JPL

ATMOSPHERIC EXCITATION OF NUTATION

by

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- **The nutations of the solid Earth**
  - In a celestial, space-fixed reference frame have frequencies associated with the orbital motions of the sun and moon
  - Because of the Earth's diurnal rotation, the nutations appear at retrograde nearly diurnal frequencies when viewed from within a terrestrial, body-fixed reference frame

$$\sigma_t = \sigma_c - \Omega \qquad \text{where } \Omega \equiv 1 \text{ cpsd}$$

  - Thus, internally forced motions of the solid Earth occurring at retrograde nearly diurnal frequencies can excite nutation
    - Diurnal ocean tidal current and sea level height variations
    - Atmospheric wind and pressure fluctuations (the S_1 thermal tide)
- **Study effect on the Earth's nutations of diurnal variations in atmospheric wind and pressure**

APPROACH

- Use the publicly available atmospheric angular momentum (AAM) excitation functions that are given at 6-hour intervals
  - US National Meteorological Center
  - European Centre for Medium-Range Weather Forecasts
  - Japan Meteorological Agency
- Convolve the AAM excitation functions with the Earth's impulse response to compute the influence of atmospheric wind and pressure fluctuations on the Earth's rotation
  - Use "broad-band" Liouville equation that accounts for resonances at both the Chandler and Free Core Nutation frequencies
- Within the resulting retrograde nearly diurnal frequency band (i.e., the nutation band), fit for periodic terms at frequencies corresponding to the
  - Retrograde semi-annual nutation (-1 cpsd - 1/1[12.621 cpd)
  - Retrograde Free Core Nutation (–1 cpsd - 1/429.8 cpd)
  - Retrograde 18.6 year nutation (-1 cpsd – 1/6798.383 cpd)
  - Prograde annual nutation (-1 cpsd + 1/365.26 cpd)
  - Prograde semi-annual nutation (-1 cpsd + 1/182.621 cpd)
- Compare predicted AAM-excited nutations with observations

AAM z-FUNCTIONS

- IN EARTH ROTATION THEORY, THE x-FUNCTIONS ARE THE FORCING FUNCTIONS THAT CAUSE CHANGES IN THE EARTH'S ROTATION AND ORIENTATION
- IN GENERAL, THEY ARE FUNCTIONS OF CHANGES IN
  - **THE EARTH'S INERTIA TENSOR**
  - **RELATIVE ANGULAR MOMENTUM**
- AAM PRESSURE TERM (INERTIA TENSOR)
  - $\chi_1^p + i\,\chi_2^p = \dfrac{-1.00\, a^4}{(C-A)\, g} \int p_s \sin\phi \cos^2\phi\, (\cos\lambda + i \sin\lambda)\, d\lambda\, d\phi$
- AAM WIND TERM (RELATIVE ANGULAR MOMENTUM)
  - $\chi_1^w + i\,\chi_2^w = \dfrac{-1.43\, a^3}{\Omega\, (C-A)\, g} \int (u \sin\phi \cos\phi + i\, v \cos\phi)\, (\cos\lambda + i \sin\lambda)\, dp\, d\lambda\, d\phi$
- AAM x-FUNCTIONS ARE COMPUTED FROM FIELDS GENERATED BY GCMs OPERATED BY WEATHER PREDICTION CENTERS:
  - **JAPAN METEOROLOGICAL AGENCY (JMA)**
  - **NATIONAL METEOROLOGICAL CENTER (NMC)**
  - **UNITED KINGDOM METEOROLOGICAL OFFICE (UKMO)**
  - **EUROPEAN CENTRE FOR MEDIUM-RANGE WEATHER FORECASTS (ECMWF)**

6-HOUR AAM

- US National Meteorological Center
  - Spans June 21, 1992 to August 29, 1994 (800 days)
  - 20 gaps exist, longest of 2-day duration
- European Centre for Medium-Range Weather Forecasts
  - Spans January 2, 1993 to September 30, 1994 (637 days)
  - Numerous gaps exist, including two of 13-day and one of 17-day duration
- Japan Meteorological Agency
  - Spans June 21, 1992 to February 8, 1994 (598 days)
    - Later values not analyzed due to presence of gap of 36-day duration
  - 8 gaps exist, longest of l-day duration
- Gaps filled by linear interpolation
  - Done separately for values at O, 6, 12, and 18 hours
- Analyze series spanning at least 1 year in order to separate annual from semi-annual nutations
  - Fit at frequency corresponding to retrograde 18.6 year nutation cannot separate it from effects of retrograde 9.3 year, prograde 9.3 year and prograde 18.6 year nutations
  - Fit at frequency corresponding to retrograde Free Core Nutation cannot separate it from effects of retrograde annual nutation

LONG PERIOD LIOUVILLE EQUATION

- Conservation of angular momentum expressed within rotating, body-fixed reference frame

$$\frac{\partial \mathbf{L}}{\partial t} + \omega \times \mathbf{L} = \tau$$

where the angular momentum vector $\mathbf{L} = 1 \cdot \omega + \mathbf{h}$

- Assume rotation is small perturbation from state of uniform rotation at rate Ω. Keeping terms to first order results in long period Liouville equation

$$\mathbf{m}(t) + \frac{i}{\sigma_{cw}} \frac{\partial \mathbf{m}}{\partial t} = \psi(t)$$

$$= \chi(t) - \frac{i}{\Omega} \frac{\partial \chi}{\partial t}$$

where: m $\equiv(\omega_1 + i\,\omega_2)/\Omega$ (terrestrial location of rotation pole)

$\psi(t)$, $\chi(t)$ are the polar motion excitation functions

σ_{cw} is complex-valued frequency of Chandler wobble

- Written in terms of reported polar motion parameters $p(t) \equiv x_p(t) - i\, y_p(t)$
  - In time domain:

$$\mathbf{p}(t) + \frac{i}{\sigma_{cw}} \frac{\partial \mathbf{p}}{\partial t} = \chi(t)$$

$$= L \frac{1.61}{?(C-A)} \left[h(t) + \frac{\Omega c(t)}{',.44} \right]$$

  - In frequency domain:

$$\mathbf{p}(\sigma) = \frac{\sigma_{cw}}{\sigma_{cw} - \sigma} \chi(\sigma)$$

FREE CORE NUTATION

- Resonance in Earth's rotation due to presence of fluid core
  - Has frequency σ_{fcn} estimated *(Mathews et aL,* 1991) to be -1.0023203 cycles/sidereal day (period of 23.88 hours) as viewed in rotating, body-fixed, terrestrial reference frame
  - Has spatial period estimated (*Mathews et al.,* 1991) to be 429.8 solar days
- Must be taken into account when studying polar motion changes at retrograde nearly diurnal frequencies
  - Effects of forcing near the FCN frequency will be resonantly enhanced
- Routinely taken into account when modeling the Earth's nutations
  - The observable nutation $n(\sigma)$ of *Sasao &* Wahr (1981) is related to the reported polar motion parameters $p(\sigma)$ by $n(\sigma) = -p(\sigma)$
  - Therefore, expressions developed by *Sasao &* Wahr (1981) to study the Earth's nutations can be used to study retrograde nearly diurnal polar motions:

$$p(\sigma) = \left[2.554 \times 10^{-4} \frac{\Omega}{\sigma_{fcn} - \sigma} + 2.686 \times 10^{-3} \frac{\Omega}{\sigma_{cw'} - \sigma} \right] \frac{\Omega\, c(\sigma)}{A\, \Omega\, \tau}$$

$$+ \left[6.170 \times 10^{-4} \frac{\Omega}{\sigma_{fcn} - \sigma} + 1.124 \frac{\Omega}{\sigma_{cn'} - \sigma} \right] \frac{h(\sigma)}{A\, \Omega}$$

where $\tau = \Omega^2 a^5 / (3GA)$

BROAD BAND LIOUVILLE EQUATION

- **Sasao & Wahr (1981) observable nutation:**

$$p(\sigma) = \left[2.554 \times 10^{-4} \frac{\Omega}{\sigma_{fcn} - \sigma} + 2.686 \times 10^{-3} \frac{\Omega}{\sigma_{Cw} - \sigma} \right] \frac{\Omega\, c(\sigma)}{A\, \Omega\, \tau}$$

$$+ \left[6.170 \times 10^{-4} \frac{\Omega}{\sigma_{fcn} - \sigma} + 1{,}124 \frac{\Omega}{\sigma_{cw'} - \sigma} \right] \frac{h(\sigma)}{A\, \Omega}$$

where $\tau = \Omega^2 a^5 / (3GA)$

  - **Written here in terms of reported polar motion parameters $p = x_p - i\, y_p$ and angular momenta associated with relative motion $h(\sigma)$ and inertia tensor $\Omega c(\sigma)$ variations**
  - **Exhibits resonances at both Chandler and FCN frequencies**
  - **Valid at all frequencies, including those in the retrograde diurnal (nutation) band**

- **Earth rotation excitation functions:**
  - **Motion (wind) term:** $\chi_w = h / (A_m \sigma_{cw})$
  - **Matter (pressure) term:** $\chi_p = (1 + k_2') \Omega c / (A_m \sigma_{cw})$

- **"Broad band" Liouville equation (Brzezinski, 1994):**
  - **In frequency domain:**

$$p(\sigma) = \left[9.509 \times 10^{-2} \frac{\sigma_{cw}}{\sigma_{fcn} - \sigma} + \frac{\sigma_{cw}}{\sigma_{cw} - \sigma} \right] \chi_p(\sigma)$$

$$+ \left[5.489 \times 10^{-4} \frac{\sigma_{cw}}{\sigma_{fcn} - \sigma} + \frac{\sigma_{cw}}{\sigma_{cw} - \sigma} \right] \chi_w(\sigma)$$

