LINJURATIONS ON THE SUBCARRIER FREQUENCY-TO-SYMBOL RATE RATIO

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ABSTRACT

For space missions in which tracking the residual carrier is absolutely critical, the use of a subcarrier to separate the data from the residual carrier is the best way to maximize residual-carrier tracking performance. In exchange for a better tracking performance, this method consumes greater bandwidth than required without a subcarrier. In this paper, the residual-carrier tracking performance for PCM/PSK/PM/NRZ for square wave and sine wave subcarriers will be determined as a function of the subcarrier-to-symbol rate ratio. Explicitly, the RF residual carrier tracking performance in terms of the symbol SNR degradation will be quantified. The upper and lower bound values for the subcarrier-to-symbol rate ratio will also be derived, and for which they can be used to determine the minimum required bandwidth for maximum residual-carrier tracking performance. The analytical results obtained from the mathematical models are validated by computer simulation.
1. INTRODUCTION

Currently, most space missions operate at relatively low data rates using residual carrier modulation, thus necessitating the separation of the data from the residual carrier to avoid interference. This can be achieved by placing the data modulation on a subcarrier, since direct modulation of the data on the carrier would cause most of the data power to fall within the bandwidth of the carrier Phase-1 locked-1 mop (PL1) and, as a consequence, interferes with its operation [1, chap. 3]. The scheme in which the data is Phase-Shift Keyed (PSK) onto a subcarrier and then base-hold-ligated (PM) onto a sinusoidal carrier is called PCM/PSK/PM modulation scheme. Furthermore, the International Consultative Committee for Space Data Systems (CCSDS) has recommended that the square wave subcarrier should be used for the deep space missions and sine wave subcarrier for the near earth missions [3].

The performance evaluation of the PCM/PSK/PM for both square wave and sine wave subcarriers has been evaluated and became textbook material [1, 4 and 7]. However, the analyses presented in [4 and 7] have always assumed that the subcarrier frequency-to-symbol rate, n, is sufficiently high enough that the interaction between the RF residual carrier and data is negligible. Recently, [2] has calculated the minimum allowable n for PCM/PSK/PM with high symbol rate and square wave subcarriers only.

The purpose of this paper is to determine the minimum (lower bound) n and to present a simplified approximate mathematical model to evaluate the upper bound of n for PCM/PSK/PM with both square wave and sine wave subcarriers. Using the available results [1-7], this paper will quantify the lower bound of n in terms of the Symbol Signal-to-Noise Ratio (SSNR) degradation and the upper bound of n in terms of power Containment in the predetection filter bandwidth. The paper is organized as follows: A brief description of the residual carrier modulation scheme...
with associated Power Spectral Density (PSD) is presented in Section 2. The performance of the PCM/PSK/PM system as a function of \( n \) and the derivation of the bounds for \( n \) are shown in Section 3 and 4, respectively. A brief description of the computer simulation to verify the theory derived in Sections 3 and 4 is presented in Section 5. Numerical results are shown in Section 6 and general discussions and conclusions are presented in Section 7.

**II. TELEMETRY FORMAT**

Telemetry format which employs PCM/PSK/PM modulation scheme with either a sine wave or a square wave subcarrier can be represented mathematically as [1, 4 and 7]

\[
S_{T}(t) = \sqrt{2P_{t}} \cos (\omega_{c}t + m d(t) p(t))
\]

where \( P_{t} \) is the total transmitted power, \( m \) is the modulation index, \( d(t) \) is the non-return-to-zero data format (NRZ), and \( p(t) \) is either a sine wave or a square wave subcarrier as shown in Eqs. (1a) and (1b) respectively.

\[
p(t) = \sqrt{2P_{t}} \cos (\omega_{c}t + \theta_{PC})
\]

\[
p(t) = \sqrt{2P_{t}} \sin (\omega_{c}t + \theta_{PC})
\]

