IDENTIFICATION OF NONLINEAR JOINTS IN A TRUSS STRUCTURE

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Abstract

This paper describe-s a technique to locate and characterize loose joints in a truss structure which are modeled as gaps in the members. The procedure involves prestressing the structure to first eliminate the gaps and then unloading the structure and monitoring a set of displacements. By comparing the calculated displacements to the set of known displacements, the gap location is identified, The loading and unloading is accomplished using a set of actuators whose locations are arbitrary. The sizes of the gaps are determined by monitoring the gap member length changes using the appropriate linear force-displacement relationship for the load level. Three numerical examples are presented to illustrate the procedure assuming ideal displacement data, A last example is presented which uses a displacement set which is corrupted to demonstrate the effect of measurement error.

Introduction

Adaptive structures are attractive alternatives to entirely passive systems in structures which support orbiting space telescopes or optical interferometers. The stringent shape requirements of a structure supporting optical equipment can be maintained with minimal cost and energy output when active members are incorporated into the structure, An accurate structural model is required to perform vibration suppression or shape control however the type of structure under consideration may have a tendency toward joint looseness. The type of structure addressed in this paper is of a size which requires a deployable or erectable design to fit the launch vehicle. When final assembly is completed in orbit, there is a possibility that any joints that were not preassembled will not possess the precision tit required. This results in a structure with joint induced nonlinearities and the true response to actuator loading requires the loose joints be located and characterized. The identification of loose joints using active members is addressed in this paper.

The research described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

Background

Numerous studies have been performed to identify the structural properties of a system. Often, the modal response of the structure is monitored by applying a dynamic excitation to the structure, then the measured frequencies and mode shapes are used to update the mass and stiffness matrices[1,2,3,4]. Chen and Garba[5] used modal information to detect the location and extent of damage in a structure. However the method is only successful for small changes in stiffness. Kim and Bartkowicz[6] also used modal data to detect damage and were able to accurately locate changes of up to 30 percent in the stiffness matrix. Their approach required extensive measurements. Lee, Hessian, and Venkayya[7] investigated correlating the system matrices using a distributed parameter scheme rather than the typical discrete or finite element approach. Their approach depends on a realistic initial estimate of the structures properties

An interesting method using a neural network approach was developed by Tsou and Shen[8] where the network "learns" the characteristics of the system and is able to generate the appropriate stiffness properties when presented with new response data, This method can detect substantial changes in the beam properties but depends on the analysis of numerous sets of data to train the system. Sanayei and Onipede[9] were able to identify changes in stiffness using a static approach with limited measurements, Using an iterative process of updating the stiffness matrix based on a sensitivity approach, they were able to successfully identify changes in properties when the measurement locations were selected properly.

The approach taken in this study also uses a static approach and requires few measurements and a minimal number of actuators. The current research differs from some of the studies mentioned in that a complete loss of stiffness is detected to determine the location of the loose joints.

Problem Description

The relationship between actuator forces and nodal displacements is clearly defined for a linear structure as well as for a nonlinear structure[10] provided an accurate analysis model is available. The identification of any loose joints is required to develop an accurate force-displacement relationship in the structure. In this paper, the joint looseness is defined as a gap associated with a particular member. The gap member does not develop any internal forces until the structure has been loaded in such a way that the relative displacement, δ, between the endpoints of the gap member changes by an amount exceeding the gap size, ε. 'J'his force-displacement relationship of the gap member is shown in Figure 1. While $|\delta| (\varepsilon/2$, the gap remains open, the member is free of internal forces and does not contribute to the system stiffness matrix. When $|\delta|) \varepsilon/2$, the gap is considered closed and the element stiffness is added to the system stiffness.

An accurate model requires knowledge of the gap member locations and the gap sizes, ε. The presence of gaps, which is equivalent to the removal of members, alters the initial stiffness matrix of the unloaded structure significantly. The difference in response between a full structure and a structure with a number of members removed is too great to use the linear structure, that is the structure with no gaps, as an approximation. The number of possible patterns of 5 gaps in 50 possible locations is over 2.1 million. Therefore any attempt to locate the gaps by comparing the structure's response to the response assuming different gap locations would be futile. Further complicating the analysis are the additional unknowns of the number of gaps and the gap sizes. In this paper, the number of gaps, their locations, and their sizes will be determined.