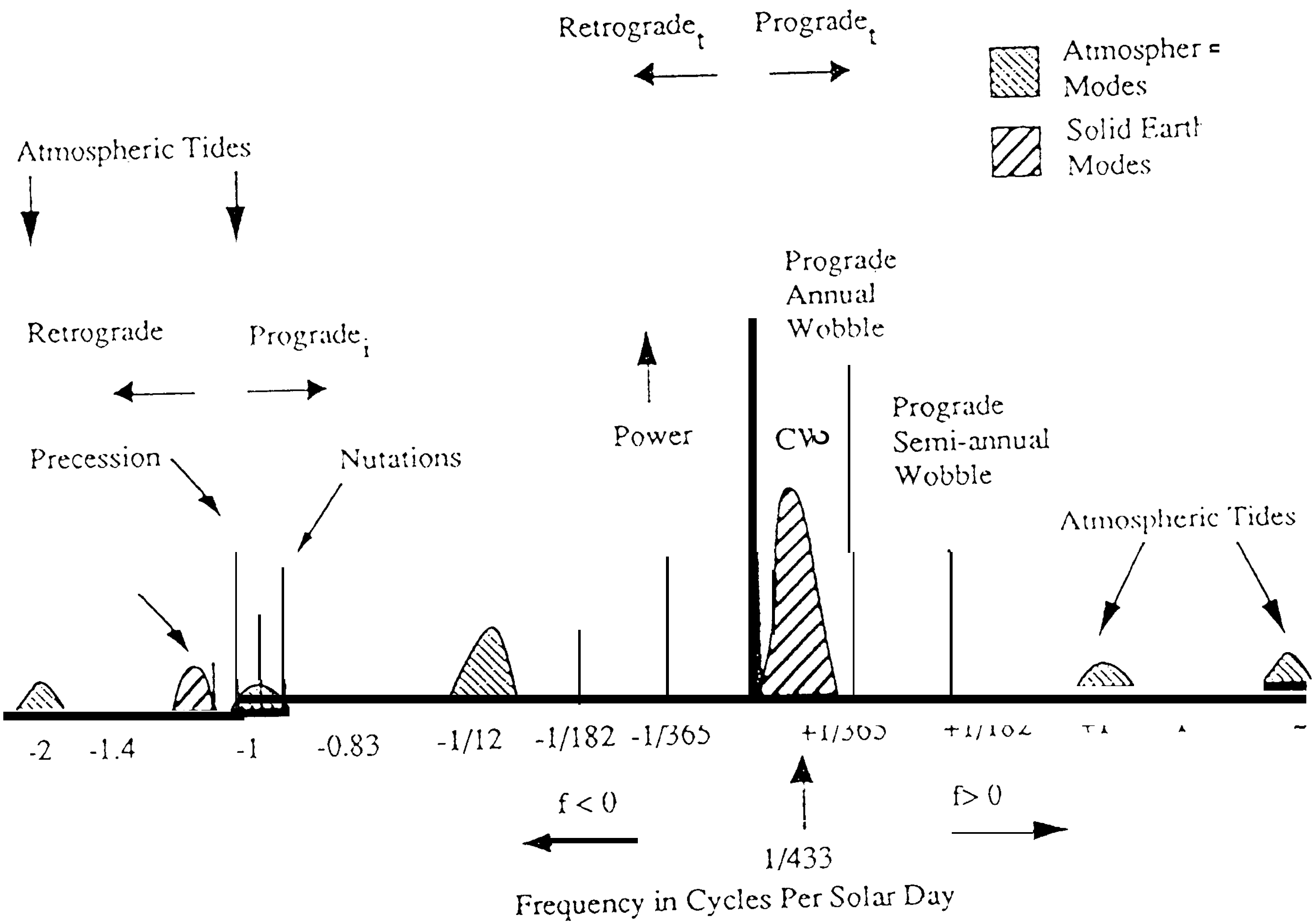
  - **In time domain:**

$$\left(\frac{\partial}{\partial t} - i\, \sigma_{cw} \right) \left(\frac{\partial}{\partial t} - i\, \sigma_{fcn} \right) p(t) =$$

$$- i\, \sigma_{cw} \left[9.509 \times 10^{-2} \left(\frac{\partial}{\partial t} - i\, \sigma_{cw} \right) + \left(\frac{\partial}{\partial t} - i\, \sigma_{fcn} \right) \right] \chi_p(t)$$

$$- i\, \sigma_{cw} \left[5.489 \times 10^{-4} \left(\frac{\partial}{\partial t} - i\, \sigma_{cw} \right) + \left(\frac{\partial}{\partial t} - i\, \sigma_{fcn} \right) \right] \chi_w(t)$$

SCHEMATIC POLAR MOTION SPECTRUM



(Eubanks, 1993)

TABLE 8. Estimated Nutation Amplitudes With Corrections to the ZMOA 1990 and the IAU 1980 Nutation series

Period, Solar Days	a_r, mas	δa_r,* mas	a_i, mas	δa_i,* mas	δa_{IAU}†, mas
-9.13	4.45	0.00	4.03	4.03	(?.01
-13.66	-3.63	0.01	0.06	0.06	-0.01
-14.77	-1.18	0.02	4.02	4.02	-0.03
-27.55	-13.77	0.04	-0.04	-0.02	0.04
-31.s1	-3.03	0.03	-0.01	-0.01	0.07
-121.75	-0.94	-0.00	0.03	0.03	4.03
-18262	-24.58	4.02	-0.07	4.02	4.05
-365.26	-33.00	–‡	0.32	0.39	-1.94
-6798.38	-8025.16	-0.46	1.33	0.47	-3.11
6798.38	-1180.73	-0.32	0.35	0.46	-0.28
365.26	25.70	-0.01	0.15	0.13	0.04
18262	-54855	-0.05	4.47	0.04	0.52
121.75	-21.54	-0.04	-0.04	-0.02	4.05
31.81	3.15	-0.03	-0.01	-0.01	4.04
27.55	14.51	0.03	0.01	0.00	-0.01
14.77	1.37	0.04	-0.02	-0.02	0.01
13.66	-94.40	-0.25	-0.02	-0.02	-0.32
9.13	-1252	-0.08	0.04	0.03	-0.08
-429.8§	0.16		-0.21		
429.8§	4.07		-0.10		

	Secular Rates	
	arc sec/cy	arc sec/cy
p, Δp¶	5038.46	-0.32
$d\varepsilon/dt$	-0.04	

One standard deviation (68% confidence interval) of the estimated coefficients is 0.04 mas except for the 18.6 year (6798 day) terms, whose standard deviations arc estimated to be 1.0 mas. The standard deviation of the estimates of the precession constant at J2000, p, and of $d\varepsilon/dt$ are 0.13 and 0.05 arc sec/cy, respectively. The derivation of the a priori theory and the standard deviations is discuss-d in the text.

* Corrections to the ZMOA 1990 nutation series arc described in the text. δa_r and δa_i are corrections to the "real" or "in-phase", and the "imaginary" or "out-of-phase" terms in this nutation series.

† Corrections to the IAU 1980 [*Seidelmann*, 1982] nutation theory. Since the IAU theory contains no out-of-phase terms, the a_i estimates are the corrections to these (absent) terms in the IAU theory.

‡ The resonance frequency of the RFCN mode has been adjusted to make the difference between the observed and theoretical values of this term zero (see text and Paper Ii').

§ The argument of the RFCN free oscillation is computed from the J2000 epoch.

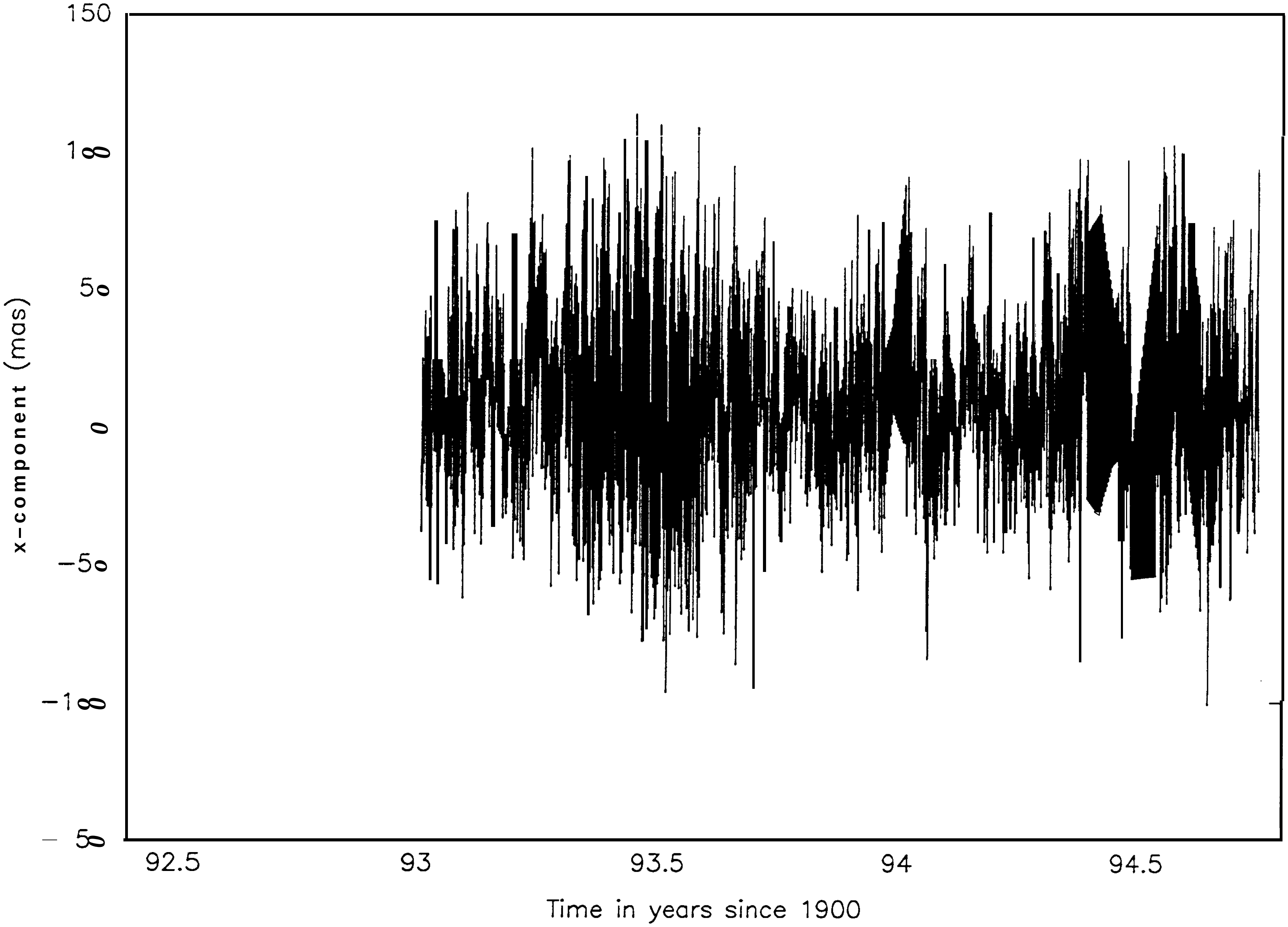
¶ Estimated luni-solar precession constant, p, and the correction. Δp, to the IAU 1976 value.