For \( p(t) = \text{sqr}(\cdot) \), one has PCM/PSK/PM/Square-wave. Using simple trigonometric expansions, Eqs. (1) and (1a) can be expressed as a sum of the residual carrier component and the data component. Explicitly,

\[
S_{T, \text{SQUARE}}(t) = \sqrt{2P_{t}} \left\{ \cos (m \sin (\omega_{c}t) + d(t)p(t)) \sin (m \cos (\omega_{c}t)) \right\}
\]

where the first term is the carrier component and the second term is the data component. In Eqn. (2), the modulation index, \( m \), allocates the total transmitted power, \( P_{t} \), between the residual carrier and the data. When \( p(t) = \text{sin}(\cdot) \), one has PCM/PSK/PM/Sin e-wave, the expansion on
Eqns (1) and (1b) yields

\[
\begin{align*}
    s_{T, SINE}(t) &= \sqrt{2P_2}\sin(\omega_c t) \left[ J_0(m) + \sum_{n=2, \text{even}}^{\infty} 2J_n(m) \cos(Im\cdot t) \right] \\
    &+ \sqrt{2P_2}d(t) \cos(\omega_c t) \sum_{n=1, \text{odd}}^{\infty} 2J_n(m) \sin(n\omega_{ec} t) 
\end{align*}
\]  

(3)

where \( J_n(m) \) is the Bessel function of the first kind with order \( n \). As before, the modulation index, \( m \), divides the total power, \( P_1 \), between the carrier and the data. From Eqn. (2), the power spectral density (PSD) for PCM/PSK/PM/Square-wave subcarrier is found to be [5]

\[
\begin{align*}
    s_{SQU, KF}(f) &= P_2 \cos^2(m) \delta(f - f_c) \\
    &+ \frac{4}{\pi^2} \sin^2(m) \sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} \left[ S_p(f - f_c - (2k-1)f_{ec}) + S_p(f - f_c + (2k-1)f_{ec}) \right] 
\end{align*}
\]  

(4)

and the PSD for PCM/PSK/PM/Sine-wave is given by:

\[
\begin{align*}
    S_{P1, \text{sine}}(f) &= P_2J_0^2(m) \delta(f - f_c) \\
    &+ \sum_{k=1}^{\infty} J_k^2(m) \left[ S_p(f - f_c - (2k)f_{ec}) + S_p(f - f_c + (2k)f_{ec}) \right] \\
    &+ \sum_{k=1}^{\infty} J_k^2(m) \left[ S_p(f - f_c - (2k-1)f_{ec}) + S_p(f - f_c + (2k-1)f_{ec}) \right] 
\end{align*}
\]  

(5)

where \( S_p(\cdot) \) is the spectrum density for the NRZ data format which is defined as [1]

\[
S_p(f) = \frac{1}{R_g} \frac{\sin^2(\pi \frac{f}{R_g})}{(\pi \frac{f}{R_g})^2} 
\]  

(6)
From Eqn. (4) and (5), the residual carrier power and the data power can be obtained directly as shown respectively for the square wave subcarrier case,

\[ P_c = P_\gamma \cos^2 (m) \]
\[ P_d = P_\gamma \sin^2 (m) \]  \hspace{1cm} (7)

and for the sine wave subcarrier case,

\[ P_c = P_\gamma j_0^2 (m) \]
\[ P_d = 2P_\gamma j_1^2 (m) \]  \hspace{1cm} (8)

The PSD functions defined in Eqns. (4), (5) and (6) are plotted in Figs. 1 (a), 1 (b), and (1c) respectively. For clarity sake, only the positive frequency and the first two harmonics are shown in figures 1(a) and 1(b). Equations (4) and (5) are symmetric functions, hence their PSDs for negative and positive frequencies are mirror of each other.

**111. PERFORMANCE ANALYSIS**

At the ground station receiver, a PI,1, is employed to track the residual carrier in the incoming signal. The performance of the PI,1, depends on how much data power is present inside the carrier tracking loop bandwidth which interferes with the carrier. The amount of interference from the data depends on the subcarrier frequency-to-symbol rate ratio, \( n \). The shaded region in Fig. 1 illustrates this effect. Interference of this type causes the PI,1, to provide imperfect reference to the tracking of residual carrier, which can be modelled mathematically as

\[ C(n, t) = \sqrt{2P_\gamma \cos(\omega_c t + \phi(n, t))} \]  \hspace{1cm} (9)

where \( \phi(n, t) \) denotes the carrier tracking phase errors induced by the data interference, and \( n \) is the subcarrier-to-symbol rate ratio which impacts the phase error. The performance of the latter has been
assessed in the past [1, 2, 4 and 7]. It degrades the Symbol Error Rate (SER) performance. Clearly, as one varies \( n \), or vary the separation between the carrier and the data, one is also varying the performance of residual-carrier tracking. One seeks, therefore, the minimum value of \( n \) such that the degradation is minimal. By analyzing the carrier tracking performance and then determining the SER performance one can determine the lower bound \( n_i \). The theory presented in [1-2] will be used to determine the lower bound \( n_i \). For PCM/PSK/PM/Square-wave with low data rate and \( n_i \); higher order harmonics (or spikes) of the carrier can fall within the carrier tracking loop bandwidth and cause potential CW interference. For this case, analytical results will be verified by computer simulation.