Approach

The number of possible patterns for a known number of gaps in the structure is immense and when adding the additional unknown of the gap size, any attempt to establish an initial structural model becomes unfeasible. Thus the approach taken here is to first completely prestress the structure with a set of actuators to enforce closure of all existing gaps. Once the structure is prestressed, it is then unloaded and the problem is reduced to a series of steps for locating one gap at a time. In the prestressing and subsequent unloading of the structure, a set of actuators at preassigned locations is used and the nodal displacements at a number of degrees of freedom are monitored. To prestress the structure, the structure is loaded until the response of the structure to an additional perturbation is consistent with the known response of the structure with no gaps. When this occurs the gaps have been closed and the actuator loads can be released to begin the detection process. Prior to prestressing the response to an incremental load is calculated which identifies the system behavior when all gaps are open.

At the start of the unloading process, the displacement response under the two conditions of all gaps open and all gaps closed is known. Starting from the prestressed structure, the actuator displacements are decreased incrementally and the nodal displacement response is determined. A vector $\Delta \mathbf{u}^n$ is generated where Au" = $\mathbf{u}^n - \mathbf{u}^{n-1}$ and **u"** **is** the vector of displacements at unloading step n. At each step Au" is compared to $\Delta \mathbf{u}^{n-1}$. When $Au^n \neq \Delta \mathbf{u}^{n-1}$, a break in linearity has occurred indicating the opening of a gap. The actuator displacements at the break in linearity are stored in a vector δ_m where m indicates the number of gaps detected. The unloading procedure continues until the next break occurs or until the structure is completely unloaded. At the conclusion of the unloading sequence, the number of breaks in linearity in the calculated response indicates the number of gaps present in the structure.

Once the number of gaps has been established, the procedure of determining their locations is initiated. To locate the first gap, displacements of $(\delta_1 + \delta_2)/2$ are applied to the actuators to bring the structure to the point where one gap is open. Then one actuator is perturbed and the difference in the displacement response between the perturbed and unperturbed state at the current load level is calculated and the stored in a vector x. The entries in x are normalized to the maximum value. At this point a matrix, S, is created consisting of column entries representing the normalized difference in displacements at the monitored degrees of freedom when one actuator is perturbed and one member has been removed from the structure. The size of S *is* the number of displacement measurements by the number of possible gap locations where the j^{th} column corresponds to the response when the j^{th} possible gap is open and the i^{th} row corresponds to the i^{th} displacement measurement.- The gap location is established by calculating the error measurement e_j over all possible gap locations:

$$e_j \cdot \sum_{i=1}^{nd} (x_i - S_{ij})^2 \qquad (1)$$
$$j=1,2,3,\ldots,npgap$$

where j represents the gap location, *nd* is the number of displacement that are monitored, and *npgap is* the number of possible gap locations. Under ideal conditions with no measurement error, when $e_j = 0$, the gap is in the j^{th} possible location.

If $e_j = 0$ at more than one location, there are multiple columns in S which match x and the gap location has not yet been identified uniquely. This is possible because the number degrees of freedom in the structure almost always exceeds the number of monitored degrees of freedom, If there is more than one matching column, a second actuator in the structure is arbitrarily y selected and perturbed. The procedure of perturbing the structure and developing the matrix S is repeated for each new actuator location however the number of columns in S *is* now reduced to the number of columns in the original matrix which were identical to x. The formulation of S and the error calculation is repeated using different actuators until a unique match is determined.

After the first gap has been identified, the numerical model is updated to reflect the opening of the first gap by removing that member from the stiffness matrix when the actuator displacements fall below δ_1. The actuator loading is then decreased to $(\delta_2 + \delta_3)/2$ and the process is repeated to identify the next gap, In this manner, when each gap location is uniquely identified, the number of static analyses is reduced to *ngap *npgap* where *ngap is* the actual number of gaps and *npgap is* the number of possible gap locations.