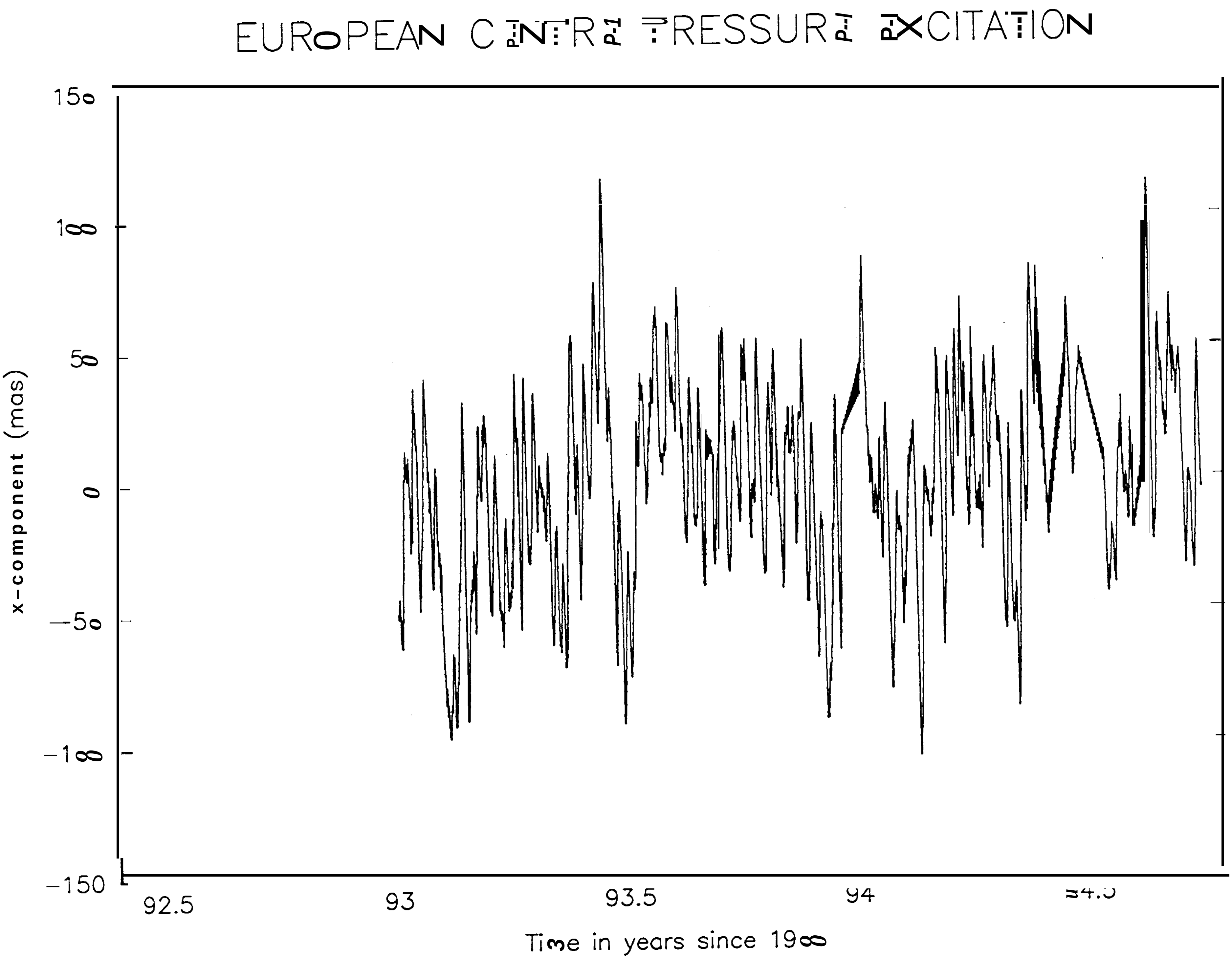
(Herring et *al.*, 1991)

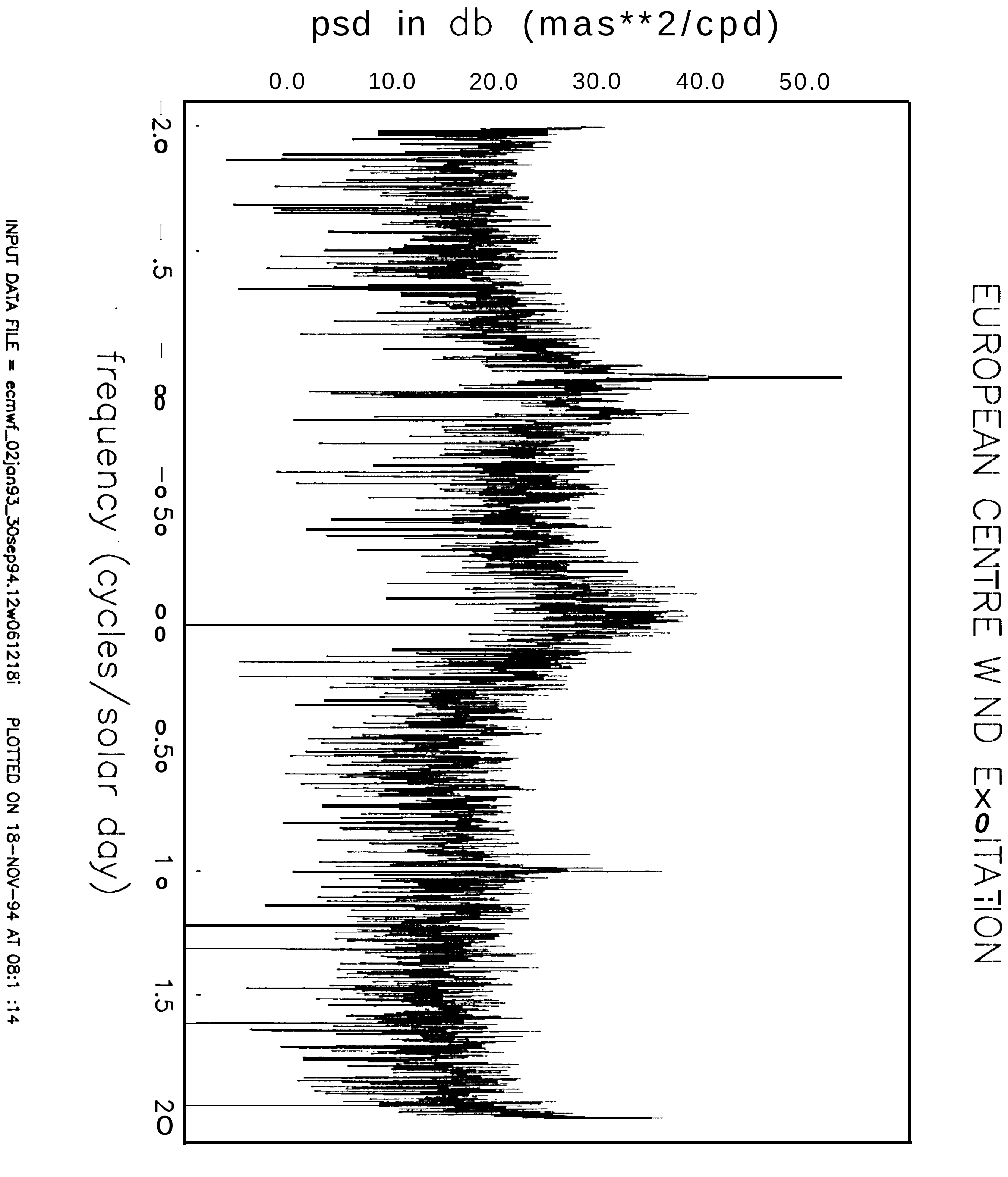
SUMMARY

- **Diurnal atmospheric pressure fluctuations are more effective in exciting nutations than are diurnal wind variations**
  - **Strength of Free Core Nutation resonance in Earth's polar motion transfer function is 170 times larger for pressure excitation than it is for wind excitation**
  - **Atmospheric pressure fluctuations load the Earth's crust, thereby deforming the solid Earth, and hence deforming the core-mantle boundary—this time-varying deformation of the core-mantle boundary excites the Free Core Nutation (Sasao and Wahr, 1981)**
- **Amplitudes and phases of atmospherically excited nutations are time-variable**
  - **S_1 tide in the atmosphere is a thermal, not gravitational, tide**
  - **Variations in thermal heating will cause variations in the thermally-induced winds and pressure, and hence in the nutations forced by these wind and pressure variations**
- **Diurnal atmospheric wind and pressure variations are energetic enough to explain observed nutation residuals**
  - **Amplitude of Free Core Nutation from VLBI observations is 0.26 ± 0.04 mas (Herring *et al.*, 1991), whereas amplitude predicted here from NMC and ECMWF wind and pressure fluctuations is as large as 1 mas**
  - **Amplitude of the VLBI observed minus ZMOA 1990 modeled prograde annual nutation residual is 0.13 ± 0.04 mas (Herring *et al.*, 1991), whereas amplitude predicted here from NMC and ECMWF wind and pressure fluctuations is as large as 0.13 mas**
  - **Caveat: VLBI observations and atmospheric predictions span different time intervals: VLBI observations span July 1980 to February 1989, whereas NMC and ECMWF results reported here span mid-1992 to September 1994**

