RELATIVITY PARAMETERS DETERMINED FROM LUNAR LASER RANGING

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Abstract

Analysis of 24 years of lunar laser ranging data has been used to test the Principle of Equivalence, geodetic precession, the PPN parameters β and γ, and $\dot{G}/G$. Recent data can be fit with an rrns scatter of 3 cm. (a) Using the Nordtvedt effect to test the Principle of Equivalence, it is found that the Moon and Earth accelerate alike in the Sun's field. The relative accelerations match to within 5×10^{3}. 'J'his limit, combined with a 7 limit from planetary time delay, gives β. Including the uncertainty due to compositional differences, the parameter β differs from unity by no more than 0.0014 ; and, if the weak equivalence principle is satisfied, the difference is no more than 0.0006. (b) Geodetic precession matches its expected 19.2 milliarcseconds/yr rate within 0.7%. This corresponds to a 1 % test of 7. (c) Apart from the Nordtvedt effect, β and γ can be tested from their influence on the lunar orbit. Theoretically it is argued that the linear combination 0.8β + 1.47 can be tested at the 1% level of accuracy. During solutions using numerically derived partial derivatives, higher sensitivity is found. Both β and γ match the values of general relativity to within 0.005, and the linear combination β -i 7 matches to within 0.003, but caution is advised due to the lack of theoretical understanding of these sensitivities. (d) No evidence for a changing gravitational constant is found, with $|\dot{G}/G| \leq 0.9 \times 10^{-11}$. 'J'here is significant sensitivity to $\dot{G}/G$ through solar perturbations on the lunar orbit.

Introduction

in July, 1969, the Apollo 11 lunar mission placed an array of 100 silica corner-cube laser retroreflectors on the Sea of Tranquility. Within a few weeks the 2.7 m telescope at the McDonald Observatory on Mt. Locke, Texas, succeeded in detecting photons returned from a laser pulse sent to the reflector. By 1970, the observatory was routinely obtaining ranges with approximate uncertainties of 20--30 cm.

Two more reflector arrays were landed by Apollo missions in 1971: one at the crater Fra Mauro and one at Hadley Rille. A French-built reflector aboard the Russian spacecraft Lunakhod II was placed near the crater Le Monnier in early 1973. These events provided an opportunity for testing relativity.

The Data Set

The data set used in this analysis consists of ranging observations from three sites covering the time from March, 1970 to Jaunary, 1994. Between 1970 and 1984 the only data used arc those from the McDonald Observatory. Then in 1984 two other stations began acquiring ranges: one on Mt. Halcakala on the island of Maui; the other at the CERGA station in Grasse, France. (In 1985 the 2.7 m McDonald instrument ceased laser ranging operation and was replaced by the McDonald Laser Ranging System, a dedicated 60 cm telescope, The Halcakala facility terminated lunar ranging operations in August, 1990.)

The lasers currently used in the ranging operate at 10 Hz, with a pulse width of about 200 picosecond; each pulse contains $\sim 10^{18}$ photons. Under favorable observing conditions a single reflected photon is detected once every few seconds. For data processing, the ranges represented by the returned photons are statistically combined into normal points, each normal point comprising anywhere from 1 to ~1 00 photons. There arc 8427 normal points used in this investigation, spanning the period from March, 1970 through January, 1994,

The ranges of the early 1970s had uncertainties of approximately 25 cm. By 1976 the uncertainties of the ranges had decreased to about 15 cm. Accuracies improved further in the mid-1980s; by 1987 they were 4 cm, and the present uncertainties arc 2-3 cm.

Estimated Parameters

Onc immediate result of lunar ranging was the great increase in the accuracy of the lunar ephemeris. Within six years, the fitting of lunar range data reduced the range error from approximately one kilometer to a fcw decimeters. Measurements at the highest level of precision also provide commensurate determination of the lunar physical libations (rotation), reflector coordinates, elastic deformation, rotational dissipation, moments of inertia, low-degree gravity field, and Love numbers, as well as the mass of the Earth-Moon system, and Earth station locations, precession and nutation of the equator, and rotation (UT1 and polar motion). Also estimated is the secular acceleration of the geocentric lunar longitude, arising principally from the interaction of the Moon with the terrestrial ocean tides.

The Mathematical Model

The simultaneous numerical integration of the Moon and planets uscs the solar-system barycenter. 'l'his approach establishes the coordinate frame used for the computation of the observable time delay or "range." Each transmit and receive time at the ranging observatory is transformed to the coordinate time for the solar-system barycenter using the vector formulation of Moyer [I]. Geocentric observatory coordinates and selenocentric reflector coordinates are modified with a Lorentz transformation. The gravity fields of the Sun and Earth delay the signal. Given a transmit time, the computed receive and reflect times arc derived from a "light-time iteration ."