The carrier tracking performance can be characterized by the tracking variance, \( \sigma_n^2 \), which is given by [2]:

\[
\sigma_n^2 = \frac{I_C R}{\rho_o}
\]

where \( \rho_o \) is the ideal PLL loop SNR (which) is defined in [2] for a square, wave and sine wave subcarriers respectively as

\[
\rho_o = \frac{P_T \cos^2 (m)}{N_o \cdot B_L} \quad (1.1)
\]

\[
\rho_o = \frac{P_T \cdot \sigma_o (m)^2}{N_o \cdot B_L} \quad (1.2)
\]

and ICR is the interference-power-to-carrier-power ratio and will be derive in the following section.

### III. EVAIUTING ICR

This section derives the ICR parameters for both PCM/PSK/PM/Square-wave and PCM/PSK/PM/Sine-wave cases. The derivation assumes that the interference spectrum is smooth.
over the carrier tracking bandwidth, hence it can be characterized as a white noise process. For the case when the interference spectrum is not smooth, computer simulation will be used in analyzing the performance of the system.

111.1.1 SQUAREWAVE CASE.

As the name suggested, ICR is the interference power that falls inside the carrier tracking loop bandwidth to the eariler power ratio. Explicitly, for square wave subcarrier case ICR is given by [2]:

\[
ICR_{\text{SQUARE}} = \tan^2(m) S_D \int_{-\infty}^{\infty} S_{\text{SQUARE}}^2(f) |\eta(f)|^2 df
\]  

where \( S_D, S_{\text{SQUARE}} \) can be found from Eq. (4), and it is given by:

\[
S_D, S_{\text{SQUARE}}(f) = 2 \sum_{k=1}^{\infty} \left[ \left( 2k - 1 \right) f_{ec} + S_D \left( f - \left( 2k - 1 \right) f_{ec} \right) \right] \tag{14}
\]

and \( |\eta(f)|^2 \) is the transfer function of the tracking loop filter given by [1]:

\[
|\eta(f)|^2 = \frac{1 + 2 \left( \frac{f}{f_n} \right)^2}{1 + 4 \left( \frac{f}{f_n} \right)^4}
\]  

where \( f_n \) is the natural frequency of the loop filter given by [1]

\[
f_n = \frac{B_f}{\pi \left( \xi + \frac{3}{4} \xi \right)}
\]  

where \( \xi \) is the damping factor, and it is assured to be 0.707.
11.1.2 SINEWAVE CASE

For modulation with sine-wave subcarrier, ICR then becomes

\[ 1 \text{CR}_{SINE} = \frac{1}{J_0^2(m)} \int_{w}^{\infty} S_{P_s, SINE}(f) |H(f)|^2 df \]  

(17)

where \( J_0(m) \) is the Bessel function, \( S_{P_s, SINE} \) is derived from Eqn. (5), and it is given by:

\[ S_{P_s, SINE} = \sum_{k=0}^{\infty} J_{2k+1}^2(m) S_{P_s}(f - (2k + 1) f_{sc}) \ ] \[ S_{P_s}(f + (2k + 1) f_{sc}) \]  

(18)

and \( H(f) \) is defined by Eqn. (15), Eqn. (17) is valid as long as \( 0 < B_t < 3f_{sc} \). As illustrated in Fig. (1b), for \( B_t > 3f_{sc} \), higher order of the carrier harmonics (see Figure 2) can fall into the tracking loop bandwidth and can caused (W interference.

11.2 DETERMINING SYMBOL ERROR IN A’I’}{ (SER)

ideally, when there is no interference and the P] \( \text{loop SNR is infinite, the SER} \) (or Pe) is computed from [1]

\[ Pe_{1PIFM} = \frac{1}{2} \text{C}r fC \left( \frac{E_s}{N_0} \right) \]  

(19)

where \( E_s/N_0 \) is denoted as the symbol SNR and it is given by [1] for the square wave and sine wave subcarriers respectively as

\[ \frac{E_s}{N_0} = \frac{P_T}{N_0} S_{\text{SINE}}^2(m) \]  

(20)

\[ \frac{E_s}{N_0} = \frac{P_T}{N_0} \frac{2}{R_s} \]  

(21)
in practice, the PLL's loop SNR is not infinite (especially when data interferes with the carrier) and 