CROPEAN CNTR WND XCTATON




EUROPEAN CENTER PRESSURE EXCITATION
x-component (mas)
150
100
50
0
-50
-100
-150
92.5
93
93.5
94
Time in years since 1900


psd in db (mas**2/cpd)
0.0
10.0
20.0
30.0
40.0
50.0
frequency (cycles/solar day)
EUROPEAN CENTRE W ND ExoITATION
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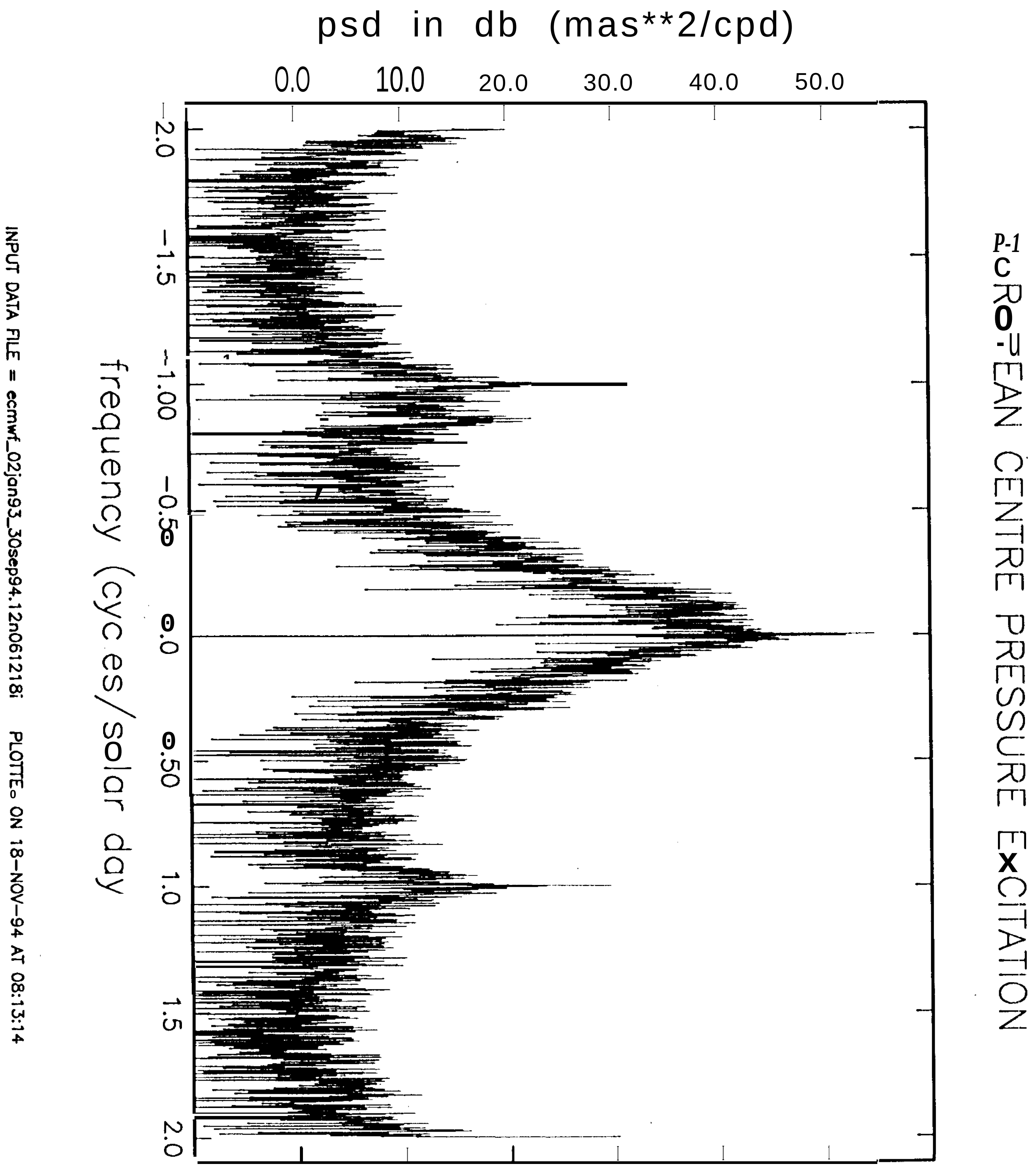
PLOTTED ON 18-NOV-94 AT 08:1 :14


P-1
CENTRE PRESSURE EXCITATION
psd in db (mas**2/cpd)
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10.0
20.0
30.0
40.0
50.0
-2.0
-1.5
-1.00
-0.50
0.0
0.50
1.0
1.5
2.0
frequency (cycles/solar day)
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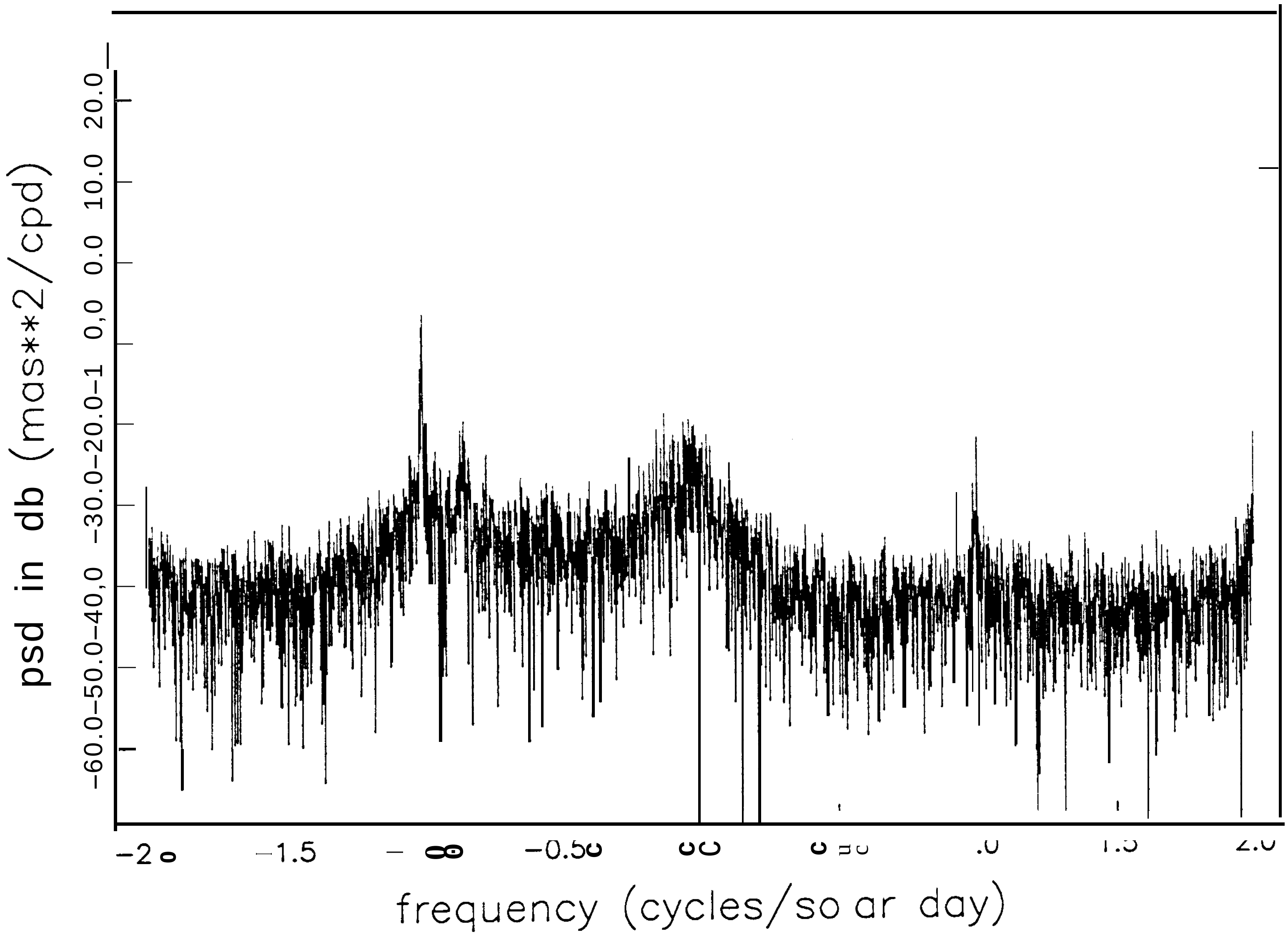
b