'The formulation of the JPL planetary ephemeris programs is used to estimate the relativity parameters. The principal gravitational force on the nine planets, the Sun, and the Moon is modeled by considering those bodies to be point masses in the isotropic, Parametrized Post-Ncwtonian (PPN) n-body metric [2]. A thorough description of the equations of motion for the planets and Moon is given in [3]. The portion of the model used in relativity analysis is the point-mass acceleration for each of the bodies:

$$
\begin{aligned}
\ddot{\mathbf{r}}_{i_{\text{point mass}}} = \sum_{j\neq i} \frac{\mu_j(\mathbf{r}_j - \mathbf{r}_i)}{r_{ij}^3} \Bigg\{ & 1 - \frac{2(\beta+\gamma)}{c^2}\sum_{k\neq i}\frac{\mu_k}{r_{ik}} - \frac{2\beta-1}{c^2}\sum_{k\neq j}\frac{\mu_k}{r_{jk}} \\
& + \gamma\left(\frac{v_i}{c}\right)^2 + (1+\gamma)\left(\frac{v_j}{c}\right)^2 - \frac{2(1+\gamma)}{c^2}\dot{\mathbf{r}}_i\cdot\dot{\mathbf{r}}_j \\
& - \frac{3}{2c^2}\left[\frac{(\mathbf{r}_i-\mathbf{r}_j)\cdot\dot{\mathbf{r}}_j}{r_{ij}}\right]^2 + \frac{1}{2c^2}(\mathbf{r}_j-\mathbf{r}_i)\cdot\ddot{\mathbf{r}}_j \Bigg\} \\
& + \frac{1}{c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}^3}\{[\mathbf{r}_i - \mathbf{r}_j]\cdot[(2+2\gamma)\dot{\mathbf{r}}_i - (1+2\gamma)\dot{\mathbf{r}}_j]\}(\dot{\mathbf{r}}_i-\dot{\mathbf{r}}_j) \\
& + \frac{(3+4\gamma)}{2c^2}\sum_{j\neq i}\frac{\mu_j\ddot{\mathbf{r}}_j}{r_{ij}}
\end{aligned}
\tag{1}
$$

where $\mathbf{r}_i$, $\dot{\mathbf{r}}_i$, and $\ddot{\mathbf{r}}_i$ are the solar-system barycentric position, velocity, and acceleration vectors of body i; $\mu_j = Gm_j$, where G is the gravitational constant and m_j is the mass of body j; $r_{ij} = |\mathbf{r}_j - \mathbf{r}_i|$; β is the PPN parameter measuring the nonlinearity in superposition of gravity; 7 is the PPN parameter measuring space curvature produced by unit rest mass; $v_i = |\dot{\mathbf{r}}_i|$; and c is the speed of light. (The remaining part of the equations of motion accounts for tides, gravitational harmonics, and the effects of the major asteroids.)

The parameter 7 also directly affects the measured range. From a geometrical point of view the Sun, Earth, and Moon each curve space in their vicinity to varying degrees. The effect of this curvature is to increase the round-trip travel time of a laser pulse. The complete relativistic light-time expression was derived in heliocentric form by Shapiro [4] in 1964 and independently by Holdridge [5] in 1967. It was formulated in expanded solar-system barycentric form by Moyer [6] in 1977. The portion of Moyer's form due to the Sun and Earth is

$$
t_j - t_i = \frac{r_{ij}}{c} + \frac{(1+\gamma)\mu_S}{c^3}\ln\left(\frac{r_i^S + r_j^S + r_{ij}^S + (1+\gamma)\mu_S/c^2}{r_i^S + r_j^S - r_{ij}^S + (1+\gamma)\mu_S/c^2}\right) + \frac{(1+\gamma)\mu_E}{c^3}\ln\left(\frac{r_i^E + r_j^E + r_{ij}^E}{r_i^E + r_j^E - r_{ij}^E}\right) \tag{2}
$$

The first term on the right is the geometric travel time duc to coordinate separation; the remaining two terms represent the curvature effects duc to the Sun and Earth. The complete equation gives the elapsed coordinate time between two photon events, where an event is indicated by the subscript i or j. Event 1 is transmission, event 2 is reflection, and event 3 is reception. A roman superscript denotes the origin of a vector: B is the solar-system barycenter, S is the Sun, and E is the Earth. In the convention used here, the subscript i represents the earlier of two photon events, j the later of the two. For the "up-leg" light time, $i = 1$ (transmission) and $j = 2$ (reflection); the "down-leg" values are $i = 2$ (reflection) and $j = 3$ (reception). In each case, $j = i + 1$.

The usc of the symbols in the equation is:

$r_i^S = |\mathbf{r}_i^S|$, the magnitude of the vector from the Sun to photon event $\mathbf{i}$ (transmission or reflection) at coordinate time t_i. r_j^S has the corresponding meaning for photon event j (reflection or reception). Superscripts B and E have the meanings stated above.

$r_{ij}^S = |\mathbf{r}_j^S - \mathbf{r}_i^S|$, the magnitude of the difference between the vector from the Sun to photon event j at time t_j and the vector from the Sun to photon event i at time t_i.

7 = the PPN parameter measuring the amount of space curvature produced by unit rest mass; $\mu_S = GM_{\mathrm{Sun}}$; $\mu_E = GM_{\mathrm{Earth}}$; and c = the speed of light.

When converted to units of lunar range, the dominant effect of space curvature is duc to the Sun and averages 7.6 m; the contribution from the Earth is about 4 cm. The ignored effect of the Moon amounts to only 0.6- 0.7 mm.