Eqn. (19) becomes

\[ P_C = \int \sqrt{\frac{E_s}{N_0} \cos \phi} \, d\phi \]  

(22)

where \( \phi \) denotes the phase error of the VCO in the PLL. The probability density function of the 
phase error is approximated by a Tikhonov density

\[ P(\phi) = 2\pi e^{-\rho \phi^2} \quad \phi \in [-\pi, \pi] \]  

(23)

where \( \rho \) is the effective loop SNR which is computed from Eqn. (10) to be

\[ \rho = \frac{1}{Q^2} \frac{1}{\rho_c} + \frac{1}{4} TCR \]  

(24)

Note that Eqn. (22) represents a mathematical model to calculate the Symbol Error Rate (SER) for the case when the data rate is high with respect to the receiver tracking loop bandwidth. For high 
data rate case, the phase error process \( \phi(n, t) \) varies slowly and is essentially constant over a symbol 
interval \( T = 1 / R_s \). However, when the data rate is low relative to the receiver tracking loop 
bandwidth, the phase process is no longer constant over the symbol interval, it varies rather fast over 
the symbol interval \( T \) and the SER can be better be approximated by [1]:

\[ P_C \approx \frac{1}{2} \int \sqrt{\frac{E_s}{N_0} \cos \phi} \, d\phi \]  

(25)
where

\[ E[\cos(\phi)] = \int \cos(\phi) p(\phi) d\phi \]  

Once the SER is determined, one can quantify the carrier tracking performance by determining the symbol SNR degradation. For a fixed SER, the symbol SNR degradation can be determined by subtracting the symbol SNR obtained from Eqn (19) for ideal case (i.e., corresponding to \( n \cdot f_{sc} / R_s = \infty \) and infinite loop SNR) from the corresponding symbol SNR obtained from Eqn (22) for non-ideal case, (i.e., with data interference and infinite loop SNR.) The difference in the symbol SNR is the degradation due to imperfect residual carrier tracking caused by data interference. Examples will be shown in the numerical results section.

IV. DETERMINING THE SUBCARRIER-TO-SYMBOL RATE RATIO BOUNDS

Using the results presented in the last two sections, one can determine the minimum value of the subcarrier-to-symbol rate ratio, \( n_1 \), to meet a specific SNR degradation. This can be done by computing the SNR using Eqn. (22) or (25) with \( n \) as parameter. Unfortunately, as one moves the subcarrier further away from the carrier (increasing \( n \)) one is also pushing the data spectrum out of the bandwidth of the predetection filter as shown in Fig. 2. This filter is usually located in front of the I’1.1, or at the first IF stage of the receiver and band limits the incoming signal. Hence, with the knowledge of the required minimum data power containment within the predetection filter bandwidth, the maximum allowable value of \( n \) can be determined. The total Power Containment (PWC) of the data within the filter bandwidth can be computed from

\[ PWC = \int_{-\infty}^{\infty} S(f) |H(f)|^2 df \]
where \( I(f) \) denotes the transfer function of a bandpass filter and \( S(f) \) is the PSD defined in Eqns. (14) and (18) for the square wave and sine wave subcarriers, respectively.

In this paper, the predetection filter assumes the same frequency responses of the filter implemented in the Deep Space Network (DSN)/Block V receiver [10]. This filter is a Surface Acoustic Wave (SAW) bandpass filter. The SAW filter has a passband of 66MHz, measured between the -1 dB drop off points. It can be modeled by a 20-pole Chebyshev bandpass filter which has a transfer function as shown

\[
|H_n(j\Omega)|^2 = \frac{1}{1 + \varepsilon^2 T_n^2(\Omega)}
\]

(28)

where \( T_n(\omega) \) is the \( n \)th Chebyshev polynomial, and \( \varepsilon \) determines the amplitude of the ripple in the passband of the filter. Reference [10] has shown that this filter can be approximated by a brick-wall bandpass filter. Therefore, Eqn. (27) can be rewritten as

\[
PWC = \int_{f_c - BW}^{f_c + BW} S(f) df
\]

(29)

where \( BW \) is the one-sided bandwidth of the SAW filter. Observe that Eqn (29) is correct since \( S(f) \) given by Eqns (14) and (18) are one-sided PSDs and the equivalent low pass filter of the bandpass filter is considered in Eqn (29). Evaluating Eqn (29) for a specified power containment (PWC) is nearly impossible since \( n \) is not known and nor is \( f_{se} \). In addition, one can interactively estimates \( n \) but for each estimate the integral in equation (29) must be evaluated from \( f_{c-BW} \) to \( f_{c+BW} \) which is a time consuming process for large bandwidth, \( BW \). One wishes, therefore, to have a mathematical model in a closed form that can provide an accurate estimate of the upper bound value of \( n_m \) that requires the less computational time (and which works for predetection filter bandwidth, \( BW \) ).
the following, a simple method is developed to approximate the upper bound value $n_M$, and the accuracy of this model will be verified later in the numerical results section by actual calculation of Eqn. (29).