EUROPEAN CENTRE WIND-DRIVEN POLAR MOTION



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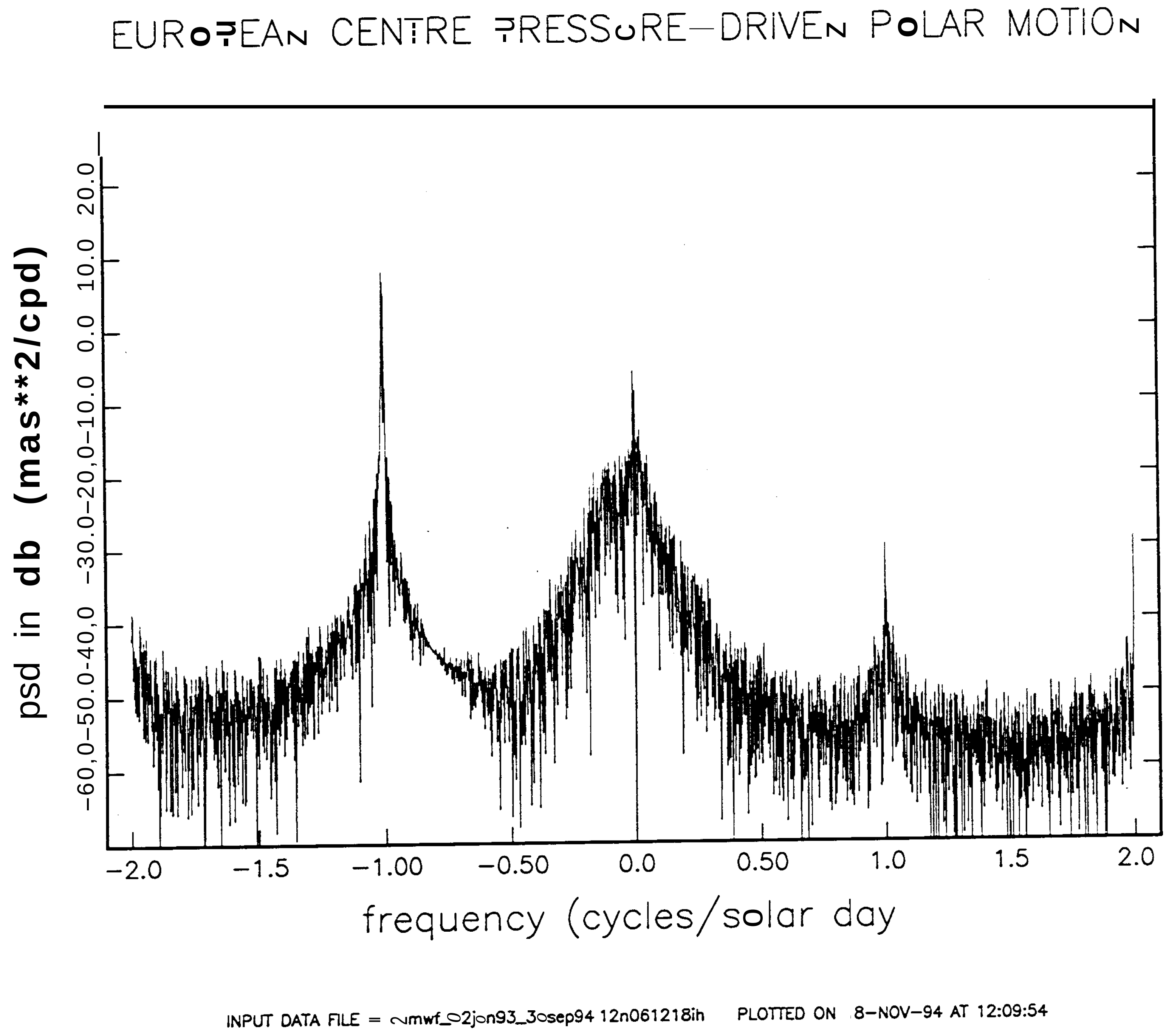
EUROPEAN CENTRE PRESSURE-DRIVEN POLAR MOTION
psd in db (mas**2/cpd)
-60,0 -50.0 -40,0 -30.0 -20,0 -10.0 0.0 10.0 20.0
-2.0 -1.5 -1.00 -0.50 0.0 0.50 1.0 1.5 2.0
frequency (cycles/solar day
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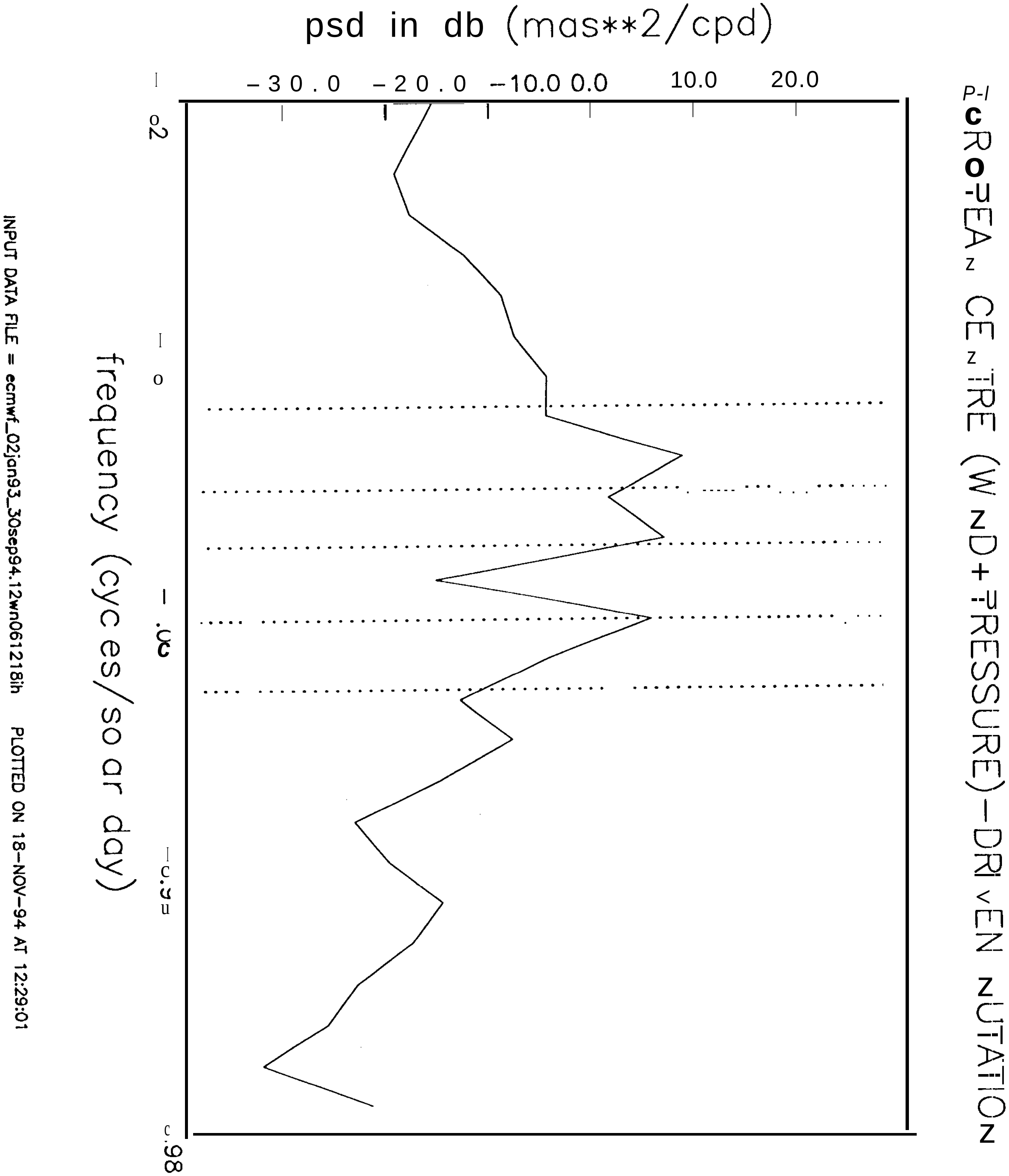


CROSS-EAR CENTRE (WIND+PRESSURE)-DRIVEN NUTATION
psd in db (mas**2/cpd)
-30.0
-20.0
-10.0
0.0
10.0
20.0
frequency (cycles/solar day)
.98
INPUT DATA FILE = ecmwf_02jan93_30sep94.12wn061218ih
PLOTTED ON 18-NOV-94 AT 12:29:01

ECMWF (WIND+PRESSURE)-DRIVEN POLAR MOTIONS AT NUTATION FREQUENCIES

Case	Retrograde semi-annual amp (mas)	Retrograde semi-annual phase (deg)	Retrograde FCN amp (nɯ)	Retrograde FCN phase (deg)	Retrograde 18.6 yr amp (mas)	Retrograde 18.6 yr phase (deg)	Prograde annual amp (nɯ)	Prograde annual phase (deg)	Prograde semi-annual amp (mas)	Prograde semi-annual phase (deg)
1.	0.0311	84.59	0.0197	81.64	0.0857	32.50	0.0687	–44.64	0.0092	30.61
	± 0.0020	± 3.67	± 0.0020	± 5.94	± 0.0021	± 1.38	± 0.0020	± 1.68	± 0.0020	± 12.55
2.	0.0310	84.71	0.0202	101.31	0.0857	32.83	0.0689	–44.68	0.0092	30.52
	± 0.0020	± 3.68	± 0.0020	± 5.77	± 0.0021	± 1.38	± %ƆOZO	± 1.68	± 0.0020	± 12.45
3.	0.0393	69.47	0.3799	178.38	0.0850	41.17	0.0812	–39.85	0.0206	59.30
	± 0.0019	± 2.80	± 0.0020	± 0.30	± 0.0020	± 1.34	± 0.0020	± 1.38	0200.07	± 5.43
4.	0.0388	66.62	0.3813	–168.12	0.0854	40.44	0.0797	–39.98	0.0199	60.70
	± 0.0018	± 2.73	L0.0019	± 0.28	± 0.0019	± 1.28	± 0.0019	± 1.36	± 0.0019	± 5.42

