

HIGH-PRECISION TIMING OF MILLISECOND PULSARS AND PRECISION ASTROMETRY

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Abstract.

We present the technique of long-term, high-precision timing of millisecond pulsars as applied to precision astrometry. We provide a tutorial on pulsars and pulsar timing, as well as up-to-date results of a long-term observations of two millisecond pulsars, PSR B1855+09 and PSR B1937+21. We consider the feasibility of tying the extragalactic and optical reference frames to that defined by solar system objects, and we conclude that precision astrometry from millisecond pulsar timing will continue to yield interesting results at an accelerating pace in the next decade.

1. Introduction

Pulsars are rapidly rotating, highly magnetized neutron stars that emit a beam of light that appears as a broad-band pulse of electromagnetic radiation once per rotation period. What is now referred to as the “slow” pulsar population includes over 600 pulsars with rotation periods in the range $33 \text{ ms} < P < 5 \text{ s}$. What is striking about most of these sources is that their rotation is extremely stable, owing to their large rotational inertia. Rotation periods with 12 significant digits are generally easy to

measure in just a few years of observations, and are typically limited only by measurement uncertainties. However, the discovery of the first millisecond pulsar by Backer *et al.* (1982) made the timing of the “slow” pulsar population seem like child’s play: the rotational period of the millisecond pulsar PSR B1937+21 has been measured with a fractional uncertainty of 10^{-15} (Kaspi, Taylor & Ryba 1994).

It is this rotational stability, combined with the Earth’s motion around the Sun, that allows high-precision astrometry to be accomplished through pulsar timing. In this paper, we review the technique of pulsar timing with an emphasis on astrometric applications. It must be noted, however, that astrometry is only one of many byproducts of timing observations. Other equally interesting and important fields addressed in high-precision timing include general relativity, cosmology, time-keeping metrology, interstellar medium physics, binary orbital evolution, as well as neutron star physics.

2. Fundamentals of Pulsar Timing

2.1. TIMING OBSERVATIONS

Although a handful of pulsars are strong enough to allow the detection of individual radio pulses, for the vast majority, the individual pulses are buried deep within the noise of the telescope. A single timing observation therefore consists of folding the digitized telescope signal modulo the expected topocentric pulse period in a sufficiently long integration to beat down the noise. This is particularly useful because a characteristic observational property of pulsars is that the coherent summation of many individual pulses always leads to the same signature, called an “average profile,” that is unique to that pulsar at that observing frequency. A fiducial point in the pulse profile, for example the pulse peak, therefore plausibly corresponds to a fixed point on the neutron star surface. Examples of two average profiles are given in Figure 1. The fiducial points chosen for the average profiles in Figure 1 are indicated by arrows. A single pulse time-of-arrival (TOA) consists of a reference epoch corresponding to the approximate half-way point of the integration, together with a fraction of the pulse period at which the fiducial point arrived.

Pulsar timing consists of obtaining a sequence of TOAs, spaced typically by several weeks, over the course of many months or years. If pulsar rotation periods as observed on Earth did not change (that is, if the Earth were not accelerating with respect to the pulsar and pulsars did not lose energy), the TOA sequence would be very simple: there would always be an integral number of periods between observations, with only very small deviations resulting from measurement errors. However, life is never simple, and before the stability of pulsar rotation can be observed, one must account for a

Figure 1. Average profiles for PSRs 111855+09 and B1937+21 at 1.4 and 2.4 GHz respectively. Arrows indicate the fiducial reference points for timing.

of systematic effects.

2.2. THE PULSAR REFERENCE FRAME

Even if one could observe from a reference frame not accelerating with respect to the pulsar, the observed period would change since the pulsar loses energy through magnetic dipole radiation. An approximate expression for the rotation phase is obtained by Taylor expansion:

$$\phi(t) = \phi(0) + \nu t + \frac{1}{2} \dot{\nu} t^2 + \frac{1}{6} \ddot{\nu} t^3 + \dots \quad (1)$$

where $\phi(t)$ is the rotation phase in cycles at time t , $\nu \equiv 1/P$ is the rotation frequency, and $\dot{\nu}$ and $\ddot{\nu}$ are the rotation frequency first and second derivatives. The coefficients in this expansion can be related to each other if some physical model for the rotation is assumed. For a rotating magnetic dipole moment M , classical electrodynamics says the total radiated power \mathcal{P} is given by

$$\mathcal{P} = \frac{2(\mathcal{M} \sin \alpha)^2 (2\pi\nu)^4}{3c^3}, \quad (2)$$

where α is the angle between the spin and dipole axes, and c is the speed of light. Setting the rate of loss of kinetic energy equal to the radiated power yields

$$\dot{\nu} = -\frac{8\pi^2 (\mathcal{M} \sin \alpha)^2}{3I c^3} \nu. \quad (3)$$