The upper bound for $n_M$ for both square wave and sine wave subcarrier cases can be computed from

$$j \hat{\omega} = h (n_H R_S) + k R_S$$

(30)

with the assumption that

$$f_{sc} = n_H R_S$$

(31)

and $n_M$ is an integer. Solving for $n_M$, one obtains

$$n_H = \left( \frac{j \hat{\omega} - k}{R_S} \right)$$

(32)

where $h$ is the harmonic order of the data spectrum to be enclosed within the filter bandwidth and $k$ is the number of right-sided sidelobes of the $h$th order harmonic to be enclosed in the filter bandwidth. One can choose $k$ as an integer or a decimal number, but the latter would result in a small deviation from the true estimate in exchange for shorter computation time. In order to compute $n_M$, one must first determine the parameters $k$ and $h$. This is possible once the percentage of power contains within the filter bandwidth is specified. To simplify the calculation of parameter $h$, one would, first, need to assume infinite number of sidelobes for each of the data harmonics in the calculation. This assumption will be rectified partially at the evaluation of the parameter $k$. With
this assumption, the parameter $h$ is determined for the sinewave subcarrier from

$$\begin{align*}
\text{PWC}_0 (h) &= \left( \frac{1}{\infty} \sum_{j=1,3,5,...}^{\infty} J_j^2 (m) \int_{-\infty}^{\infty} S_h (f - j f_{sc}) \, df \right) \quad (33)
\end{align*}$$

and for squarewave subcarrier it is determined by

$$\begin{align*}
\text{PWC}_2 (h) &= \left( \frac{8}{\pi^2} \sum_{j=1,3,5,...}^{h} \frac{1}{j^2} \int_{-\infty}^{\infty} S_h (f - j f_{sc}) \, df \right) \quad (34)
\end{align*}$$

Evaluating integrals in Eqs. (33) and (34), one obtains the simplified forms as shown respectively below

$$\begin{align*}
\text{PWC}_0 (h) &= \frac{1}{\infty} \sum_{j=1,3,5,...}^{\infty} J_j^2 (m) \\
\text{PWC}_2 (h) &= \frac{8}{\pi^2} \sum_{j=1,3,5,...}^{h} \frac{1}{j^2} \\
\end{align*}$$

Evaluating integrals in Eqs. (33) and (34), one obtains the simplified forms as shown respectively below

$$\begin{align*}
\text{PWC}_0 (h) &= \frac{1}{\infty} \sum_{j=1,3,5,...}^{\infty} J_j^2 (m) \\
\text{PWC}_2 (h) &= \frac{8}{\pi^2} \sum_{j=1,3,5,...}^{h} \frac{1}{j^2} \\
\end{align*}$$

From Eqs. (35) and (36), one can determine the minimum parameter $h$ that is required to obtain approximately the given PWC. To correct the assumption that the pre-filtering bandwidth is unlimited one observes that, as shown in Fig. (2), only $k$ numbers of sidelobes of the data's $h$th harmonics are needed to meet the required PWC. One can determine this parameter simply by iteratively evaluating Eqn (37) until a value for $k$ can be found to satisfy the specified PWC.
where \( S_p(f) \) is the data PSD at baseband defined in Eqn. (6). Once the parameters \( h \) and \( k \) are obtained, the upper bound \( n_m \), can be calculated easily from Eqn. (32). The estimated total power containment within the specified pre-filter bandwidth (BW), \( PWC_{est} \), corresponding to \( n_m \) is given by

\[
PWC_{est} = \left( PWC_j (h, \frac{1}{2}) \right) \cdot PWC_{correct} \quad ; \quad h = 1
\]

\[
PWC_{est} = \left( PWC_j (h, 2) \right) \cdot PWC_{correct} \quad ; \quad h > 1
\]

for \( j = 0 \) corresponds to sine wave, anti 1 for square wave subcarrier. The actual value of the total power containment for a given \( n_m \) can be obtained by substituting \( n_m \) into Eqn. (31) and evaluating Eqns (31) and (27) for PWC. Numerical results will be shown in the following sections to confirm the accuracy of the model. The specifications used in the calculation pertains to the DSN/Block IV and V receiver systems.