LLR and Relativity

The Moon orbits the Earth at a mean distance of 385,000 km. Solar perturbations distort the orbit from an idealized geocentric ellipse at about 1% of that figure. Since the earliest development of the classical theory of gravitation, the Moon has been an important test of that theory. Now that laser range observations to the Moon have accuracies of 3 cm, tests of relativistic gravitational theory arc practical. This paper presents the results of tests of the principle of equivalence, geodetic precession, the PPN quantities β and γ, and the time-rate of change of the gravitational constant G.

In the analysis and results which follow, the standard errors given for the estimated relativistic parameters are '(realistic" rather than the formal values derived from the estimation process. The reasons arc (1) for a large number of data points, systematic errors can corrupt solutions by more than the formal error (which assumes random errors) without producing obvious signatures in post-fit residuals. There arc known dynamical systematic effects, such as solar radiation pressure and internal lunar viscous dissipation, that arc not modeled; and (2) both the density and orbital sample space of the data arc non-uniform. Most of the lunar range data arc obtained near the first and last quarter phases of the Moon; such preferential distribution is of concern.

This paper presents the results of the determination of relativity parameters using LLR data. During the solution process, however, approximately 140 additional parameters are estimated, including the ephemerides of all the planets and the masses of selected asteroids. Reliable estimation of the planetary orbits and asteroid masses requires the inclusion of more than 64000 planetary observations. '1'hose data do indeed provide a strong determination of the aforementioned parameters, but their presence is not directly used to estimate relativity parameters. This paper is intended as a lunar test of relativity.

The Principle of Equivalence

Nordtvedt [7], [8], [9], [10] has published an analysis of the effects of a violation of the Principle of Equivalence. (A consequence of this principle is that the gravitational mass

M_G of any object is identical to its inertial mass M_I.) The Earth and Moon are accelerated by the gravitational field of the Sun. Failure of the Principle of Equivalence would cause a differential acceleration between the two bodies, giving a dipole term in the expansion of the Sun's gravitational field at the Earth. Nordtvedt points out that a failure of the principle would lead to an anomalous radial perturbation with the 29.53-day synodic period between the Moon and Sun. (The synodic period is the reciprocal of the difference between between the inverse sidereal periods: 29.53 = $[1/27.32 - 1/365.24]^{-1}$.) The argument (designated D) with synodic period is the mean longitude of the Moon minus the mean longitude of the Sun and is zero at new Moon. Any anomalous radial perturbation will be proportional to Cos D.

A breakdown of the Principle of Equivalence gives an acceleration of the Moon with respect to the Earth of $GM'E\mathbf{r}'/r'^3$, where G is the gravitational constant, M' is the is the mass of the Sun, r' is the vector from the Sun to the Earth-Moon center of mass, r' is the magnitude of $\mathbf{r}'$, and $E = (M_g/M_i)_{\mathrm{Earth}} - (M_g/M_i)_{\mathrm{Moon}}$ is the difference between the Earth and Moon gravitational-to-inertial mass ratios.

The lunar mean anomaly is l; its rate is the natural frequency for radial perturbations. Nordtvedt's original first-order expression for a near-circular orbit can be written

$$\Delta r = -a'E\frac{\dot{L}'^2(2\dot{l} + \dot{D})}{\dot{D}(\dot{l}^2 - \dot{D}^2)}\cos D. \tag{3}$$

In the conventional notation of lunar theory, L is the mean longitude of the Moon, L' is the mean longitude of the Sun (180 degrees from the heliocentric mean longitude of the Earth-Moon barycenter), a' is the heliocentric semimajor axis of the Earth-Moon barycenter orbit, and $D = L - L'$. (Dots over quantities indicate rates; primes denote quantities referring to the Sun.)

As a check of Nordtvedt's original result a somewhat different derivation based on perturbations of orbital elements was performed. It gives

$$\Delta r = -a'E\frac{\dot{l}\dot{L}'^2(2\dot{l} + \dot{D})}{\dot{L}\dot{D}(\dot{l}^2 - \dot{D}^2)}\cos D. \tag{4}$$

When evaluated in meters, the two coefficients are $-2.08 \times 10^{10}E$ and $-2.05 \times 10^{10}E$, respectively. The difference between the two is only 1.4%. It will be noted that the denominator contains the combination $\dot{l} - \dot{D}$, which is the difference between the solar mean motion and the lunar perigee precession $\dot{L}' - \dot{\omega}$. A breakdown of the equivalence principle would also give rise to a perturbation in longitude proportional to sin D. For $a\Delta L$, where a is the semimajor axis, the companion to Eq. (4) has a coefficient -2.1 times larger.

Recently Nordtvedt [11] has demonstrated that the earlier-derived coefficients of cos D need to be increased by about 40% over the values given above. This amplification arises because of the strong solar influence on the lunar orbit. The synodic period of the perturbation interacts with the $2D$ tidal expansion of the solar field at the Earth. With this correction the radial perturbation in meters is

$$\Delta r = -2.87 \times 10^{10} E \cos D$$

The longitude perturbation also needs to be increased by about 40%.