Pulsar timing allows the measurement of the parameters ν , $\dot{\nu}$, and occasionally $\ddot{\nu}$. These in turn can be used to estimate various other quantities of interest. For example, the characteristic spin-down age τ of a pulsar can

be estimated from $\tau = -\nu/2\dot{\nu}$, and the magnetic dipole moment M can be estimated via Equation 3. It can easily be shown that $\dot{\nu}$ from dipole radiation is negligible for most pulsars, including all millisecond pulsars.

2.3. THE OBSERVATORY REFERENCE FRAME

The accurate measurement of the intrinsic spin parameters ν and $\dot{\nu}$ depends crucially on the transformation to the neutron star's rest frame. If this is not done correctly, the spin parameters will be contaminated by the relative acceleration. A transformation to the solar system barycenter is most expedient and insufficient for all disk pulsars that are not in binaries (those in globular clusters suffer significant accelerations in the cluster potential that are difficult to model); the approximation that the barycenter is not accelerated with respect to the pulsar rest frame is more accurate for closer pulsars. An excellent review of the transformation is given in Backer and Hellings (1986). We briefly discuss the main points here.

An infinite frequency arrival time t_b at the solar system barycenter is specified by

$$t_b = t + \frac{\mathbf{r} \cdot \hat{\mathbf{n}}}{c} + \frac{(\mathbf{r} \cdot \hat{\mathbf{n}})^2 - |\mathbf{r}|^2}{2cd} - \frac{D}{f^2} + \Delta_{E(\odot)} - \Delta_{S(\odot)} \quad (4)$$

where t is the observed topocentric TOA, \mathbf{r} is a vector from the barycenter to the phase center of the telescope, and $\hat{\mathbf{n}}$ is a unit vector from the barycenter to the pulsar. The terms featuring the vector \mathbf{r} are also referred to as the solar system "Roemer delay." The observing frequency f is measured in the barycentric rest frame, and the dependence of the time delay on f^{-2} is a result of the ionized interstellar plasma. The "Einstein delay" term, $\Delta_{E(\odot)}$, is a combination of gravitational redshift and time dilation effects due to the motion of the Earth and other objects in the solar system. The "Shapiro delay" term, $\Delta_{S(\odot)}$, characterizes the general relativistic curvature of spacetime near the Sun. The correct calculation of all the above terms requires an accurate planetary ephemeris; we use the Jet Propulsion Laboratory's DE200 ephemeris (Standish 1982).

2.4. HANDLING BINARY PULSARS

For pulsars having binary companions, an additional transformation is required since it is the binary center of mass that has a small acceleration with respect to the solar system barycenter. A total of five Keplerian parameters are necessary to describe a binary orbit: the orbital period, the orbital eccentricity, the longitude of periastron, the epoch of periastron, and the projected semi-major axis of the pulsar orbit. In some pulsar binary systems, "Post-Keplerian" parameters, such as time-derivatives of the

Keplerian parameters, must also be included. Details of the parametrization using various relativistic binary models are summarized by Taylor and Weisberg (1989).

3. Pulsar Data and Precision Astrometry

What should be clear from Equation 4 is that the position of the pulsar plays a key role in the timing model. Imagine a pulsar in the ecliptic at some ecliptic longitude. If the TOAs from that pulsar are not corrected for Earth's motion, delays and advances will be observed such that the differences between the model-predicted arrival times and the observed arrival times, that is, the "residuals," will contain a systematic sine wave of period 1 yr with a phase that depends on the longitude. The amplitude of that sine wave will be about 480 s (corresponding to the Earth-Sun distance), much larger than all known pulse periods. By fitting out this sine wave, a very accurate ecliptic longitude is obtained. If the pulsar is not exactly in the ecliptic, the sine wave amplitude is reduced, and the fit determines the ecliptic latitude as well. Small amplitude 1-yr sine waves in residuals can be used to fine tune the fit position.