V. SIMULATION

A direct approach to obtain the lower bound \( n_i \) is by computer simulation. This approach will be used to verify the presented mathematical models. The communication systems under investigation are simulated using Comdisco's Signal Processing Workstation (SPW) software. Figures 3 and 4 show the block diagram for the systems of square wave and sine wave subcarriers respectively. Basic operation of these systems are described below.

The PIJ receives the PCM/PSK/PM/NRZ/Square-wave or PCM/PSK/PM/NRZ/Sine-wave signal from the Test Signal Generator (TSG). Due to data interfering with the carrier, the PIJ will output an imperfect reference of the carrier as defined in Eqn. (9). The imperfect carrier is then
used to demodulate the data signal which, using Eqns. (2) and (3), can be modeled for the square wave and sine wave subcarriers respectively as

\[ I(t)_{\text{square}} = \sqrt{2P_s} d(t) \sin(m) \cos(\omega_c t) + n(t) \]  

\[ I(t)_{\text{sine}} = \sqrt{2P_s} d(t) \left( \sqrt{2} J_1(m) \sin(\omega_c t) \right) + n(t) \]

where \( n(t) \) is a white Gaussian noise. The demodulated signals are then filtered to produce the corresponding baseband signals

\[ I(t)_{\text{square}} = \sqrt{P_s} \sin(m) d(t) \cos(\phi(n, t)) + n_{FP}(t) \]  

\[ I(t)_{\text{sine}} = \sqrt{P_s} (\sqrt{2} J_1(m)) d(t) \cos(\phi(n, t)) + n_{FP}(L) \]

These baseband signals are then filtered by a match filter to reproduce the transmitted data symbol. Since there is noise in the signal and the carrier tracking is imperfect, the reproduced data symbol may not match the transmitted data symbol. In this case, an error has occurred; hence, an error counter is employed to count the number of times this happens. Knowing this number, the SER can be computed using the equation

\[ P_{\text{err}} = \frac{N_{\text{ERR}}}{R_s t f_s} \]

where \( N \) denotes the number of symbol error detected, \( R_s \) is the symbol rate, \( I \) is the total number of iteration points run for the simulation, and \( f_s \) is the sampling frequency. The subsequent section presents simulation results obtained from the simulation systems shown in Figs. (3)-(4). Note that by simulating the systems shown in Figs. (3)-(4) one has assumed that the subcarrier tracking is perfect which is consistent with the mathematical models presented in Section 3.
VI. NUMERICAL RESULTS

Two types of data will be presented here to determine the bounds for n, one comes from the theories described above and the other comes from computer simulation. These data will be presented jointly for comparison purposes.

For the square wave subcarrier, the simulated case is for $B_L/R_s \approx 0.5$, tracking loop bandwidth is 150 Hz [9], and the corresponding bit rate is 300 bit-per-second. Using Eqs. (22) and (23) the bit error rate is computed. Figures (5a) and (5b) shows the SIR versus the symbol SNR from the simulation and simulation plus calculation respectively. As illustrated by the arrows, the differences in the symbol SNR between the curves of “Ideal” and the curve of “$n = \infty$” represent the radio loss as defined in [1]. Note that the curve labeled “Ideal” denotes for the case when the loop SNR and n are infinite. The symbol SNR degradation due to data interference can be found by measuring the horizontal distance between the curve of "$n = \infty$" to the corresponding curve of n for a fixed value of SIR. In Fig. (5b), the simulation and the theory datum are plotted jointly to show that the as n gets larger, the results from theory and simulation are more agreeable.

Figures (6a) and Fig. (6b) depict SIR curves for $B_L/R_s = 2.5$, plotted for the simulation and theory plus simulation data respectively. At this $B_L/R_s$ ratio, the data interference can not be considered as white noise for low value of n. In these cases, results from the simulation will be used to determine the minimum value of n. Figures (7a) and (7b) depict SIR curves for $B_L/R_s = 4.69$. Note that for the latter two cases, the theoretical results for the SIR are obtained from calculating, Eqs. (25) and (26).

Using the loop bandwidth-to-symbol rate ratio ($B_L/R_s$) values of the square wave subcarrier case, the SIR for the sine wave subcarrier case were calculated and plotted against symbol SNR as shown in Figures 8, 9, and 10. Note that for the two cases of $B_L/R_s = 2.5$ and 4.69, higher order of
the carrier harmonics do fall into the carrier tracking loop bandwidth for low values of n (i.e. n = 1, 2, or 3.) For these cases, the SER were obtained from calculating equation (25) and (26) and the results were verified by computer simulation.