The above equations apply to any breakdown of the Principle of Equivalence, A breakdown of the Strong Equivalence Principle, where gravitational self-energy U_g can influence the gravitational interaction, is possible for bodies the size of the Earth and Moon. Nordtvedt gives

$$\frac{M_g}{M_i} - 1 = \eta \frac{U_g}{Mc^2}.$$

The quantity η depends on the PPN parameters β and γ according to

$$\eta = 4\beta - \gamma - 3$$

and is zero for General Relativity. Numerically,

$$\begin{aligned} E &= (-4.64 \times 10^{-10} + 0.19 \times 10^{-10})\eta \\ &= -4.45 \times 10^{-10}\eta. \end{aligned} \tag{5}$$

Expressed in terms of η, the radial perturbation in meters is $\Delta r = 12.8\eta \cos D$.

The above values in Eq. (5) are the same as used in [12], where the Earth's self-energy is based on the result [13] for a structured interior, and the Moon's self-energy is based on a homogeneous interior. Adelberger *et al.* [14] have suggested a 10% larger value for the Earth. Our own computation for the self-energies for radially structured interiors for both bodies recovered the earlier values to the number of digits given in Eq. (5).

Apart from the Nordtvedt effect, there are other causes of cos D signatures in the lunar distance. From the classical expansion of the lunar orbit ([15], [16]) there is a 109 km cos D term in the radial coordinate. The amplitude depends on mass ratios, mean motions, and the mean distance to the Moon, but these are well enough known that no significant error occurs for this coefficient. There is also a relativistic contribution apart from the Nordtvedt effect which has been computed by [11], [17], [18], [19], [20], [21], and [22]. This relativistic contribution is given as -6 cm cos D in [19]. The numerical integration of the relativistic equations of motion should include classical and relativistic orbit signatures in our lunar ephemeris. Williams *et al.* [12] mention that the interaction between the Earth's gravitational second harmonic J_2 and the Sun gives rise to a -5 cm cos D effect (-7 cm with the 40% increase). This force is included in our equations of motion. Solar radiation pressure also gives rise to a small signature [11], [23]. This effect is estimated to be -0.35 cm cos D. The software contains a model for the solar gravity field but not for solar radiation pressure, The differences between the transmit, reflect, and receive times are computed by iteration, and the time delay of Eq. (2) is modeled, implying that there should be no anomalous signatures due to these sources [24].

The partial derivative for M_g/M_i is generated by numerical integration (prior to the results in [25] wc used the cos D formulation). The solution gives E = (4.3 ± 4.6) x 10^{-13}. This is equivalent to --1.2 ± 1.3 cm in the coefficient of cos D or, for a violation of the Strong Equivalence Principle, to η = - 0.0010 ± 0.0010. The argument D is unevenly sampled. Ranges arc never acquired near new Moon bccausc of the bright Sun. The former 2.7 m McDonald Observatory ranging system could acquire ranges near full Moon, but the newer, more accurate, lower-energy-per-pulse systems have acquired full-Moon ranges only during an eclipse. Wishing to bc cautious about uncertainties, the procedure of [26] has been used. in a root-surn-squared sense, 1 crn has been added to the uncertainty in the coefficient of cos D, 3.5 x 10^{-13} to E, and 0.0008 to η. If the unmodeled 0.3 cm effect from solar radiation pressure is applied as a correction, then E = (3.2 ± 4.6)x 10^{-13}, the cos D coefficient is - 0.9 ± 1.3 cm, and η = -0.0007 ± 0.0010. In the solution for E, the largest correlations of .4 OCCUr with $GM_{\mathrm{Earth+Moon}}$, lunar semimajor axis a, and eccentricity e. These occur because a cos $2D$ term is important for the GM determination and will not be independent of the cos D term because of nonuniform sampling and of the facts that the semimajor axis is connected to GM through Kepler's third law and that the product ae is better determined than e.

The results for E and the cos D coefficient apply to the Equivalence Principle, weak or strong. Earlier results for the Nordtvedt effect have been interpreted in terms of the Strong Equivalence Principle, the laboratory results for the Weak Equivalence Principle being able to rule out effects duc to composition. Limits as low as those given above require considera-ion of the Weak Equivalence Principle [20]. Adelberger *et al.* [14], [27] have combined their Eötvös results with the Princeton [28] and Moscow [29] Eötvös experiments. For accelera-tion in the solar field they place limits on the fractional acceleration due to composition. Su *et al.* [30] have used test bodies which simulate the compositional differences of the Earth and Moon. Their compositional contribution to E is (-1.6 ± 2.2) x 10^{-12}. It should bc noted that the Nordtvedt test is a null result. It would bc necessary to have compensating violations of the Strong and Weak Equivalence Principles.

We wish to derive β from η and γ using $\beta = (\eta + \gamma + 3)/4$. The compositional constraints from the preceding discussion contribute to η and β. Combining the compositional [30] and LLR results gives η = -0.0043 ± 0.0051. The uncertainty for γ is taken to bc 0.002 from the interplanetary time delay [31]. Including the Weak Equivalence Principle, β = 0.9989 ± 0.0014. Under the assumption that the Weak Equivalence Principle is satisfied, β = 0.9998 ± 0.0006.

Previously reported results for the Nordtvedt effect are given in [1 2], [25], [26], [32], [33], [34], [35], and [36]. The last two results have uncertainties comparable to this paper. The uncertainty in determinations of the Nordtvedt effect has decreased by a factor of 30 during 18 years.