This measurement can be done with unprecedented precision: if the input pulsar position is wrong by $\sim 1''$, the annual sinusoid has amplitude ~ 1 ms, so for pulsars with ~ 0.4 ms uncertainties of a few ms, the position can be known ideally to within a few arcseconds after only one year, with the uncertainty decreasing as the square root of the length of the data span. For millisecond pulsars, typical timing uncertainties are three orders of magnitude smaller, so far greater accuracy is easily achieved. For example, after 9 yr of timing PSR B1937+21 with individual TOA uncertainties of $\sim 0.2 \mu\text{s}$, the position uncertainty is around $50 \mu\text{arcsec}$ in the ICRF reference frame defined by the DE200 planetary ephemeris.

If the pulsar has a substantial proper motion, the result in the residuals will be a sine wave of linearly increasing amplitude. At a distance of 1 kpc, a pulsar moving with typical transverse velocity 200 km s^{-1} has proper motion 40 mas yr^{-1} . Thus, it is quite difficult to detect proper motions of slow pulsars from timing measurements, however those of millisecond pulsars are easily obtained. Because pulsars are effectively point sources, pulsar proper motions are also measurable using interferometric techniques (e.g. Harrison, Lyne and Anderson 1993).

Also, if the pulsar is close enough, a moment's thought can convince you that geometric parallax, which has its greatest effect when the Earth-Sun line is perpendicular to the pulsar-Sun line, that is twice per year, results in a sine wave of period 0.5 yr. (This effect is precisely described by the second Roemer term in Eq. 4). The parallax effect is tiny even by pulsar standards:

Figure 2. Post-fit residuals for PSR B1855+09 at 1.4 GHz.

for a pulsar in the plane of the ecliptic at a distance of 1 kpc, the amplitude of the parallax signal π is only $1.2 \mu\text{s}$. A measurement of π is quite valuable however, since it allows a direct, geometrical determination of the distance d to a pulsar, via $\pi = 1/d$. A timing parallax has been measured for only one pulsar, PSR B1855+09, discussed below. Pulsar parallaxes can also be measured directly using VLBI (Gwinn *et al.* 1986).

4. Recent Results from Arecibo

We now describe recent results from a long-term, high precision millisecond pulsar timing project, done at the 305-m dish at Arecibo, Puerto Rico. Bi-weekly timing observations for two millisecond pulsars, PSRs B1855+09 and B1937+21, are described in detail in Kaspi, Taylor and Ryba (1994) (hereafter KTR94). High-precision timing of PSR B1855+09 was begun in 1986 at Arecibo at 1.4 GHz, and has yielded daily-averaged arrival time uncertainties of $\sim 1 \mu\text{s}$. Data for PSR B1937+21 goes back to 1981 at 1.4 and 2.4 GHz, with daily-averaged TOA uncertainties of $\sim 0.2 \mu\text{s}$. The data sets analyzed by KTR94 include arrival times obtained through the end of 1992, although here we include data through the end of 1993. All TOAs in the analysis are referred to UTC.

PSR B1855+09 is a 5.4 ms pulsar in a 12 day circular binary orbit with a white dwarf. The pulsar's celestial coordinates and proper motion are determined with uncertainties of 0.13 mas and 0.07 mas yr^{-1} respectively, in the reference frame defined by the JPL DE200 planetary ephemeris. PSR B1855+09 is the only pulsar for which a significant measurement of timing parallax has been made. The pulsar's astrometric parameters are listed in Table 1. The residuals after removal of the best spin, astrometric and binary parameters are shown in Figure 2. The residuals are clearly

dominated by random, Gaussian measurement uncertainties which indicate that the 111 model describes the data well. The slight hint of a cubic signal is apparent if the plot is held at a distance; this will be discussed below.