From these Figures, one can see that in order to minimize symbol SNR degradation due to data interference, one would need a certain minimum value of n. This is the lower bound n₁. Table I summarizes the minimum value of the subcarrier frequency-to-bit rate ratio, nₛ, that result in the Symbol SNR degradation of 0.1 dB or less.

**Table 1. Minimum Value of the Subcarrier Frequency-to-Symbol Rate Ratio, nₛ, for SER ≤ 10⁻³ With the Symbol SNR Degradation of ≤ 0.2 dB or Less**

<table>
<thead>
<tr>
<th>Modulation Type</th>
<th>PCM/PSK/PM/Square-Wave</th>
<th>PCM/PSK/PM/Sine-Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>nₛ = fₛₐ₉/Rₛ</td>
<td>4.69</td>
<td>4.69</td>
</tr>
<tr>
<td>Rₛ/Rₛ</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>sSNR Degradation in dB</td>
<td>≤ 0.19 dB</td>
<td>≤ 0.1 dB</td>
</tr>
<tr>
<td></td>
<td>≤ 0.1 dB</td>
<td>≤ 0.16 dB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt; 0.1 dB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt; 0.1 dB</td>
</tr>
</tbody>
</table>

The upper bound of n, nₐ, can be calculated by Eqn. (30). Here, the pre-filter is taking as the pi-c-filtering filter of the block V receiver [8], the bandpass SAW filter. This filter has a pass band of 66MHz; measured between the -1 dB drop off points. Using Eqns. (32) and (38), the maximum value for the parameter of n was calculated for two values of the data power containment and the results are verified by using Eqn. (29). The results are tabulated in Tables (2)-(5) for both sine-wave and square-wave subcarrier. Note that the column contained the “Estimated Value” is calculated from Eqns (32) and (38). While the column labeled “Actual Value” is computed from Eqn. (29). Depending on the amount of the desired power containment, the upper bound for the subcarrier frequency-to-bit rate ratio can be selected from these tables.
### Table 2. Maximum (Upper Bound) Value of the Subcarrier Frequency-to-Symbol Rate Ratio, $n_M$, for Two Values of Power Containment at 500 K Symbols per Second (SPS)

![Table 2](image)

### Table 3. Maximum (Upper Bound) Value of the Subcarrier Frequency-to-Symbol Rate Ratio, $n_M$, for Two Values of Power Containment at 300 Symbols per Second (SPS)

![Table 3](image)
Table 4. Maximum (Upper Bound) Value of The Subcarrier Frequency-to-Symbol Rate Ratio, \( n_M \) for Two Values of Power Containment At 60 Symbols Per Second (SPS)

<table>
<thead>
<tr>
<th>( n_M )</th>
<th>PCM/PSK/PM/Square Wave</th>
<th>PCM/PSK/PM/Sine Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWC %</td>
<td>Estimated Value</td>
<td>Actual Value</td>
</tr>
<tr>
<td>190</td>
<td>78571</td>
<td>23912</td>
</tr>
<tr>
<td>94.93</td>
<td>95.31</td>
<td>98.31</td>
</tr>
</tbody>
</table>

Table 5. Maximum (Upper Bound) Value of The Subcarrier Frequency-to-Symbol Rate Ratio, \( n_M \) for Two Values of Power Containment At 32 Symbols Per Second (SPS)

<table>
<thead>
<tr>
<th>( n_M )</th>
<th>PCM/PSK/PM/Square Wave</th>
<th>PCM/PSK/PM/Sine Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWC 0/o</td>
<td>Estimated Value</td>
<td>Actual Value</td>
</tr>
<tr>
<td>750</td>
<td>147321</td>
<td>33266</td>
</tr>
<tr>
<td>94.93</td>
<td>95.31</td>
<td>98.73</td>
</tr>
</tbody>
</table>

VII. Summary and Conclusions

This paper has analyzed the performance of the PCM/PSK/PM receiver for both square and sine wave subcarrier. The performance of the receivers under investigation is characterized by (a) the SIR performance, and (b) the amount of the data recovered from the first IF stage filtering (or the total data power contained in the filtered bandwidth). Both SIR and total data power containment are computed as a function of the subcarrier frequency-to-symbol rate ratio, \( n \). Based on the SIR performance, one
can establish the lower bound (or minimum) value $n_l$. On the other hand, the upper bound (or maximum) $n_u$, is determined from the required data power containment in the first IF stage filter bandwidth. The specifications on the loop bandwidth, minimum and maximum data rate, and the bandwidth of the first stage IF filter are obtained from the DSN/Block IV and V receivers. These specifications are used in the theoretical calculations of the SER and data power containment (or recoverable data power) in the filter bandwidth. The theoretical results are then verified by computer simulation.