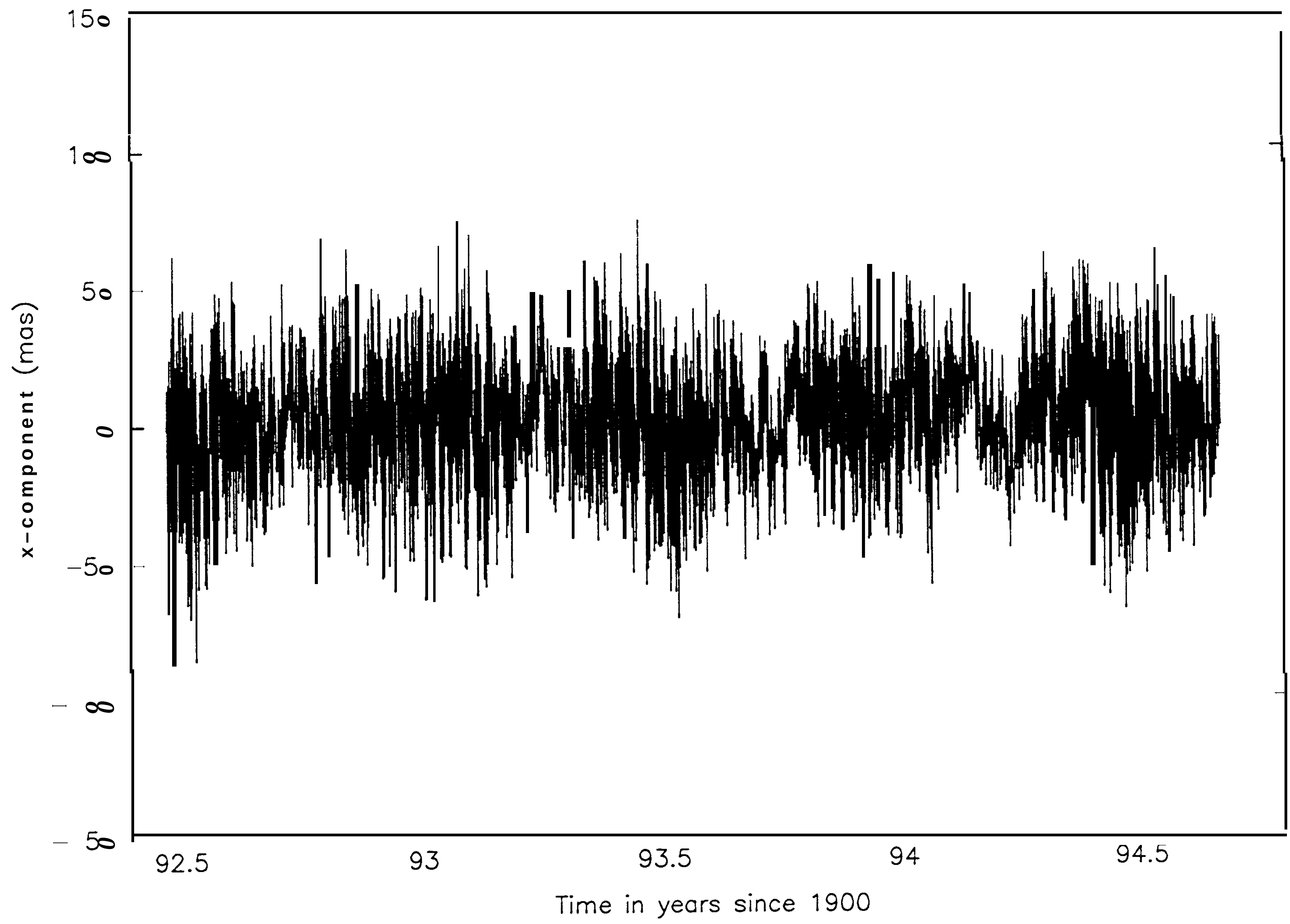
Case 1. Fit to data spanning 02JAN93 to 30SEP94 assuming FCN period = –429.8 solar days, Q = ∞ (Mathews *et al.*, 1991)
Case 2. Fit to data spanning 02JAN93 to 30SEP94 assuming FCN period = –434.1 sidereal days, Q = 53821 (Dehant *et al.*, 1994)

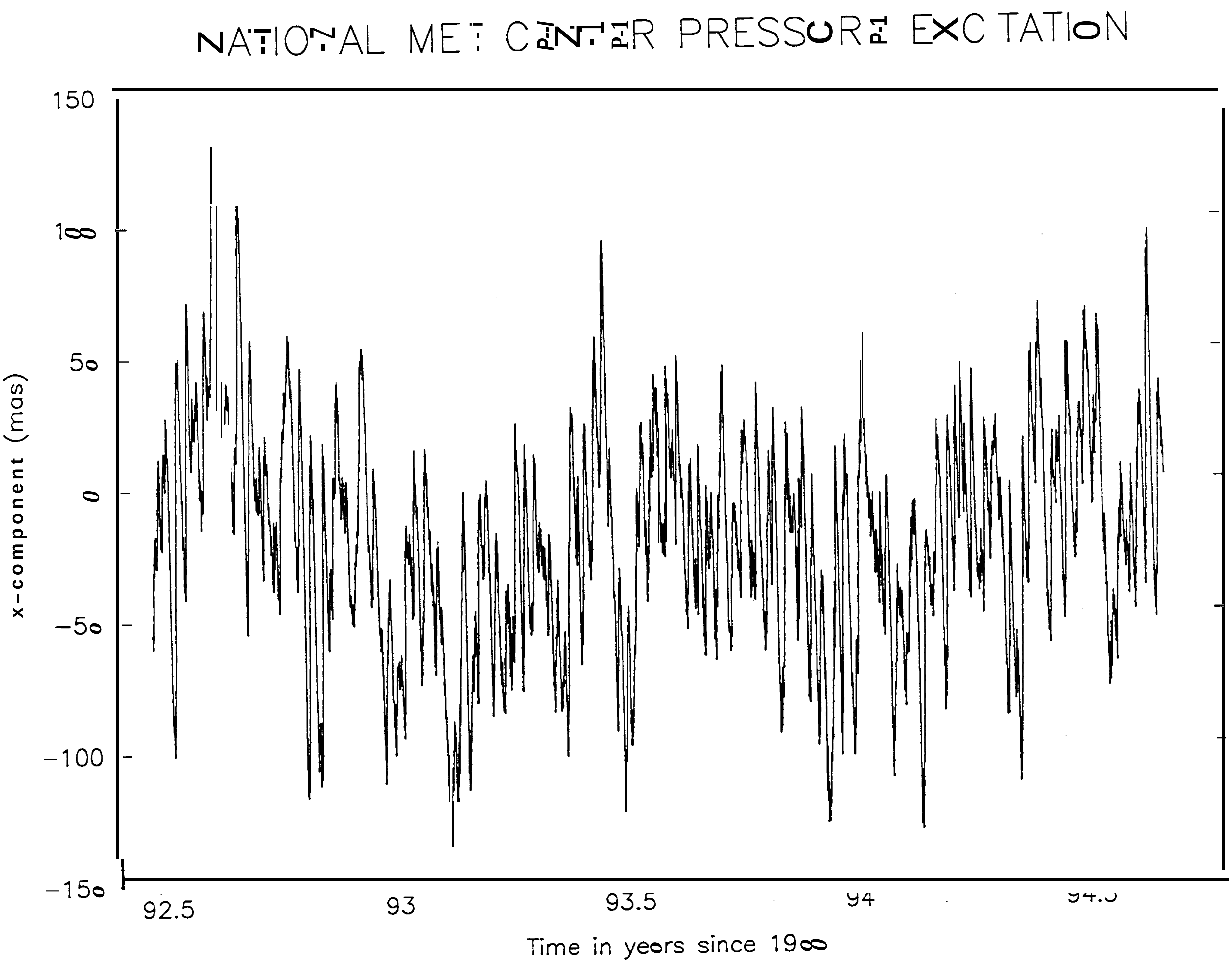
Case 3. Fit to data spanning 02JAN93 to 29AUG94 assuming FCN period = –429.8 solar days, Q = ∞ (Mathews *et al.*, 1991)
Case 4. Fit to data spanning 02JAN93 to 29AUG94 assuming FCN period = –434.1 sidereal days, Q = 53821 (Dehant *et al.*, 1994)

Prograde and retrograde polar motion amplitude and phase defined by: $p(t) = x_p(t) - i\,y_p(t) = A_p e^{i\alpha_p} e^{i\sigma t} + A_r e^{i\alpha_r} e^{-i\sigma t}$

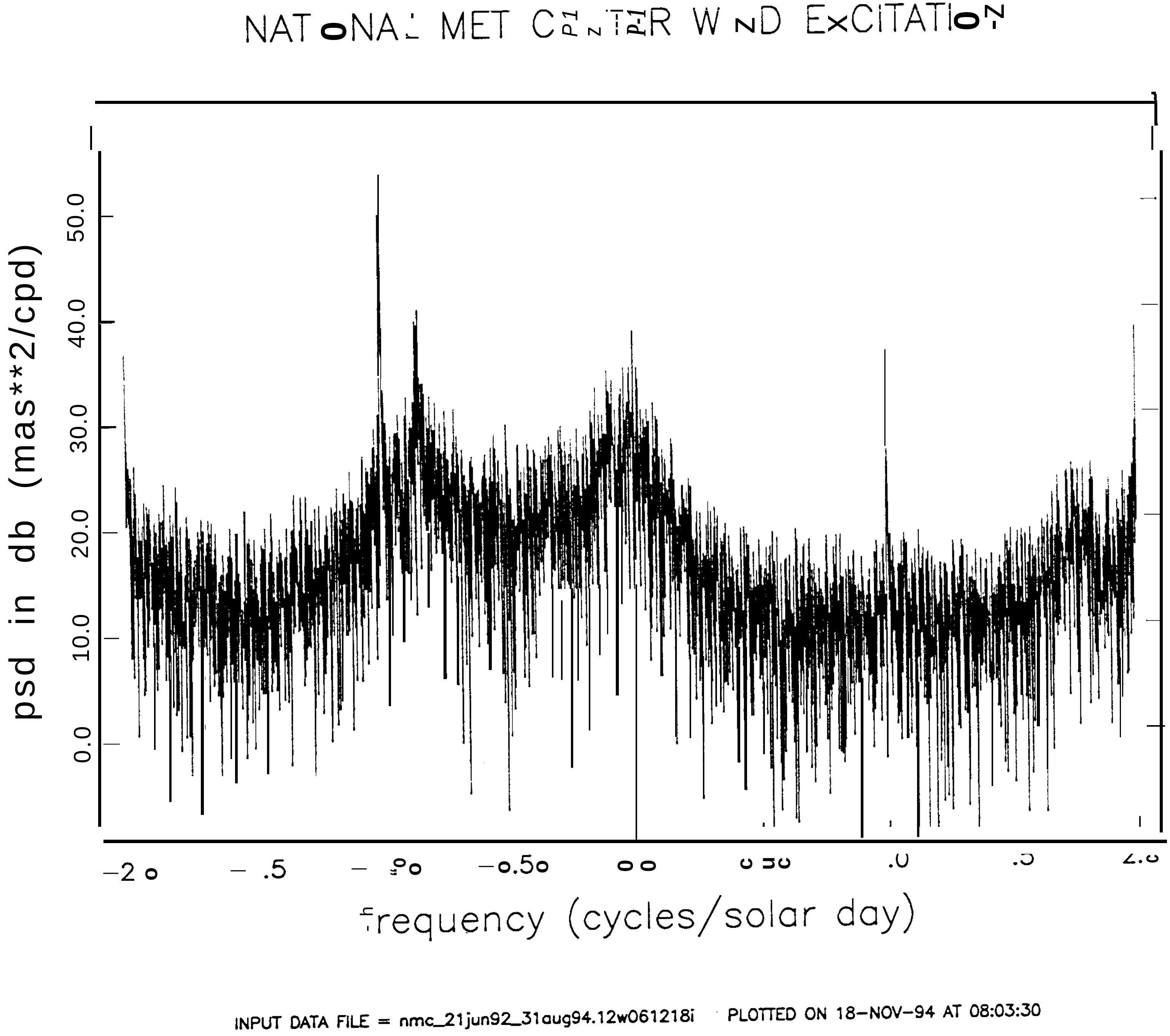
Amplitudes and phases tabulated above are the fitted retrograde nearly diurnal polar motion amplitudes and phases