TABLE 1. Astrometric Parameters for PSRs B1855+09 and B1937+21

	PSR B1855+09	PSR B1937+21
α (J2000)	18 ^h 57 ^m 36 ^s .393515(4)	19 ^h 39 ^m 3 ^s .5(0211 (2)
δ (J2000)	+09 [°] 13'17".32(70(1 2)	+21 [°] 34'59".14170(4)
μ_α (mas yr ⁻¹)	-2.91(4)	-0.131(8)
μ_δ (mas yr ⁻¹)	-5.48(6)	-0.45(9)
π (mas)	1.0(3)	<0.20

PSR B1937+21 was the first discovered and is still the fastest known millisecond pulsar, having $P = 1.5$ ms. It shows no evidence for a binary companion. 111,200 celestial coordinates and proper motion were measured with uncertainties of 0.05 mas and 0.01 mas yr⁻¹, and the annual parallax was found to be $\pi < 0.20$ mas. The best fit astrometric parameters are shown in Table 1, and the residuals after subtraction of the best model including astrometric and spin parameters are shown in Figure 3. There is an obvious cubic trend in the residuals that indicates our model does not completely model the rotation of the neutron star. KTR94 address this issue in detail, and demonstrate that although there are several possible origins of the "noise," including planetary ephemeris errors, a primordial background of gravitational waves, clock errors, or some poorly understood interstellar propagation phenomenon, the most likely origin is intrinsic to the pulsar itself. The lower-level cubic of opposite sign in the residuals for PSR B1855+09 support this argument.

It should be noted that we have minimized the effect these deviations from the simple spin-down model have on the astrometric parameters for PSR B1855+09 and especially PSR B1937+21 by "pre-whitening," that is, by including sufficiently many higher order frequency derivatives to render the residuals random when determining the astrometric parameters.

5. Frame Ties and the Future

The high precision astrometry available through millisecond pulsar timing is particularly valuable in tying the various astronomical reference frames. An interferometric position for a pulsar together with a timing position, for example, tie the extragalactic reference frame with that defined by solar

Figure 3. Post-fit residuals for PSR B1937+21 at 2.4 GHz.

system objects. At 400 MHz, PSR B1937+21 is a 240 mJy source and so is easily observable with VLBI techniques. Bartel *et al.* (1990) have observed the pulsar and obtained a VLBI position suitable for a frame tie. PSR B1855+09 by contrast is only 31 mJy at 400 MHz, and so is not well-suited to this technique.

An optical detection of a pulsar or, more likely, of the companions to binary millisecond pulsars, can in principle provide a tie with the optical reference frame. Although under 10% of the slow pulsar population are in binary systems, well over 75% of disk millisecond pulsars are in binaries, with the majority having a potentially optically observable companion. The detection of optical companions to millisecond pulsars is also interesting from an evolutionary point of view (Kulkarni *et al.* 1991). The advent of HST and other sensitive instruments such as the Keck Telescope will be a great boon to the detection of even very faint companions, as well as to improved astrometry for brighter sources.

One newly discovered millisecond pulsar deserves notice. PSR J0437-4715, discovered by Johnston *et al.* (1993), is a 5.7 ms pulsar in a 5 day circular binary orbit with a white dwarf. It is a very bright 600 mJy source at 400 MHz, and is also one of the closest pulsars to the Earth, with an estimated distance of only 140 pc. The source will be easily detectable by VLBI, and furthermore, the pulsar's companion has already been detected optically by Danziger *et al.* (1993). After only two years of timing, the timing position and proper motion have been measured with uncertainties of 4 mas and 3 mas yr^{-1} respectively (Bell *et al.* 1995).

The number of millisecond pulsar discoveries has sky-rocketed in recent years (see Fig. 4) owing principally to faster computers. There is every reason to believe that millisecond pulsar timing will be a booming industry in the years to come, and that precise positions and proper motions will