At the completion of each stage, the gap location is known and the actuator displacements at which the gap opened is known. To determine the gap size, the linear force-displacement relationships for a truss structure are used. The key equations are taken from the displacement method of analysis and starts with the compatibility equation:

$$\Delta = \beta u \qquad (2)$$

where A is the member length change, u *is* the nodal displacements and β is the matrix of direction cosines which relate the two. The next relationship used is the force-displacement relationship for a truss structure:

$$P = K(\Delta - \Delta_0) \qquad (3)$$

where P is the vector of member forces, K is a diagonal matrix of member stiffnesses and Δ_0 is the vector of initial member length changes. The final equation used is the force equilibrium relationship:

$$F = \beta^T P \qquad (4)$$

where F *is* the vector of external forces. Combining these three equations and recognizing that the external forces, F, are zero, gives:

$$A = \beta K_G^{-1} \beta^T K \Delta_0 \qquad (5)$$

where $KG = \beta^T K \beta$ is the global stiffness matrix. This provides the relationship between the initial actuator length changes and all member length changes, The independent variable in this formulation is Δ_0, the vector of initial member length changes which is not a measurable quantity. The vector, Δ_0, contains the actuator displacements in a free-free condition. The actual actuator length changes that are generated when the actuators are incorporated into the structure are found in A. To relate δ, the actuator length changes to Δ_0, a matrix T_{aa} is created by selecting the rows and columns of $\beta K_G^{-1}\beta^T K$ which correspond to the actuator locations. Then the measured actuator length changes, δ, are expressed by:

$$\delta = T_{aa}\Delta_0 \qquad (6)$$

Using equations (5) and (6) gives the gap member length changes:

$$\Delta_g = T_{ga}T_{aa}^{-1}\delta \qquad (7)$$

where Δ_g is the vector of gap member length changes and T_{ga} is a submatrix of T created by selecting the rows corresponding to the gap locations and the columns corresponding to the actuator locations, During the unloading procedure, the gap member length changes are monitored and summed to determined the cumulative value of Δ_g at each break in linearity to determine the gap size.

Numerical Examples

A modified version of a support structure for a space-based segmented reflector was used as the basic structure in the numerical examples. This structure consists of 72 members and 63 degrees of freedom with member sizes which range from ,77 meters to .92 meters in length. The gap sizes were all set at 100 microns although uniformity in size is not necessary for the procedure. The number of actuators and gaps varies in each example but the set of monitored displacements remains the same. The structural displacements used to generate Δu_n and x are a set of twelve out of plane displacements at the surface. In each case the gap locations and the actuator locations were selected at random. The results are presented in terms of the actuator length changes and the maximum actuator forces generated.

Case 1 - 2 actuators, 4 gaps

In the first case, four gaps were arbitrarily placed in members 10, 42, 53, 62 and five actuators were located in members 15, 23, 43, 60, and 71. The solid circles near the joints represent the gaps and the darkened members indicate the actuator locations in Figure 2. The prestressing was accomplished by exercising only actuators 15 and 23. The prestress actuator displacements and the actuator displacements at which each gap opens are shown in Table 1. The gaps opened in the order 53, 10, 62, and 42, and the all gap sizes were calculated at 100 microns. In this case, each time a gap opened and the response to the actuator perturbation, x, was compared to the columns of S, a unique match was found and no additional actuator perturbations were performed. Therefore in this example, five actuators were selected to perform the gap identification but *only* two were used with a maximum actuator force of 102 lbs compression. The length changes that occurred at actuators 43, 60 and 71 were in response to the displacements enforced in actuators 15 and 23. Although only two actuators were driven, the remaining three were retained in the analysis to monitor the resultant forces and displacements and insure excessive forces are not generated during the process.

Table 1. Case 1- Actuator Length Changes for Prestressing and Gap Opening

	actuator length changes, microns				
actuator no.	prestress	gap 53 open	gap 10 open	gap 62 open	gap 42 open
15	473.8	386.1	304.4	245.2	201.4
23	179.3	146.1	115.2	92.8	76.2
43	17.8	14.5	11.4	9.2	7.6
60	-3.5	-2,9	-2.2	-1.8	-1.5
71	-4.9	-4.0	-3.1	-2.5	-2.1

Case 2 - 4 actuators. S **gaps**

In case ***2,*** four actuators were placed in members 18, 23, 45, and 66, and **five** gaps were located in members 2, 3, 4, 43, and 44 (Figure 3). In this case, the maximum actuator force was 114 lbs in compression and the actuator length changes at **prestress** and gap openings are shown in Table **2.** This case also resulted in a unique match of ***x*** to the appropriate column of S at each gap opening. In this case, the gaps sizes were also correctly identified at 100 microns.

