Noninteracting Control of Robotic Space Vehicles

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Abstract

This paper develops methods for noninteracting control of articulated, possibly flexible, multibody space vehicles based on the following diagonalized equation of motion: $\dot{\nu} + \mathcal{C}(\theta, \nu) = \epsilon$. Noninteraction implies that at each fixed time instant the cent rol for each degree of freedom is decoupled from all oft he other degrees of freedom. The diagonal equations are obtained by using the recently developed mass matrix factorization $\mathcal{M}(\theta) = m(\theta)m^*$ (0), which is readil y implemented using the spatially recursive filtering and smoothing methods advanced by the authors in recent years.

1 Introduction

The diagonal equations of motion result by combining Lagrangian mechanics with the mass matrix factorization

$$M = [I + H\phi K]D[I + H\phi K]^* \tag{1.1}$$

in which H, ϕ, D and K are spatial operators mechanized recursively by suitably defined [1]- [2] spatial filtering and smoothing algorithms. Use of this in the system kinetic energy $\mathcal{K}(\theta, \dot{\theta}) = \frac{1}{2}\dot{\theta}^* \mathcal{M}(\theta)\dot{\theta}$ results in

$$\mathcal{K}(\theta, \dot{\theta}) = \frac{1}{2}\nu^*\nu = \sum_{k.1}^{\mathcal{N}} \nu^2(k) \tag{1.2}$$

where $\nu =$ [v(I), . . . , $\nu(\mathcal{N})$] is a new set of variables related to the joint-angle rates $\dot{\theta}$ by

$$\nu = D^{\frac{1}{2}}[I + H\phi K]\dot{\theta} \tag{1.3}$$

In these new variables, the kinetic energy is diagonalized in the sense that it is a simple sum of the squares of the total joint rates $\nu(k)$ over all of the $\mathcal{N}$ joints, This is in contrast to the original expression $\mathcal{K}(\theta, \theta) = \frac{1}{2}\theta\, \mathcal{M}(\theta)\theta$ which involves the mass matrix M(0) as a weighting matrix. The diagonal equations of motion $\dot{\nu} + \mathcal{C}(\theta, \nu) = \epsilon$ are obtained in this paper by applying classical Lagrangian mechanics methods to the above diagonalized kinetic energy. The new variables ν have a physical interpretation as time-derivatives of Lagrangian quasi-coordinates, similar to those typically encountered [.3, 4] in analytical d ynamics. A nother key term in the new equations of motion is the forcing "input" $\epsilon = [\epsilon(1), \ldots, \epsilon(\mathcal{N})]$ appearing on the right side of the equation. This term is related to the applied moments T by means of the configuration dependent relationship

$$\epsilon = m^{-1}(\theta)T = D^{-\frac{1}{2}}(I - H\psi K)T \tag{1.4}$$

The operators H, ψ, K and D are mechanized by an inward filtering operation [1]. The inputs ϵ also have a physical interpretation. The input $\epsilon(k)$ at the k^{th} joint can be thought of as being that part of the applied moment ?'(k) that dots mechanical work at this joint.

The quasi-coordinates v appearing in the diagonalized equations of motion are closely analogous to the innovations process extensively investigated [5] in the area of linear filtering and estimation for state space systems. The innovations process [5] is a central ingredient in factoring, diagonalizing, and inverting state-sl)ace system covariance matrices by means of Kalman filtering and smoothing algorithms. The innovations process plays a similar role in the dynamics of mechanical systems [1 ,6,7]. The analogy between estimation theory and robot dynamics has been one of the central themes investigated by the authors [1, 2].

2 Diagonalization in Velocity Space

A diagonalizing transformation can be found in velocity space. This transformation replaces the joint-angle velocities θ with a new set of velocities ν, without replacing the configuration variables θ. The search for this transformation begins with the following factorization of the mass matrix.

Assumption 1 *There exists a smooth, differentiable and invertible function $m(\theta)$, with inverse denoted by k?{(?), which factors the mass matrix as $\mathcal{M}(\theta) = m(\theta)m^*(0)$ for all configurations.* ***The function $m(\theta)$ need not be the gradient of any junction,***

Lemma 1: *The equations of motion using the (θ, ν) coordinates are*

$$\dot{\nu} + \mathcal{C}(\theta, \nu) = \epsilon \tag{2.5}$$

with the new Coriolis force vector $\mathcal{C}(\theta,\nu) = \ell(\dot{m}\nu - \frac{1}{2}\dot{\theta}^* \mathcal{M}_\theta \dot{\theta})$ and $\epsilon = \ell(\theta)T$. The corresponding kinematic equation to obtain the joint-angle rates $\dot{\theta}$ is $\dot{\theta} = \ell(\theta)\nu$ ∎

These equations of motion are considerably simpler than the original ones. The mass matrix here is equal to the identity matrix. The most critical element leading to the above diagonalized equations is the mass matrix factor $m(\theta)$. A numerical (e.g. Cholesky-like) factorization of the mass matrix at each configuration can be used to obtain a candidate factor $m(\theta)$. However, it may not be easy to interpret physically tile corresponding transformed] variables.

Recent results [1, 6] have established that the mass matrix can be factored and inverted using methods widely used in linear filtering and estimation theory. These results arc summarized by the following; identities, whose proof can be found in [1, 6].