Figure 4. Number of known disk millisecond pulsars versus year

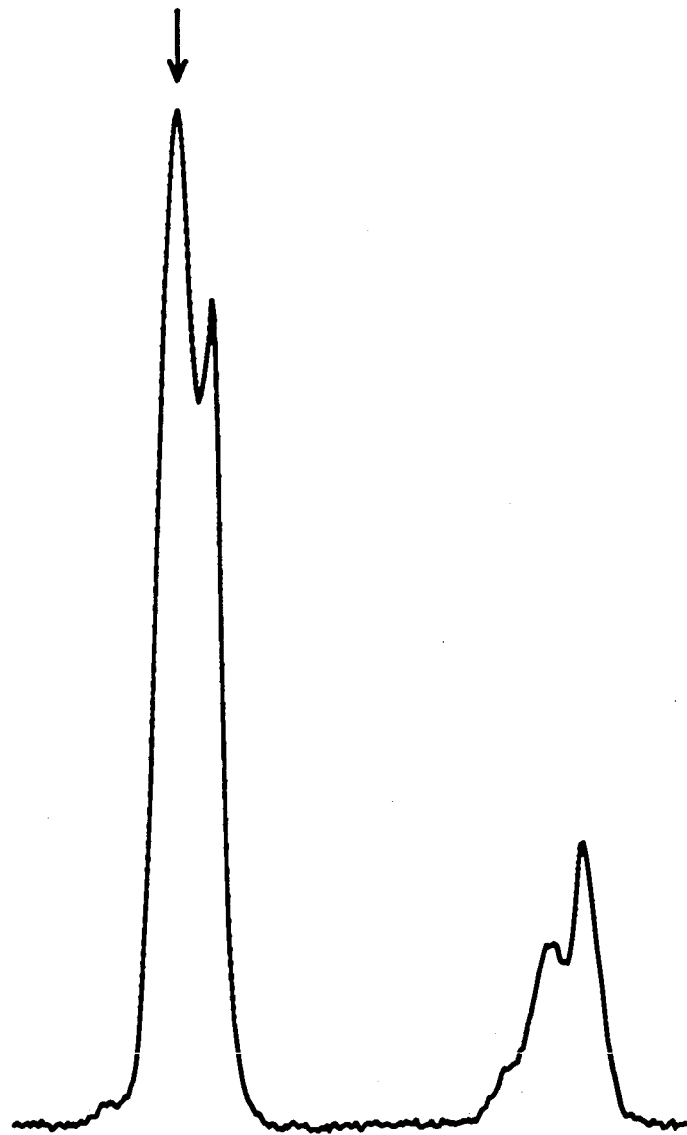
be available in the next decade for a large number of sources. Furthermore, as more millisecond pulsars are discovered, it is becoming clear that they are distributed isotropically on the sky, and so the possibility of a complete tie of optical and extragalactic reference frames to the dynamical reference frame from observations of millisecond pulsars will soon be within reach.