Reference epoch for fit is J2000 Stated uncertainties are ± 1 sigma (68% confidence interval)


NATIONAL M C ER WND XC TAT ON
x-component (mas)
Time in years since 1900
92.5
93
93.5
94
94.5



NATIONAL MET CENTER PRESSURE EXCITATION
x-component (mas)
150
100
50
0
-50
-100
-150
92.5
93
93.5
94
Time in years since 1900


NATIONAL MET CENTER WIND EXCITATION
psd in db (mas**2/cpd)
0.0
10.0
20.0
30.0
40.0
50.0
−2.0
−.5
−0.50
0.0
.5
frequency (cycles/solar day)
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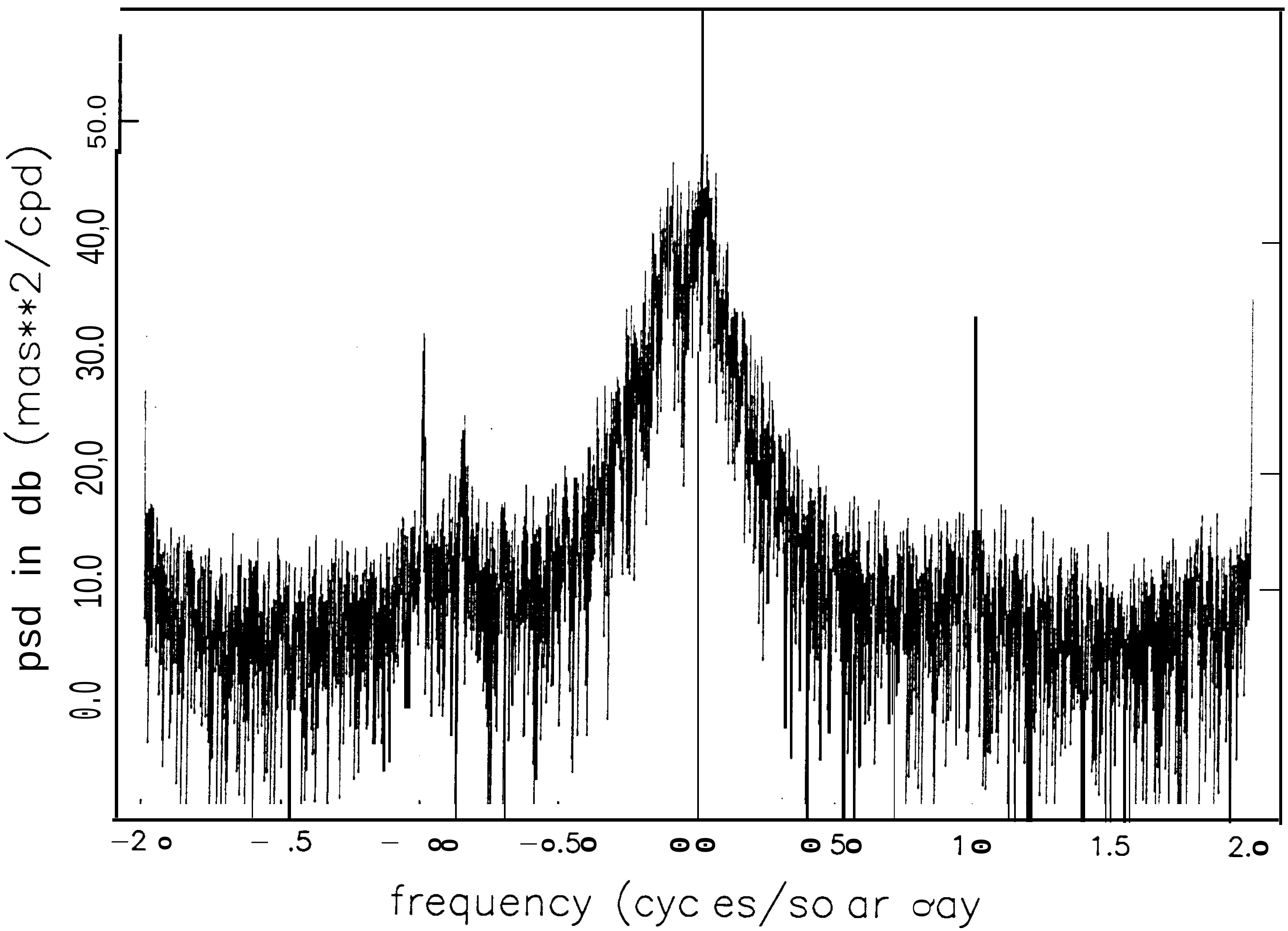
PLOTTED ON 18−NOV−94 AT 08:03:30