Identity 1

$$M = H\phi M\phi^* H^* \tag{2.6}$$

$$M = [I + H\phi K]D[I + H\phi K]^* \tag{2.7}$$

$$[I + H\phi K]^{-1} = I - H\psi K \tag{2.8}$$

$$M^{-1} = [I - H\psi K]^* D^{-1}[I - H\psi K] \tag{2.9}$$

The innovations factorization in Identity 1 leads to a set of diagonal equations of motion. To this end, define the operators $m(\theta)$ and $\ell(\theta)$ as

$$m(\theta) = [I + H\phi K]D^{\frac{1}{2}} \qquad \ell(\theta) = m^{-1}(\theta) = D^{-\frac{1}{2}}[I - H\psi K] \tag{2.10}$$

and

$$\mathcal{M}(\theta) = m(\theta)m^*(\theta); \qquad \mathcal{M}^{-1}(\theta) = \ell^*(\theta)\ell(\theta) \tag{2.11}$$

The function m(0) so defined satisfies all of the conditions in Assumption 1, although verifying the condition of differentiability requires the following more careful argument. The operators H and ϕ are smooth and differentiable functions of the coordinates, S0 the only potential troublespot is in the differentiability of the articulated body quantities, particularly the inverse D^{-1} of the diagonal operator $D = HPH^*$. This diagonal matrix D is always positive definite and invertible. Consequently, D^{-1} is always a smooth and differentiable function of θ. 'i'bus, $m=[I+H\phi K]D^{\frac{1}{2}}$ is also a smooth and differentiable matrix function. Thus, $m(\theta)$ satisfies all the conditions in Assumption 1.

$O(\mathcal{N})$ Algorithms for $\mathcal{C}(\theta,\nu)$

An algorithm to compute C(O, ν) recursively is described below. It is assumed that V as well as the various articulated body quantities have been computed and are available prior to these computations.

Algorithm I

$$\dot{P}(0) = 0$$

for $k = 1 \ldots \mathcal{N}$

$$\begin{aligned}
X(k) &= \Omega_\delta(k)P(k) \\
\dot{P}(k) &= \psi(k,k-1)\dot{P}(k-1)\psi^*(k,k-1)-\{X(k)+X^*(k) \\
y(k) &= \psi(k,k-1)y(k-1)+2[V(k)\times M(k)-X(k)+X^*(k)]V(k)+ \\
&\quad \psi(k,k-1)\dot{P}(k-1)V(k-1)-\dot{P}(k)\psi^*(k+1,k)V(k+1) \\
\mathcal{C}(k) &= \frac{1}{2}D^{\frac{1}{2}}H(k)y(k)
\end{aligned}$$

end loop

This $O(\mathcal{N})$ algorithm proceeds from tip-to-base and also solves the forward dynamics problem in the (0, ν) coordinates.

Coriolis Force Does No Work

The Coriolis term $\mathcal{C}(\theta,\nu)$ is orthogonal to the generalized velocities ν and therefore dots no mechanical work.

Lemma 2: $\nu^*\mathcal{C}(\theta,\nu)=0$ ∎

The orthogonality of the nonlinear Coriolis forces is similar to the orthogonality condition w . w $\times I\cdot\dot{\omega}=$ O of the gyroscopic force term in the equations of motion for a single rigid body moving with the angular velocity w. In contrast, the corresponding Coriolis forces term $\mathcal{C}(\theta,\dot{\theta})$ in the regular equations of motion in dots work, i.e., $\dot{\theta}^*\mathcal{C}(\theta,\dot{\theta})\neq$ O. The non-working nature of the Coriolis forces has an interesting implication.

***Lemma* 3:** *The rate of change of the kinetic energy is the dot product of the generalized forces and generalized velocities*

$$\frac{d}{dt}\mathcal{K}(\theta,\nu)=\frac{1}{2}\nu^*\dot{\nu}=\nu^*[\epsilon-\mathcal{C}(\theta,\nu)]=\nu^*\epsilon \tag{2,12}$$

B

Noninteracting Control

The diagonal equations can also be used for to design controllers that are decoupled or non-interacting in a. quasi-static. sense. The term quasi-static. denotes the idea that for a fixed time instant the control is decoupled. The analysis and control design is simpler because the equations of motion used in this design are decoupled,

Control 1 (a) *The* **rate** *feedback control $\epsilon=-c\nu$ in which c is a positive diagonal control gain matrix renders the system stable in the sense of Lyapunov.*
(b) The feedback control $\epsilon = -c_1\nu - c_2H\psi B[\hat{Y} - Y(\theta)]$ in which c_1 and c_2 are positive, digonal control gain matrices, causes the system to reach the prescribed configu -

ration $\hat{Y}$ and drives the velocities to zero.

3 Conclusions

The diagonalized equations of motions presented here are very closely related to the body of knowledge [1,2,6,8] recently developed by the authors on spatially recursive algorithms for inverse and forward dynamics. The present paper complements and builds upon the previous work by writing explicitly the diagonalized Lagrangian equations of motion which correspond to the recursive algorithms. The focus here is on the new equations of motion, on the diagonalizing transformations required to obtain them, and on the physical interpretation of the transformed variables. The results presented embed in a single diagonalized equation several of the spatially recursive algorithms previously developed. This provides an additional step toward an increasingly more succinct statement of the equations of motion for articulated multibody systems.

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