Acknowledgements

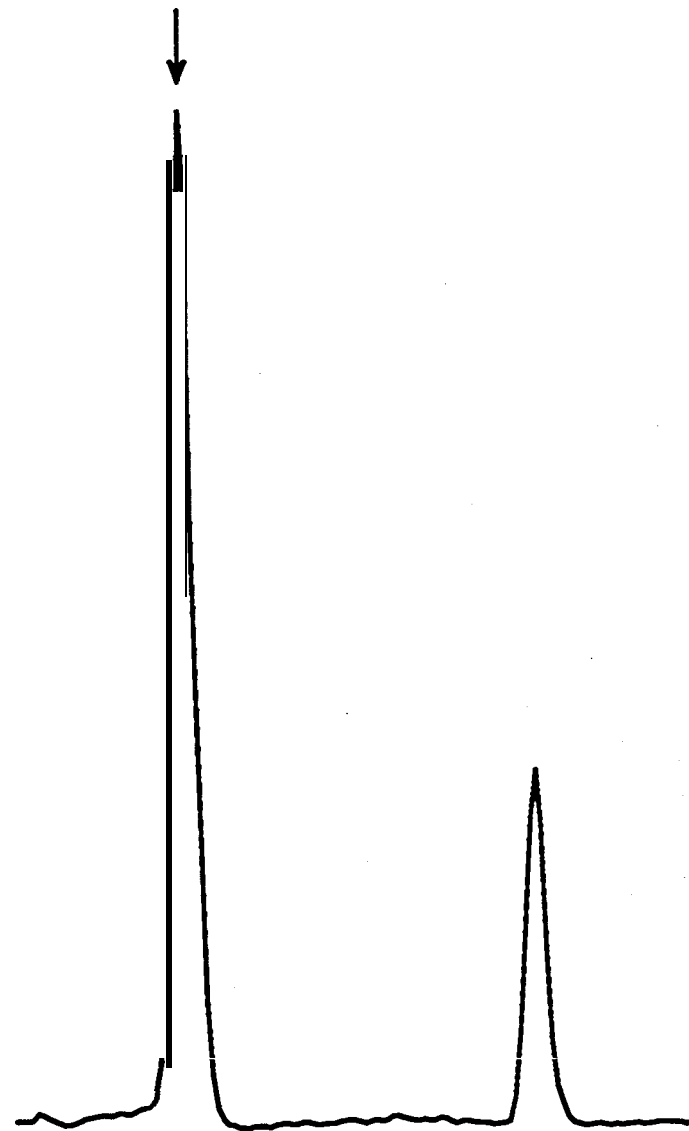
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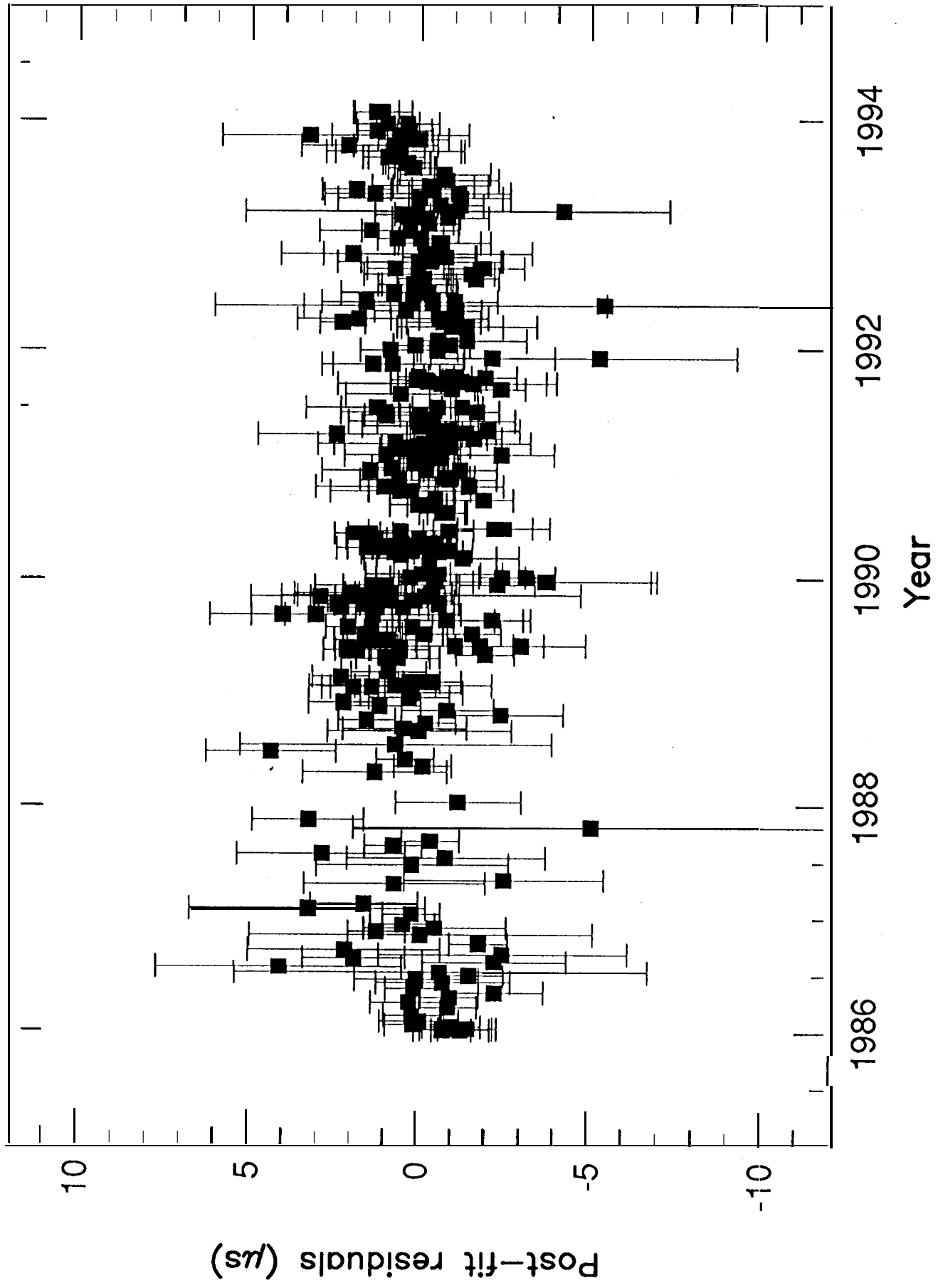