Case 3 - ***4*** **actuators, 5 gaps**

In case 3, with four actuators and five gaps, the problem of detecting **more** than ***one*** possible gap location was encountered. The gaps are located in members 29, 48, 53, 57, and 62, and the actuators are placed in members 3, 20, 46, and 54 (Figure 4). In this case, the first two gaps, located at members 48 and 29, were identified correctly in **terms** of location and size. In attempting to locate the third gap, three columns of S provided an error measure of O when **compared** to the displacement vector, ***x*****,** indicating possible gaps at members 26, 35, or 53. After perturbing each remaining actuator, the same three possibilities remained.

The next step was to take each possible gap location at a time, assume it to be the correct one and continue with the procedure. In doing so, when assuming a gap at location 26, no matches were detected on attempting to locate the fourth gap, thus eliminating the possibility of a gap in member 26. Using 35 and 53 one at a time to detect the fourth gap both resulted in duplicate results of the next gap occurring at member 57. At this point the fourth gap, in member 57, has been determined but the location of the third gap remains undefined

Once more, two cases were continued, one assuming four gaps at members 48, 29,"35, and 57 and another assuming four gaps at 48, 29, 53, and 57. Both of these cases produced a result of a fifth gap in member 62. At this point, all actuators had been used for perturbations, and two different paths had been followed in **the hopes that** one would lead to an error. Since gap number three still had not been determined, a new unloading pattern was used.

This time, instead of uniformly decreasing the actuator displacements until the structure is completely unloaded, the actuators were unloaded sequentially. In doing so, different nodal displacements are obtained and all of the gaps were then uniquely **identified** in location and size. In this case, the maximum actuator force in prestressing was 279 **lbs** at actuator 20.

Case 4 - **include measurement error**

The example described here begins with the same gap and actuator placement as that in Case 1. In the present case, however, **the** displacement vector ***x*****,** was perturbed with a random error vector to determine when the process of gap identification deteriorates, In the first error analysis, the error vector contains a random distribution of numbers in

Table 2. Case 2- Actuator Length Changes **for Prestressing and Gap Opening**

	actuator length changes, microns					
actuator no.	**prestress**	**gap 2 open**	gap 3 **open**	gap 4 **open**	**gap** 43 **open**	gap 44 **open**
18	198.4	156.4	114.2	112.2	100.8	96.3
23	75.2	83.5	67.5	66.7	62.4	60.5
45	381.4	316.1	235,0	231.2	209.3	199.8
66	200.2	_ 156.0	113.4	111.4	9 9 . 9	94.9

the range of O to .001 and is added to x, which has been normalized to the maximum displacement. The original vector $\boldsymbol{x}$, and the displacement vector with measurement error, $\boldsymbol{x}_e$ are:

x	X_e
-0.2230	-0.2228
-0.0857	-0,0856
0.1274	0.1281
-1.0000	-0.9993
-0.7592	-0.7583
-0.0477	-0.0473
-0.3559	-0.3554
-0,2676	-0.2668
-0.0484	-0.0483
0.5771	0.5771
0.3644	0,3650
-0.2303	-0.2297

With this level of error added, the procedure successfull y identified all gaps correctly, by selecting j at which the minimum value of e_j occurs. The error level was increased and the gap detection process repeated until the program could no longer accurately identify the gap locations. At an error with a maximum value of .04, the joint identification process remained successful. The original vector and vector with errors in this case were:

x	x,
-0.2230	-0.2210
-0.0857	-0.0552
0.1274	0.1582
-1.0000	-0,9669
-0.7592	-0.7542
-0.0477	-0.0471
-0.3559	-0.3284
-0.2676	-0.2329
-0.0484	-0.0232
0.5771	0.6065
0.3644	0.3934
-0.2303	-0.1903

The procedure failed at an error distribution with a maximum value of .05. The vectors x and x, in this case were:

x	x,
-0.2230	-0.2226
-0.0857	-0.0665
0.1274	0.1307
-1.0000	-0.9791
-0,7592	-0,7249
-0.0477	-0.0182
-0.3559	-0.3094
-0.2676	-0.2253
-0,0484	-0.0220
0.5771	0,5817
0.3644	0.3971
-0.2303	-0.2095

If the error addition is small enough such that the gap location is still identifiable, as when using a maximum error of ,04, the gap size will still be calculated correctly. The gap size is dependent on the member length changes at the break in linearity in the unloading process and the gap location. It is unaffected by the error in x.