Both theoretical and simulation results show that, for $B_i/R_s \leq 0.5$ and $SER \leq 0.3$, the lower bound for $n$ is 3 (i.e., $n_l = 3$) for both square and sine wave subcarriers. For $0.5 < B_i/R_s \leq 4.7$, the lower bound for $n$ is found to be 10. When the subcarrier frequency is chosen to be exactly at $n_lR_s$, a symbol SNR degradation of 0.2 dB or less occurs. Table 6 summarizes the results for lower bound values of $n_l$.

The maximum (or upper bound) value $n_u$ is determined from a 95% of total data power containment in the first stage IF filter bandwidth. Using the results presented in Tables 2-5, the log of $n_u$ as function of the symbol rate is plotted in Fig. 11 for both square wave and sine wave subcarriers. For a fixed IF filter bandwidth, figure 11 shows that as one increases the data rate one has smaller upper bound $n_u$. 
Table 6. Recommended Minimum Values for the Subcarrier Frequency to-Symbol-Rate Ratio for SFR ≤ 10-3 with Symbol SNR Degradation of 0.2 dB or Less.

<table>
<thead>
<tr>
<th>Operating Conditions</th>
<th>B₁/Rₛ ≤ 0.5</th>
<th>0.5 &lt; B₁/Rₛ ≤ 4.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recommended Minimum n₁</td>
<td>3</td>
<td>10</td>
</tr>
</tbody>
</table>

ACKNOWLEDGMENT

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REFERENCES


[8] "Digital filter to simulate the SAW bandpass filter in the Block V receiver", M. Aung, J. K. Holmes, IOM 3338-91-021

Figure 1. Power Density Spectrum of PCM/PSK/PM/NRZ: (a) with squ. wave subcarrier; (b) with sinewave subcarrier; (c) Data spectrum at baseband.
NOTE THAT THE POWER SPECTRUM ON THE LEFT OF THE RESIDUAL RF CARRIER $f_c$ IS NOT SHOWN FOR THE SAKE OF CLARITY.

Fig. 2.
Figure 4: CGS3S Demo of PCM/PSK/FM/S-32-WAVE Transceiver
Figure 5a. Simulation SER vs Symbol SNR for PCM/PSK/PM/NRZ/Squarewave
Figure 5b. Theory and Simulation SER vs Symbol SNR for PCM/PSK/PM/NRZ/Squarewave
Figure 6a, Simulation SER vs Symbol SNR for PCM/PSK/PM/NRZ/Squarewave
Figure 6b. Theory and Simulation SER vs Symbol SNR for PCM/PSK/PM/NRZ/Squarewave
Figure 7a, Simulation SER vs Symbol SNR for PCM/PSK/PM/NRZ/Squarewave
Figure 7b. Theory and Simulation SER vs Symbol SNR for PCM/PSK/PM/NRZ/Squarewave

- $B_L = 150 \text{ Hz}$
- $R_s = 32 \text{ bps}$
- $m = 0.35 \text{ radian}$
Figure 8a. Simulation SER vs Symbol SNR for PCM/PSK/PM/NRZ/Sinewave
Figure 8b. Theory and Simulation SER vs Symbol SNR for PCM/PSK/PM/NRZ/Sinewave
Figure 9a, Simulation SER vs Symbol SNR for PCM/PSK/PM/NRZ/Sinewave.
Figure 9b. Theory and Simulation SER vs Symbol SNR for PCM/PSK/PM/NRZ/Sinews.ve
Figure 10a. Simulation SER vs Symbol SNR for PCM/PSK/PM/NRZ/Sinewave
Figure 10b. Theory and Simulation SER vs Symbol SNR for PCM/PSK/PM/NRZ/Sinewave

$B_L = 150$ Hz
$R_s = 32$ bps
$m = 0.35$ radian

$B_L = 150$ Hz
$R_s = 32$ bps
$m = 0.35$ radian
Figure 11. MAXIMUM SUB CARRIER FREQUENCY-TO-SYMBOL RATE RATIO AS A FUNCTION OF SYMBOL RATE