PSR B1855+09 at 1.4 GHz

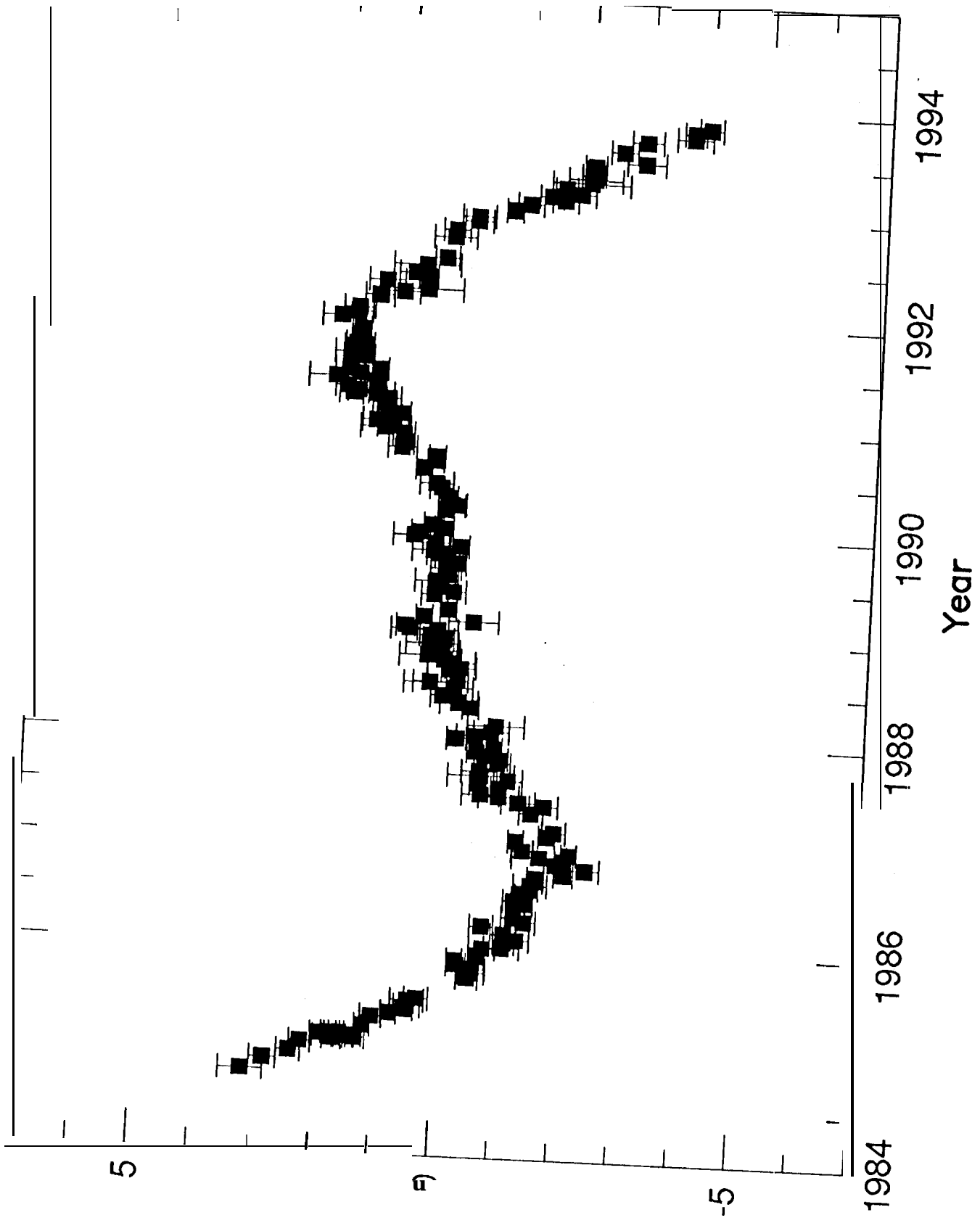


PSR B1937+21 at 2.4 GHz

2



Post-fit residuals (μs)



