we propose to use the data catalogue more fully and quantify the spread of the measurements about the average formula, then incorporate this information into the inversion algorithm. This paper describes how we accomplish this using a Bayesian approach. In fact, our method allows us to

1) make an optimal estimate of ϵ and h

2) place a quantitatively honest error bar on each estimate, as a function of the actual values of the remote sensing measurements

3) fioc-tune the inital formulas expressing the dependence of the remote sensing data on the soil parameters

4) take into account as many (or as few) remote sensing measurements as we like in making our estimates of ϵ and h, in each case producing error bars to quantify the benefits of using a particular combination of measurements.

MATHEMATICAL APPROACH

Because the laws of scattering from randomly rough natural surfaces are quite complicated, especially at microwave frequencies, empirical models have often been used to help express the observed remote sensing measurements as a function of the surface parameters. A typical approach, adopted in (Oh et al 1992), is to start with a "training set" consisting of a catalogue of carefully collected data: in the case of Oh et al 1992, this catalogue consists of 1-, C- and X-band polarimetric radar backscattering measurements for various bare soil surfaces, along with laser-profiler and dielectric-probe measurements of the corresponding r.m.s. surface height and dielectric constant values. Guided by the physics that govern electromagnetic scattering, and using the data at hand, an approximate model relating the radar backscatter to the surface parameters can be established. One way to proceed is to disregard the data catalogue from this point on: given a particular radar measurement, a deterministic method can be used to invert the approximate model and retrieve the corresponding surface parameters. The accuracy of the retrieved parameters would naturally depend on the inversion method used, and would be cliff icult to quantify.

Another approach, that can potentially make fuller use of the data catalogue, is to model not only the approximate dependence of the radar backscatter on the surface parameters, but also the *spread* of the actual data about the approximate model. Indeed, the approximate model can be more or less accurate over certain intervals. Using this information about its accuracy, and how it depends on the values of the surface parameters, as evidenced by the carefully collected data, can only help in the inversion problem. In fact,

1) a Bayesian approach can indeed use this information to produce an optimal algorithm, i.e. an algorithm which, among all possible algorithms, and on average, makes the smallest error in its estimates of the surface parameters.

2) Moreover, such an approach can quantify the accuracy of its estimates, depending, naturally, on the values of the measured radar backscatter in every case.

3) It also turns out that the approach allows one to fine-tune the initial approximate model to better fit the data.

4) Finally, the approach does not restrict one to a r,resl,ecified number of input measurements: indeed, it can use any combination of inputs to produce an estimate of the surface parameters that is based on these inputs. Moreover, it can quantify the uncertainty of these estimates. This is important because it provides a natural means of evaluating the usefulness of using one or another combination of measurements to estimate one or another surface parameter.

For definiteness, we consider the following specific problem: Given two measurements m and n, representing respectively the ratio of HH to VV L-band radar cross-section and the ratio of HV to VV cross sections, respectively, of a single radar resolution element, we would like to make an "optimal" estimate of the correct pair (ϵ, h) that gave rise to the particular (m, n) observed. By "optima]", we mean that the r.m.s. difference between the

optimal estimates and the actual values of ϵ and h should be smallest among all the errors made by any candidate estimators: the optimal method is the one which, *on average*, i.e. **over** many (all) observations, makes the smallest error.

A natural way to proceed is to look for expressions of the form $m = f(\epsilon, h)$, $n = g(\epsilon, h)$. Yet it is unlikely that any given candidate functions f or g can make these equalities exactly true, ever, because of noise and other uncertainties inherent to radar data. In fact, modeling the average dependence of m and n on ϵ and h is not sufficient, in itself, to allow one to determine which ϵ and h best correspond to given measurements m, n: one must still model the dependence of m and n on the many remaining factors, whose omission from the deterministic equations is indeed the reason that these equations are never exactly verified. In order to find an optimal procedure, one needs to make an effort to mathematically account for the discrepancy between the left- and right- hand- sides of the deterministic relations. We therefore replace these by stochastic equations

$$m = f(\epsilon, h) \, M_1$$
$$n = g(\epsilon, h) \, M_2 \qquad (1)$$

where (f, g) represent the deterministic "typical" or "average" way in which (m, n) depend on ϵ and h, and where the M_j are random variables that do not depend on ϵ or h and which represents the remaining randomness in (m, n).

Once we have succeeded in establishing (1), and identifying the joint behavior of the random variables (M_1, M_2), there remains to compute the conditional density function $\mathcal{P}_{(\epsilon, h)|(m, n)}$ for ϵ, h given the measured values of m and n. Using Bayes's theorem, the unnormalized version of this conditional density satisfies

$$\mathcal{P}_{(\epsilon, h)|(m, n)} = \frac{\mathcal{P}_{(\epsilon, h)}}{f(\epsilon, h) g(\epsilon, h)} \mathcal{P}_{(M_1, M_2)} \left(\frac{m}{f(\epsilon, h)}, \frac{n}{g(\epsilon, h)} \right) \qquad (2)$$

where $\mathcal{P}_{(\epsilon, h)}$ is the a priori joint density for (ϵ, h), in which one includes all the a priori information about ϵ and h (such as estimates based on other instruments - in case one does not know anything a priori about them except their physical range of values, $\mathcal{P}_{(\epsilon, h)}$ would just be the uniform density function over *the* product of the corresponding intervals).