Geodetic Precession

The geodetic precession is also called both the geodesic precession and the de Sitter-Fokker precession. It contributes a 19.2 mas/yr (milliarcseconds/year) rate to the lunar node, longitude of perigee, and mean longitude. The geodetic precession is prograde and is computed from [37], [38], [39] to be

$$P_g = (3/2)\frac{(n'a'/c)^2 n'}{(1-e'^2)} \tag{6}$$

where c is the speed of light and, for the orbit of the Earth-Moon system about the Sun, n' is the mean motion, a' the semimajor axis, and e' the eccentricity.

We review and extend the discussion of [40], which proposes testing for the geodetic precession using LLR data. The distance from the center of the Earth to the center of the Moon can be represented by the series with largest terms (in kilometers):

$$r = 385001 - 20905\cos l - 3699\cos(2D - l) - 2956\cos 2D + \ldots \tag{7}$$

The first term is the mean distance, the second results from the eccentricity of the orbit, and the third and fourth are from solar perturbations. The lunar mean anomaly is l (27.56-day period), and D is the mean elongation of the Moon from the Sun (29.53-day period).

For purposes of explanation, we can imagine that the least-squares solutions are equivalent to determining amplitudes, phases, and phase rates of individual terms in Eq. (7). More exactly, there are a limited number of free parameters in a solution, so that the amplitudes, phases, and phase rates are not all independent. Typically, the amplitude of a well sampled frequency can be measured to about 1 cm accuracy. From the second term one expects to determine the mean anomaly to 0.1 mas and the anomalistic mean motion $\dot{l}$ with correspondingly high accuracy. Limitations, which increase the uncertainty, include the need to also determine quadratic and long-period (18.6 yr) tidal contributions to the mean anomaly [41], terms at nearby frequencies which require 6.0-year and 8.9-year data spans to separate fully, and a span of the most accurate data, which is 7 years long. From the two solar perturbation terms and the mean anomaly, one gets D with sub-mas accuracy and its rate with corresponding accuracy. It is presumed that the planetary data give $\dot{L}'$. Since $D = L - L' = \varpi + l - L'$, the longitude-of-perigee rate can be determined. The geodetic precession can be thought of as being detected through its influence on the lunar longitude-of-perigee precession rate. In addition to the errors in $\dot{l}$ and $\dot{L}'$, we must ask what errors are present in the longitude-of-perigee rate.

The lunar perigee precession rate is dominated by solar perturbations. While the classical contributions to the perigee precession rate from lunar and solar orbital parameters are mostly very well known, two influences merit discussion. An error in the inclination of the lunar orbit plane to the ecliptic plane of 1 mas would introduce a 0.18 mas/yr uncertainty in the perigee precession rate. The orientation of the planes of the lunar orbit, ecliptic, and the Earth's equator are determined by the LLR data, but it takes 18.6 years to get a full separation of these parameters. Thus the uncertainty in the lunar inclination has been decreasing strongly with time, and a good test of the geodetic precession is a benefit of the long data arc. The error in the first LLR tests of geodetic precession ([25], [42]) was dominated by the inclination uncertainty. Because the highest quality data extend over only the past 7 years, the accuracy of the inclination should continue to improve in the future. The inclination uncertainty is now less than 1 mas, and this source of error should continue to decrease.

The second significant source of perigee precession error comes from the lunar second-degree gravitational harmonics J_2 and C_{22}. Since the ratio is accurately known from the LLR analyses, we will refer to the error in J_2 only. Until recently we have used a 0.6% uncertainty for the lunar J_2, corresponding to a precession error of 0.14 mas/yr (0.7% of the geodetic precession). This J_2 uncertainty came from [33], which combined the analysis of Lunar Orbiter Doppler data and LLR data in a single solution. It was the Lunar Orbiter data which determined the J_2 in that combination. There have been two recent developments: LLR can now determine the second-degree lunar harmonics as accurately as the earlier Lunar Orbiter analysis [26], and the Lunar Orbiter data have been extensively reanalyzed [43] with an improvement in accuracy. The two results are concordant. As we now include the lunar J_2 as a solution parameter, the J_2 uncertainty, like the inclination error, is accounted for during the least-squares solutions.

The equations of motion for the numerical integration of the lunar and planetary ephemerides in Eq. (1) are those of General Relativity. They contain the inherent geodetic precession effects.

We isolated the terms in the relativistic equations of motion which give rise to the geodetic precession, and we included a scale factor K_{gp} representing a possible departure from the prediction of General Relativity:

$$\ddot{\mathbf{r}}_M - \ddot{\mathbf{r}}_E = K_{gp}\frac{2\mu_S}{c^2 r_B^3}\{-\gamma[\dot{\mathbf{r}}_B\cdot(\dot{\mathbf{r}}_M - \dot{\mathbf{r}}_E)]\mathbf{r}_B + (1+\gamma)[\mathbf{r}_B\cdot(\dot{\mathbf{r}}_M - \dot{\mathbf{r}}_E)]\dot{\mathbf{r}}_B\}$$

where the quantities $\mathbf{r}_E, \mathbf{r}_M$, and $\mathbf{r}_B$ denote the solar-system barycentric positions of the Earth, Moon, and Earth-Moon barycenter, respectively, and time is referenced to the solar-system barycenter.