1994

Number of Known Disk Millisecond Pulsars

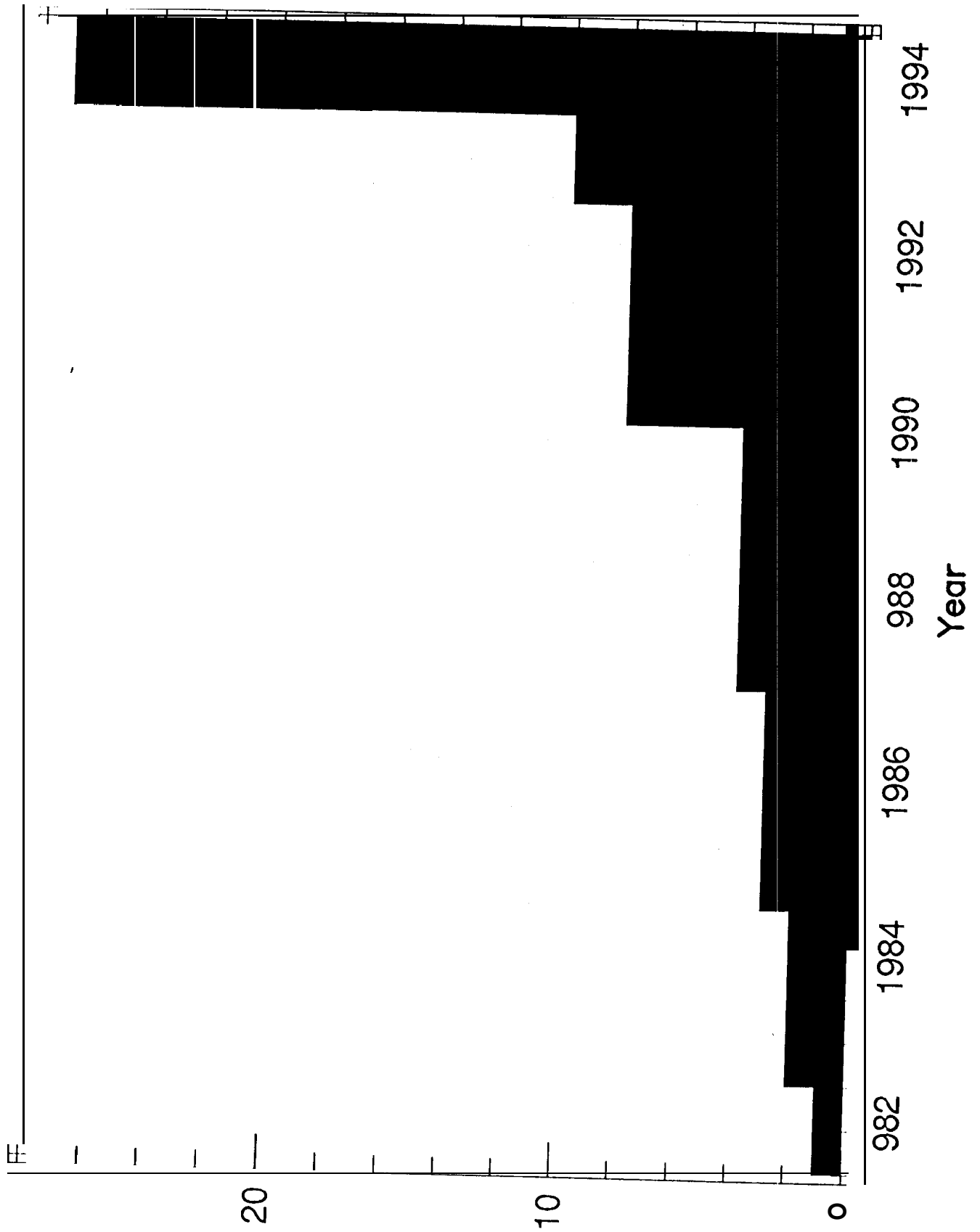


Fig 1