Using the conditional density given by equation (2), the optimal unbiased estimator $\hat{\epsilon}$ for ϵ that has minimum variance (i.e. that minimizes the r.m.s. error) is the conditional mean

$$\hat{\epsilon} = \mathcal{E}\{\epsilon|(m, n)\} = \int \epsilon \mathcal{P}_{(\epsilon, h)|(m, n)}(\epsilon, h) \, d\epsilon \, dh . \qquad (3)$$

Similarly, the optimal unbiased estimator $\hat{h}$ for h is the conditional mean $\mathcal{E}\{h|(m, n)\}$. Formula (3) is quite easy to discretize (in ϵ and h) and **evaluate numerically.** Thus, if we can replace the deterministic equations by stochastic equations (1) in such a way that we also know the joint **density** function $\mathcal{P}_{(M_1, M_2)}$ of (M_1, M_2), we have a straightforward method of obtaining the optimal estimate, and of calculating all its moments.

We applied our approach to the case where the model and data to be used are the University of Michigan Radiation Laboratory Model (Oh et al 1992), and the corresponding set of radar cross-sections measured by the LCX polarimetric scatterometer POLARSCAT (Oh et al 1992). Specifically, we assume that

$$f(\epsilon, h) = \left(1 - \left(\frac{2\theta}{\pi} \right)^{1/3\Gamma_0} \epsilon^{-kh} \right)^2$$

$$g(\epsilon, h) = 0.23 \sqrt{\Gamma_0} (1 - \epsilon^{-kh}) \qquad (-1)$$

where O is the incidence angle of the radar beam, k is the wave number, and $\Gamma_0 = \Gamma_0(\epsilon) = \left| \frac{1 - \sqrt{\epsilon}}{1 + \sqrt{\epsilon}} \right|^2$ is the Fresnel reflectivity of the surface at normal incidence. We further assume that the functions f, g model m, n in the sense that $m/f(\epsilon, h) = M_1 = N_1/N_3$ and $n/g(\epsilon, h) = M_2 = N_2/N_3$, where N_1, N_2 and N_3 are independent, Γ-distributed random variables. Practically, this means that

$$\mathcal{P}_{(M_1, M_2)}(x, y) = \frac{1}{\xi \nu} \frac{(x/\xi)^{\alpha - 1}(y/\nu)^{\beta - 1}}{(x/\xi + y/\nu + 1)^{\alpha + \beta + \gamma}} \frac{\Gamma(\alpha + \beta + \gamma)}{\Gamma(\alpha) \Gamma(\beta) \Gamma(\gamma)} \qquad (5)$$

where the parameters $\alpha, \beta, \gamma, \xi$, v are to be determined. The model we used for M_j was based on a study of the nature of the randomness present in (m, n). One source for this randomness is the non-δ-function distribution of the background power level. Another is that the candidate functions f and g may well turn out to be poor approximations of the true means. Indeed, ever] if one used a very accurate method to estimate the sample mean, one remains vulnerable to measurement error, and to contamination of the measurements try unknown scatterers on the surface ("debris", etc). Taking all these considerations into account, it is neither unreasonable nor arbitrarily restrictive to assume that the measurements m and n are related to ϵ and h by equation (1), in which each M_i is distributed like the ratio of the two corresponding (independent) Γ distributions. With the Michigan data (Oh et al 1992), a χ^2-test for goodness of fit shows that $\alpha = \beta = 5$, $\xi = 1.04$, u $=$. 0.82 do fit the data well, these values having appropriately been determined using the maximum likelihood approach.

RESULTS

Figure 1 is a contour plot of the optimal estimate $\hat{\epsilon}$ ϵ, as a function of the cross section ratios m and n at O $= 40°$. The values of $\hat{\epsilon}$ were obtained using our Bayesian approach, and starting with an a priori density function $\mathcal{P}(\epsilon, h)$ that is uniform over the rectangle $2 \le \epsilon \le 20$, $0 \le kh \le 1$. Overlaid on the contour plot of figure 1 are those samples of the Michigan data that were collected at 40" incidence, each accompanied by the value of ϵ computed according to the inversion algorithm proposed by Oh et al 1992, as well as the measured values, At 40°, the value of ϵ calculated by the direct inversion algorithm falls within 25% of the measured value in four out of the eight samples, but it misses by 100% in three cases. Figure 2 shows the estimated variance of our estimates of ϵ. In this case, the model consisting of the function f and g of equation (4) can be considered 'useful" if the r.m.s. uncertainty of the estimates that we obtain with it is smaller than the a priori uncertainty made by assuming that ϵ and kh are uniformly distributed, i.e. if the r.m.s. uncertainty in $\hat{\epsilon}$ is smaller than $(20 - 2)/\sqrt{12} \simeq 5.2$. Figure 2 shows that this is indeed achieved everywhere. In fact, the numerical results show that the r.m.s. relative uncertainty in the case of ϵ is never worse than 50%. Although somewhat high, this value seems encouraging.