The solution for the geodetic precession coefficient is

$$K_{gp} = -0.0034 \pm 0.007$$

The uncertainty corresponds to a precession error of 0.14 mas/yr. The largest correlation of 0.56 is with the lunar J_2; this parameter is now a more important error source than the orbit inclination. As the coefficient of the 19.2 mas/yr geodetic precession is $(1+2\gamma)/3$, the precession due to γ is 12.8 mas/yr. The above result for K_{gp} corresponds to a 1% test of γ.

Bertotti *et al.* [40] did not fit data but argued that geodetic precession was being satisfied *(a)* from the small size of the LLR residuals, and *(b)* from the agreement between LLR and VLBI Earth rotation rate. Direct fits to the LLR data ([25], [42]) confirmed geodetic precession to 2%. More recently Dickey *et al.* [26] give a 0.9% test. The result above reduces the uncertainty to 0.7%. All results are consistent with 7 = 1 and General Relativity.

The PPN Parameters

The PPN parameters of interest are β, measuring the non-linearity in the superposition of gravity, and 7, measuring the amount of space curvature produced by unit rest mass. In General Relativity both parameters arc unity. Estimates of 7 and β have been obtained by other investigators. Shapiro *et al.* [44], Cain *et al.* [45], and Hellings [46] used the Viking orbiter and lander data to determine γ. Reasenberg *et al.* [31] estimate the uncertainty in 7 to be 0.002, using Viking lander data. The test of the geodetic precession can be taken as a 1% test of 7, but this statement ignores additional sensitivity to PPN parameters, discussed below.

The lack of detection of Nordtvedt's term has been used to imply a small uncertainty on β. Tests of β using the planetary range data ([34]) yield a β uncertainty of 0.003. 'J'here is value in attempting to test β and 7 in an alternate manner.

Distinct from the Nordtvedt term, the relativistic point mass interactions of Eq. (1) give sensitivity of the lunar orbit to β and 7. Partial derivatives for β and 7 have been generated from Eq. (1) by numerical integration. The orbit perturbations include the geodetic precession. 'J'bus, one expects solutions for 7 to have accuracies comparable to, or better than, the above 1 % test resulting from the geodetic precession.

LLR solutions for β and 7 using the sensitivity from the relativistic point mass interactions and the gravitational time delay Eq. (2), but not the Nordtvedt term, show a smaller uncertainty for 7 than would be predicted from the geodetic precession alone, and nearly identical accuracies for both β and 7. Solving for β and 7 simultaneously: both uncertainties arc 0.005, there arc no significant deviations from 1, and the linear combination β + 7 is better determined with an uncertainty of 0.003. The challenge is to understand the source of this sensitivity and whether it is valid.

The discussion of the geodetic precession presented the view that the sensitivity to that precession comes from the solar perturbations in combination with the elliptical radial variations. In that discussion, the determination of the rate of the angle $D = L - L' = \tilde{\omega} + l - l'$ was presented as giving sensitivity to the geodetic precession, and hence 7, through the lunar perigee rate. When the relativistic point-mass interactions arc considered, the rate of D also gives sensitivity to β and γ as they influence $\dot{L}'$. In the near circular approximation, the angular rate of the Earth-Moon system about the Sun is given by

$$\dot{L}' = [(GM')^{1/2}/A'^{3/2}][1 - S(\beta + \gamma/2)]$$

where A' is not the semimajor axis a' but rather the mean distance from the Sun when relativistic perturbations are included, G is the gravitational constant, and M' is the solar mass $(GM' = n'^2 a'^3)$. The scale for relativistic effects is set by

$$S = GM'/a'c^2 = (n'a'/c)^2 = 0.98706 \times 10^{-8}$$

with $Sa' = 1.4766$ km and $Sn' = 12.792$ mas/yr. The relativistic contribution to the angular rate is $-Sn'(\beta$ -ı 7/2). 'J'he relativistic contribution to $\dot{\omega} - \dot{L}'$ is then $Sn'(0.5 + \beta +$ 1,57) when the geodetic precession is included, but is closer to $Sn'(0.65 + 0.8\beta +$ 1.47) when a more exact expression for the perigee rate is included ([1 1], [17], [1 8)). From the experience with the geodetic precession, the linear combination of β and 7 should be determinable to 1%, This argument assumes that the lunar mean anomaly rate is well determined by the LLR data and the mean distance from the Sun is well determined by the planetary range data,

The solutions include both the LLR and planetary ranging data. A normal set of solution parameters is used for the initial conditions of the Moon and planets, but the relativity solution parameters were "turned on" only for the LLR data. Both data sets arc sensitive to the heliocentric Earth-Moon orbit. in an attempt to isolate the relativistic sensitivities of the LLR data from those of the planetary data, a double standard is being applied to the heliocentric orbit. 'l'he planetary data arc included so that appropriate uncertainties in the heliocentric orbit will be propagated into the lunar orbit during the solutions. The double standard is not perfect, but we do not see another way to isolate the contributions of the lunar data from the planetary data, We have done a variety of solutions, including those with planetary relativity parameters turned on, and they do support strong sensitivity of the LLR data to relativity.