NATIONAL MET CENTER PRESSURE EXCITATION



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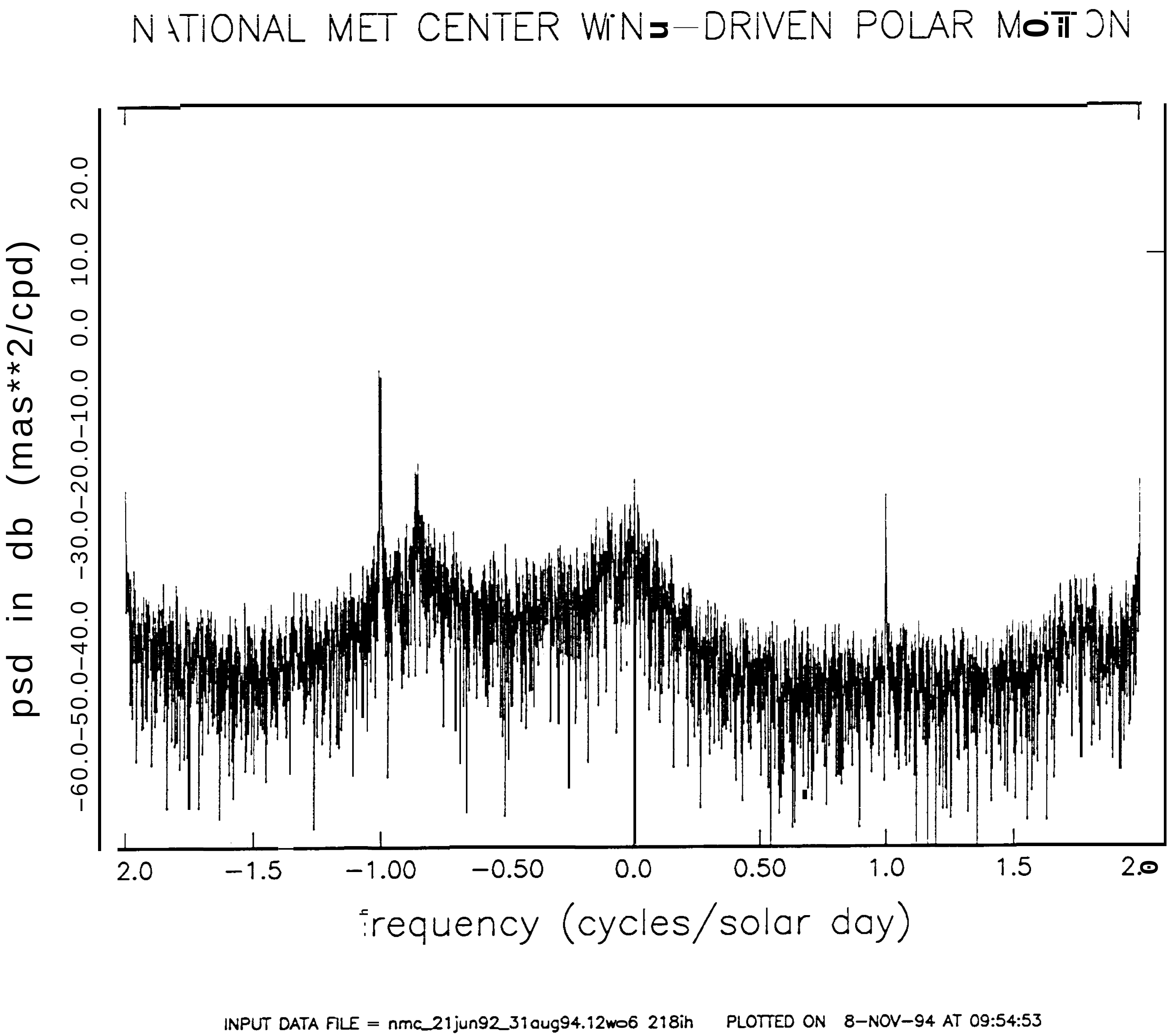
NATIONAL MET CENTER WIND-DRIVEN POLAR MOTION
psd in db (mas**2/cpd)
-60.0 -50.0 -40.0 -30.0 -20.0 -10.0 0.0 10.0 20.0
2.0 -1.5 -1.00 -0.50 0.0 0.50 1.0 1.5 2.0
frequency (cycles/solar day)
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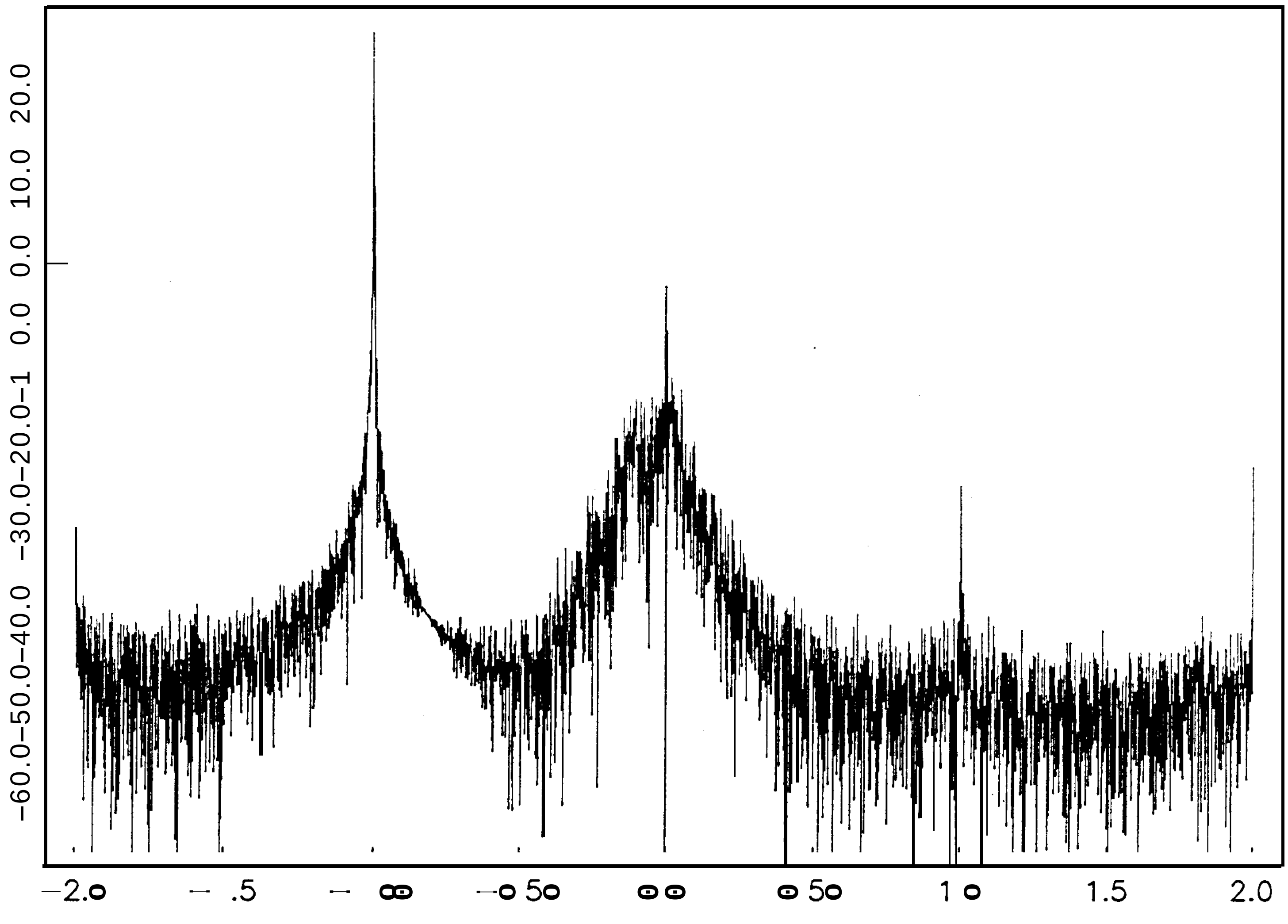
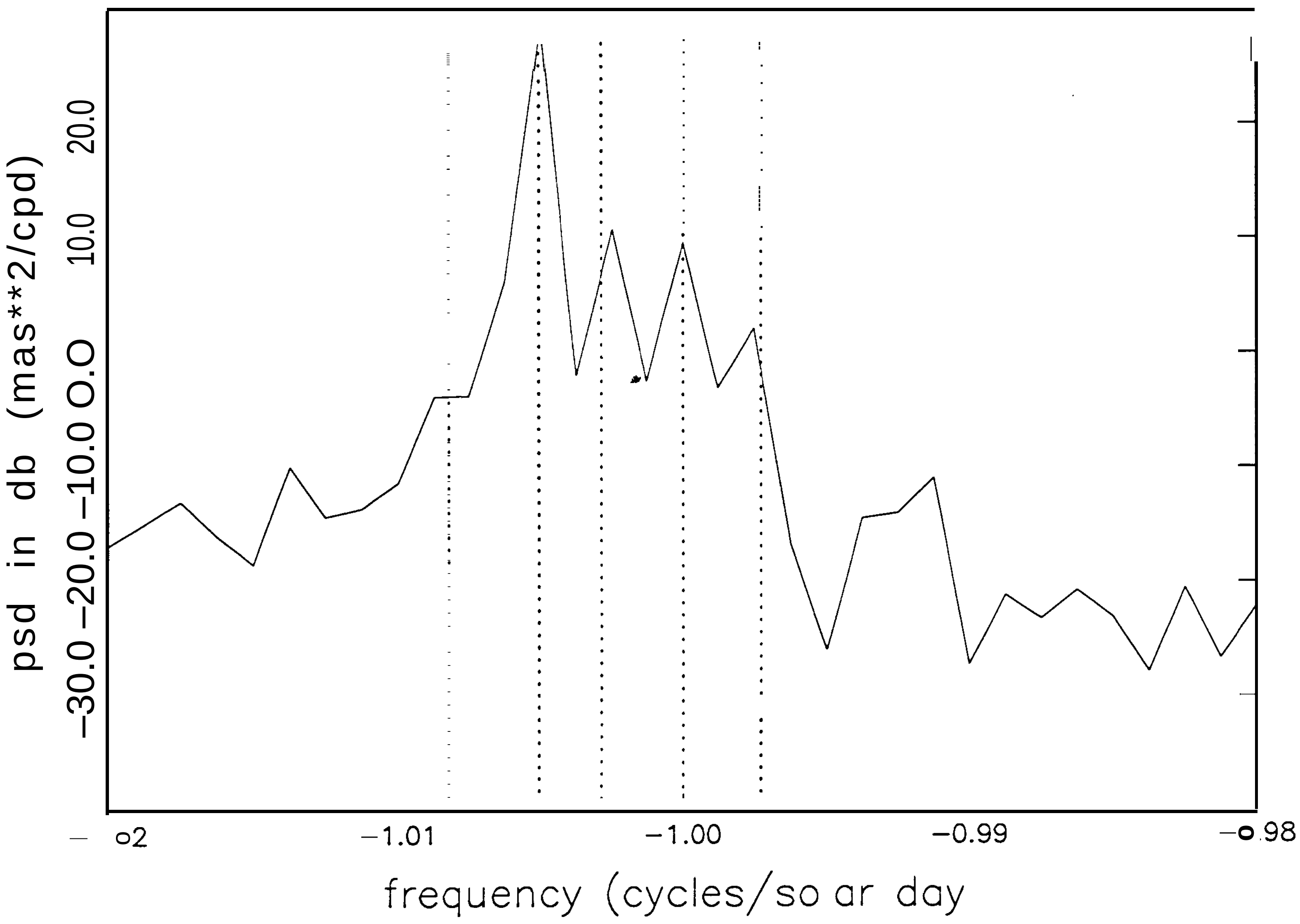
NATIONAL MET CENTER PRESSURE–DRIVEN POLAR MOTION



INPUT DATA FILE = nmc_21jun92_3 oug94.12ø06 218ih PLOTTED ON 18–NOV–94 AT 11:53: 6

NATIONAL MET CENTER (WIND+PRESSURE)-DRIVEN NUTATION



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NMC (WIND+PRESSURE)-DRIVEN POLAR MOTIONS AT NUTATION FREQUENCIES

Case	Retrograde semi-annual amp (mas)	Retrograde semi-annual phase (deg)	Retrograde FCN amp (mas)	Retrograde FCN phase (deg)	Retrograde 18.6 yr amp (mas)	Retrograde 18.6 yr phase (deg)	Prograde annual amp (mas)	Prograde annual phase (deg)	Prograde semi-annual amp (mas)	Prograde semi-annual phase (deg)
1.	0.0300 ± 0.0023	106.79 ± 4.31	1.0264 ± 0.0023	155.09 ± 0.13	0.1530 ± 0.0023	21.02 ± 0.85	0.1014 ± 0.0023	−71.58 ± 1.27	0.0415 ± 0.0022	148.03 ± 3.10
2	0.0300 ± 0.0020	106.56 ± 3.84	1.0304 ± 0.0020	169.27 ± 0.11	0.1463 ± 0.0020	21.07 ± 0.79	0.0989 ± 0.0020	−71.88 ± 1.16	0.[illegible]10 ± 0.[illegible]20	148.84 ± 2.79
3	0.0624 ± 0.0023	121.23 ± 2.11	0.5520 ± 0.0023	179.49 ± 0.24	0.1342 ± 0.0024	37.22 ± 1.02	0.1348 ± 0.0024	−68.51 ± 1.00	0.0258 ± 0.0023	122.78 ± 5.21
4	0.0597 ± 0.0022	120.07 ± 2.10	0.5538 ± 0.0022	−167.00 ± 0.23	0.1349 ± 0.0023	36.55 ± 0.96	0.1331 ± 0.0022	−69.06 ± 0.96	0.0259 ± 0.0022	125.62 ± 4.93

Case 1. Fit to data spanning 21JUN92 to 29AUG94 assuming FCN period = −429.8 solar days, Q = ∞ (Mathews *et al.*, 1991)
Case 2. Fit to data spanning 21JUN92 to 29AUG94 assuming FCN period = −434.1 sidereal days, Q = 53821 (Dehant *et al.*, 1994)

Case 3. Fit to data spanning 02JAN93 to 29AUG94 assuming FCN period = −429.8 solar days, Q = ∞ (Mathews *et al.*, 1991)
Case 4. Fit to data spanning 02JAN93 to 29AUG94 assuming FCN period = −434.1 sidereal days, Q = 53821 (Dehant *et al.*, 1994)

Prograde and retrograde polar motion amplitude and phase defined by: $p(t) \; = \; x_p(t) \; - \; i\, y_p(t) \; = \; A_p\, e^{i\,\alpha_p}\, e^{i\,\sigma\, t} \; + \; A_r\, e^{i\,\alpha_r}\, e^{\;i\,\sigma}$

Amplitudes and phases tabulated above are the fitted retrograde nearly diurnal polar motion amplitudes and phases

Reference epoch for fit is J2000 Stated uncertainties are ± 1 sigma (68% confidence interval)