One way to reduce the uncertainty in our estimates is to tune the parameters in the model to the situation at hand. Specifically, one can postulate a model of the form

$$f(\epsilon,h) = \left(1 - \left(\frac{2\theta}{\pi}\right)^{a/\Gamma_0} e^{-kh}\right)^2$$

$$g(\epsilon, h) = b\Gamma_0^c (1 - e^{-kh}) \tag{6}$$

together with a probability density function $\mathcal{P}_{(M_1, M_2)}$ for the observed ratios $(m/f,\ n/g)$ of the form

$$\mathcal{P}_{(M_1,M_2)}(x,y) = \frac{x^{N-1} y^{N-1}}{(x + y + 1)^{3N}} \cdot \frac{\Gamma(3N)}{\Gamma(N)^3} \tag{7}$$

as before, then go on to determine a, *b*, *c* and N in order to maximize the likelihood of observing the radar cross sections reported in the Michigan data. At [,-band, we found that the optimal values fo r a, *b* and *c* were $a = 0.33675$, $b = 0.12344$, $c = 0$ and $N = 15$, The accuracy of the ϵ- and h- estimates does improve, as does the significance of the spread between the optimal estimates on the dry and wet cases for the other samples, especially at the steeper incidence angles. Yet the variance between of the ϵ-estimate remains uncomfortably large compared with the difference between the wet and dry cases.

We can further reduce the uncertainty in our estimates by trying to fuse data collected from different "instruments", or different bands in the case at hand, namely 1,-, C- and X-bands. 10 make use of the three polarizations from the three bands simulta- neously, we first replace kh and ϵ by two frequency-independent parameters, say h itself and the moisture content μ. The expres- sion for the conditional density function $\mathcal{P}$ for (μ,h), conditioned

Actual μ	$\bar{\mu}_{40°}$	$\sigma(\mu_{40°})$	$\bar{\mu}_{50°}$	$\sigma(\mu_{50°})$
0.29	0.25	0.064	0.24	0.060
0.14	0.18	0.058	0.21	0.069
0.30	0.32	0.053	0.34	0.047
0.09	0.13	0.057	0.13	0.063
0.31	0.30	0.074	0.31	0.067
0.15	0.23	0.092	0.17	0.103
0.19	0.23	0.107	0.24	0.102
0.16	0.17	0.109	0.21	0.107

Table 1: Soil moisture estimates using HH/VV and HV/VV ratios at 1,, C and X bands

on these 3 pairs of observations, is derived as above, and the con- ditional means of $\mathcal{P}$ is the optimal estimates for μ and h given the 6 measurements at hand. Table 1 summarizes the results for μ. The estimated r.m.s. uncertainty almost never reaches 50% of the estimated moisture content anymore. In fact, except in the roughest case, the wet-dry spread is typically as large as the uncertainty in the estimate, whereas in the L-band-only case it was very much smaller. This is very important because it implies that the combination of L-, C- and X-bands does allow rme to estimate the soil moisture content accurately enough to consistently discriminate between wet and dry conditions.

ACKNOWLEDGEMENTS

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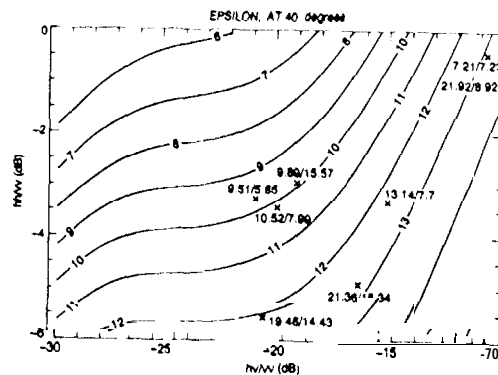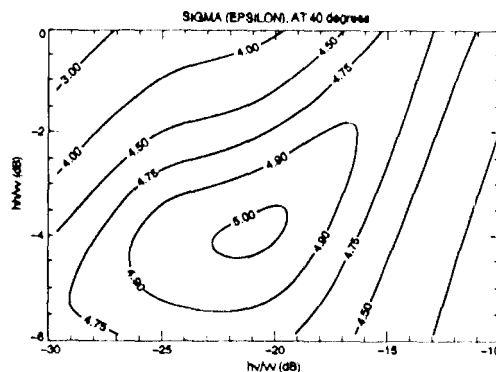
Figure 1

Figure 2