The solution cannot be finding all of its β and 7 sensitivity through the argument D, or the two would not separate. Other terms with smaller amplitudes give less sensitivity to other arguments; for example, sensitivity to the mean anomaly 1' of the orbit about the Sun is an order of magnitude less than the mean longitude sensitivity. Sensitivity through the amplitudes are possibilities. Brumberg and Ivanova [1 7], [18] and Nordtvedt [11] have investigated the sensitivities of the amplitudes to β and 7. When one considers observable amplitudes, the β and 7 sensitivities are mostly at the few-centimeter level. Brumberg and Ivanova show two notable possibilities. 'J'he annual cos 1' term shows a (-- 16-I 26β - 67)-cm relativistic contribution to its amplitude, and the cos D term has (33 $- 48\beta +$ 107) cm in its amplitude. Nordtvedt dots not compute the former term; for the latter term he gets a similar-sized sensitivity. For General Relativity (β = 7 = 1), the Brumberg and Ivanova solution can also be compared to the solution by Lestrade and Chapront-Touzé [19]. The agreement is good except for a few terms involving the annual argument 1'; this discrepancy seems to be traceable to the 1.66-msec annual term in the time transformation between the solar-system barycenter and the Earth-Moon barycenter. 'J'he amplitudes might give sensitivity to β approaching 1%.

Concerning the sensitivity of the LLR data to β and 7 through point-mass interactions, it should be possible to determine the combination ($0.8\beta +$ 1.47) to 1% accuracy through an argument rate. 'J'here is additional sensitivity to β and 7 through amplitudes. There is numerical evidence that the sensitivity may be less than 1%, for which we cannot find theoretical support. It is clearly worthwhile to combine the relativistic solutions from both the LLR and planetary ranging data. Since the LLR data have their sensitivity through the lunar and heliocentric orbit, while the planetary data have their main sensitivity to γ through the time delay in the solar gravity field and their main sensitivity to β through the precession of Mercury's perihelion, the combination of data types offers an interesting possibility. A combined solution would improve the accuracy of separating β and the solar J_2, which do not separate well when using the existing planetary data alone.

Change in the Gravitational Constant

Analyses of the LLR data have the potential to determine the rate of change of the gravitational constant G. Tides on the Earth dissipate energy and transfer angular momentum from the Earth's spin to the lunar orbit. This causes the mean distance and orbital period to increase. A decreasing G would also cause both distance and period to increase ($2\dot{n}/n + 3\dot{a}/a = \dot{G}/G$), but not in the same ratio as tides. Since the tidal effect is relatively large ($\dot{n}/n = 1.5 \times 10^{-10}$/yr), and since we are interested in $\dot{G}/G$ less than 10^{-11}/yr, accurate tidal modeling is a necessity.

Our present tidal model includes dissipation by both diurnal and semidiurnal tides on the Earth and dissipation in the Moon. From recent solutions [26] these Contribute to the total tidal $\dot{n}$ or $\dot{a}$ in the proportions 16%, 86%, and - 2%, respectively. The uncertainty in the total is 2%. More important than the linear increase in distance, the major tidal acceleration effect comes from the - t^2 change in mean anomaly causing a - t^2 sin 1 signature in range, The diurnal and semidiurnal terms are separable by a small 18.6 yr term in mean anomaly [41]. The dissipation in the Moon is mainly observed through its influence on the lunar rotation and not the orbit,. The influence on the orbit is inferred from the lunar dissipation model. 'l'here are two possible sources of dissipation in the Moon: solid-body dissipation, and viscous dissipation at a liquid-core/solid-mantle interface [47]. The former source is programmed in our software; the latter is not. The two sources do not give rise to the same orbital effects, so the lunar contribution is uncertain by most of its present 1.5% effect. Expecting that changes in the lunar model would leave the total $\dot{n}$ and $\dot{a}$ the same, the present tidal model should be capable of supporting tests for $\dot{G}/G$ with an accuracy of 0.6×10^{-11}/yr, which corresponds to 2% of the tidal effect, or better. Programming the alternative lunar dissipation model would improve the tidal acceleration computation and benefit future tests.

A G/G rate of -10^{-11}/yr causes a 3.9 mm/yr increase in the lunar mean distance ($\dot{a}/a = -\dot{G}/G$), but if the t^2 term in mean anomaly is indistinguishable from tidal acceleration, then –1/3 of the radial change, or --1,3 mm/yr, would be distinct from tidal acceleration. A change in G also causes accelerations in the angular motion about the Sun ($\dot{n}'/n' = 2\dot{G}/G$), and the solar perturbation terms in Eq. (7) contribute additional terms. The contribution from the acceleration in the heliocentric orbit through the solar perturbation terms gives coefficients of periodic terms which are quadratic. in time. With the linear term, the $\dot{G}$ contribution to radial distance that is distinct from tidal acceleration is

$$\frac{1}{3} r \frac{\dot{G}t}{G} - 2\frac{\dot{G}}{G} n' t^2 \, [3699 \sin 2D + 2956 \sin(2D - l) + \ldots] \text{ km}.$$

Some small linear terms from the sensitivity of the coefficients to mean-motion changes have been ignored.

For a $\dot{G}/G$ rate of -10^{-11} /yr, the major terms are

$$-1.28t + (0.46 \sin(2D - l) + 0.37 \sin 2D)t^2 \text{ mm}, \tag{8}$$

with t in years. For data spans of more than a decade, the nonlinear terms surpass the linear term in importance (The envelope, rms, and average signature due to Eq. (8) are shown in Figure 1). Thus, an increasing data span has the potential to strikingly improve the G/G determination.