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The locations and sizes of the loose joints in the structure were successfully located using an actuator-induced static loading and unloading procedure. Four numerical examples were presented, three of which assumed no deterioration in the assumed displacement measurements. In one of these three cases, the initial unloading procedure did not provide complete information on the location of the loose joints. However, by altering the unloading sequence, the last gap was correctly located. The last case presents the effect of introducing an error into the structure's displacement vector. The level of error was increased until an error in gap identification occurred.

The procedure described in this paper has the advantage of using a small number of actuators whose positions in the structure are not critical. In each case, the locations of the actuators and gaps were randomly assigned and the gap location and sizes were correctly identified.

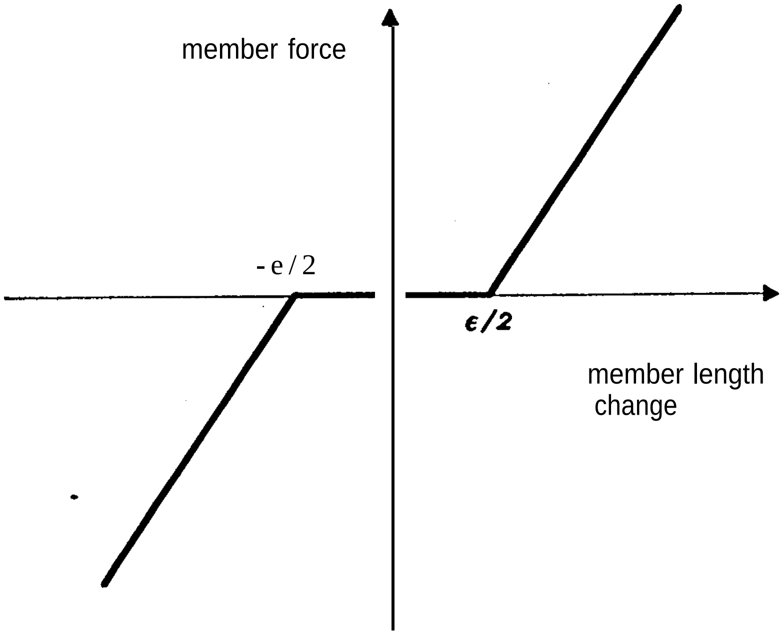

Figure 1. Gap Member Force-Displacement Relationship

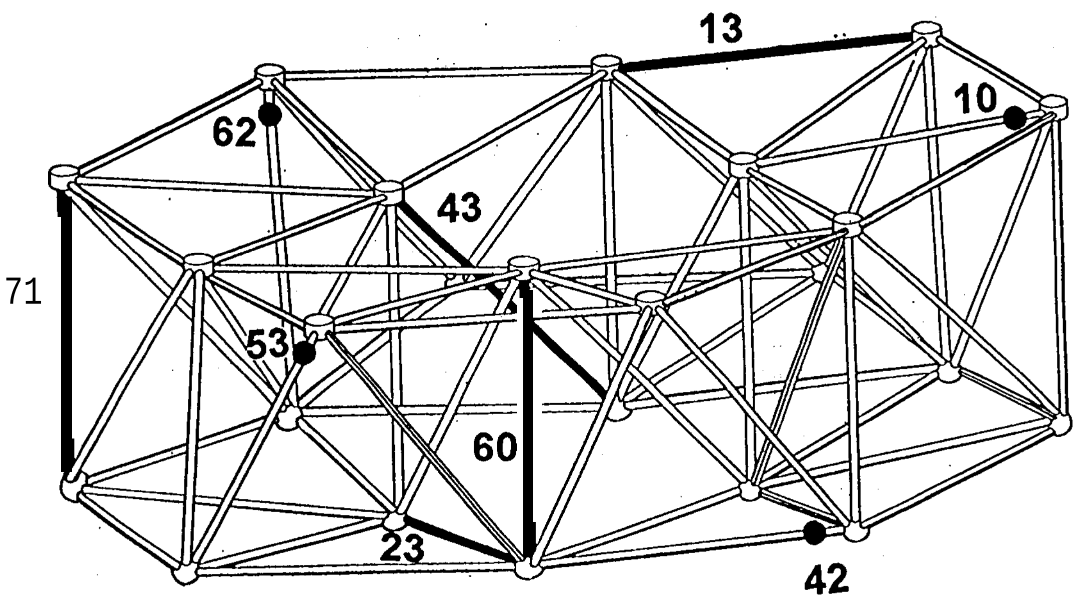

Figure 2. Actuator and Gap Locations for Case 1

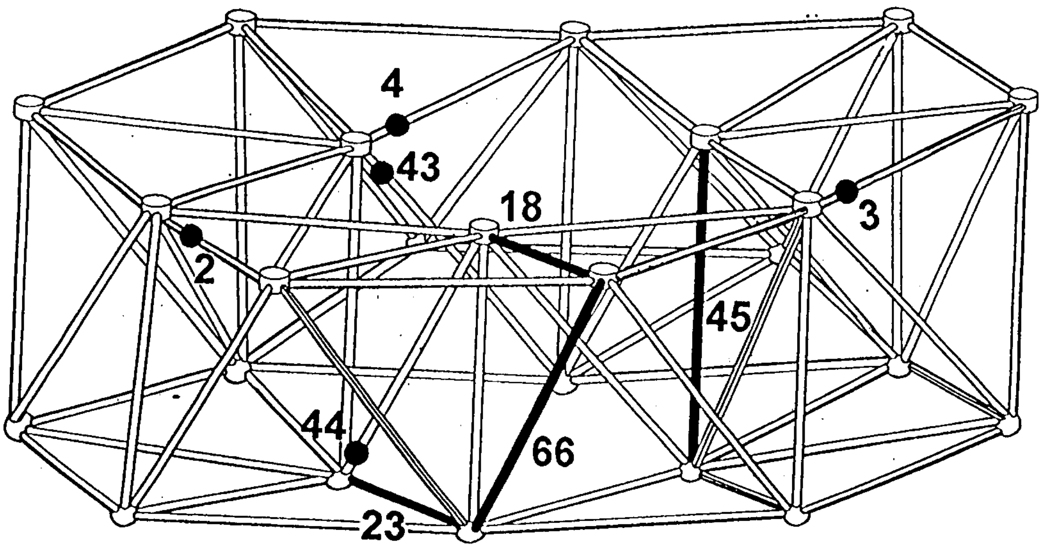

Figure 3. Actuator and Gap Locations for Case 2

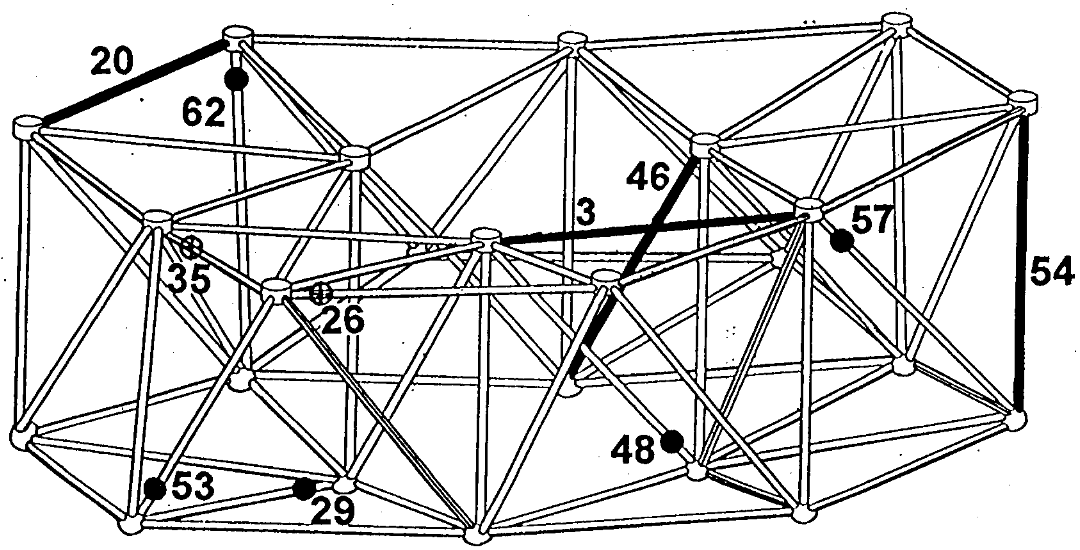

Figure 3. Actuator and Gap Locations for Case 3

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