The LLR data have been used to estimate $\dot{G}/G$ with a null result. A rate of 10^{-11}/yr would yield signatures from the solar perturbations exceeding 10 cm rms in the early 1970s and reaching 1 cm rms in 1993; the lack of such signatures demonstrates the importance of the early data in conjunction with the more accurate data in limiting G. The size of the signature justifies an uncertainty of 0.7×10^{-11}/yr. Including an uncertainty of 0.6×10^{-11}/yr from the 2% tidal acceleration error gives 0.9×10^{-11}/yr total uncertainty. The LLR G/G result is $(0.1 \pm 0.9) \times 10^{-11}$ /yr. The largest correlation is .67 and is with the semidiurnal tidal component.

As a check of the linear effect, a separate solution has estimated a rate in the mean distance with uncertainty 3.5 mm/yr, equivalent to 2.7×10^{-11} /yr for $\dot{G}/G$. The former solution implies a smaller G/G uncertainty, illustrating the dominance of the nonlinear solar perturbation terms.

The present LLR results for G/G do not improve significantly on the planetary ranging results ([34], [48]). Recent results have also been given for planetary data combined with LLR data [49] and the binary pulsar [50], [51], [52], [53].

comparison

Brief references to individual tests have been given in the separate sections of this paper. Broad analyses of the LLR data for several relativistic effects have been given by Müller *et* al. ([35], [54], [55]). The Earth-Moon orbit about the Sun contributes uncertainty which was not considered in the solutions given in [35] and the uncertainties given in [54] and [55] are more realistic (J. Müller private communication, 1994). The results and uncertainties of those later papers are in general agreement with this paper.

Conclusions

Solutions using 24 years of lunar laser data have been used for three tests of relativity and a check of the constancy of the gravitational constant. The LLR data have improved with time. The data since 1987 are particularly accurate with 1987 ranges showing a weighted rms residual of 4 cm and 1993 residuals scattering by 3 cm.

The Nordtvedt effect gives strong sensitivity to any violation of the equivalence principle. Using a numerically derived partial derivative for the gravitational to inertial mass ratio,

$|(M_G/M_I)_{\mathrm{Earth}} - (M_G/M_I)_{\mathrm{Moon}}| \leq 5 \times 10^{-19}$. Since any violation of the strong equivalence principle depends on β and 7, and since there are good determinations of 7 from interplanetary time delay measurements, then including the uncertainty due to compositional differences between the Earth and Moon $|\beta - 1|$ <0.0014. If it is assumed that the weak equivalence principle is satisfied, then $|\beta - 1|$ <0.0006.

The geodetic precession is within 0.7% of its expected value of 19.2 mas/yr. Since this precession depends on the factor (1 + 27)/3, this result is also a 1% test of γ under the assumption that other relativistic effects are known. The lunar J_2 is the most important correlated source of uncertainty.

Independent of the Nordtvedt effect, but including the geodetic precession, there arc orbit perturbations depending on β and γ. The time delay gives some sensitivity to γ. The LLR solutions usc numerically derived partial derivatives for the orbit perturbations and indicate sensitivity to β and γ beyond that expected from theoretical work. It is certain that the linear combination $0.8\beta + 1.4\gamma$ is tested at the 1 % level since it arises through the same solar perturbation terms which give the geodetic precession test. The work of Brumberg and Ivanova indicates additional sensitivity to β and 7 through annual and synodic monthly terms, and Nordtvedt's work supports sensitivity in the latter term. Neither work would support β and 7 accuracy better than 1%. From the LLR solutions β and *7* match the values of general relativity within the uncertainty of 0.005, and the linear combination β -1 7 matches within 0.003. For the LLR solutions it must be cautioned that usc is made of the planetary ranging data to determine the distance of the Earth-Moon orbit from the Sun, without allowing those data to directly contribute to the determination of the PPN parameters. It is important to understand this test better, since the sensitivity to $4\beta - \gamma$ from the Nordtvedt effect in combination with the sensitivity to $\beta + 7$ gives a test of γ with uncertainty 0.003, which is second in accuracy only to the interplanetary time delay, and it can be expected to improve in the future.

On the question of a changing gravitational constant, solutions show no significant change, with $|G/G| \leq 0.9 \times 10^{-11}$. It previously has been understood that G and tidal acceleration both influence the lunar period and mean distance, but $\dot{G}$ and tidal acceleration would be separable from a linear term in time. Here it is shown that the influence of $\dot{G}$ also causes nonlinear time signatures, through the solar perturbations, which arc already dominant.

'J'he lunar orbit is highly perturbed by the Sun. This paper's tests of relativity and $\dot{G}$ all depend on the solar perturbations. Reasoning from two-body theory is insufficient for the lunar orbit. All of the tests will improve with additional data of present quality. The geodetic precession test, depending on a secular effect, will benefit from increased data span. The tests of β and γ through orbit perturbations (apart from the Nordtvedt effect) arc the least well understood, but hold promise. In combination, the lunar and planetary ranging data should be able to improve on the dynamical determination of the solar J_2. Finally, there are lunar G terms, nonlinear in time, which should permit significant future improvements in testing the constancy of G.

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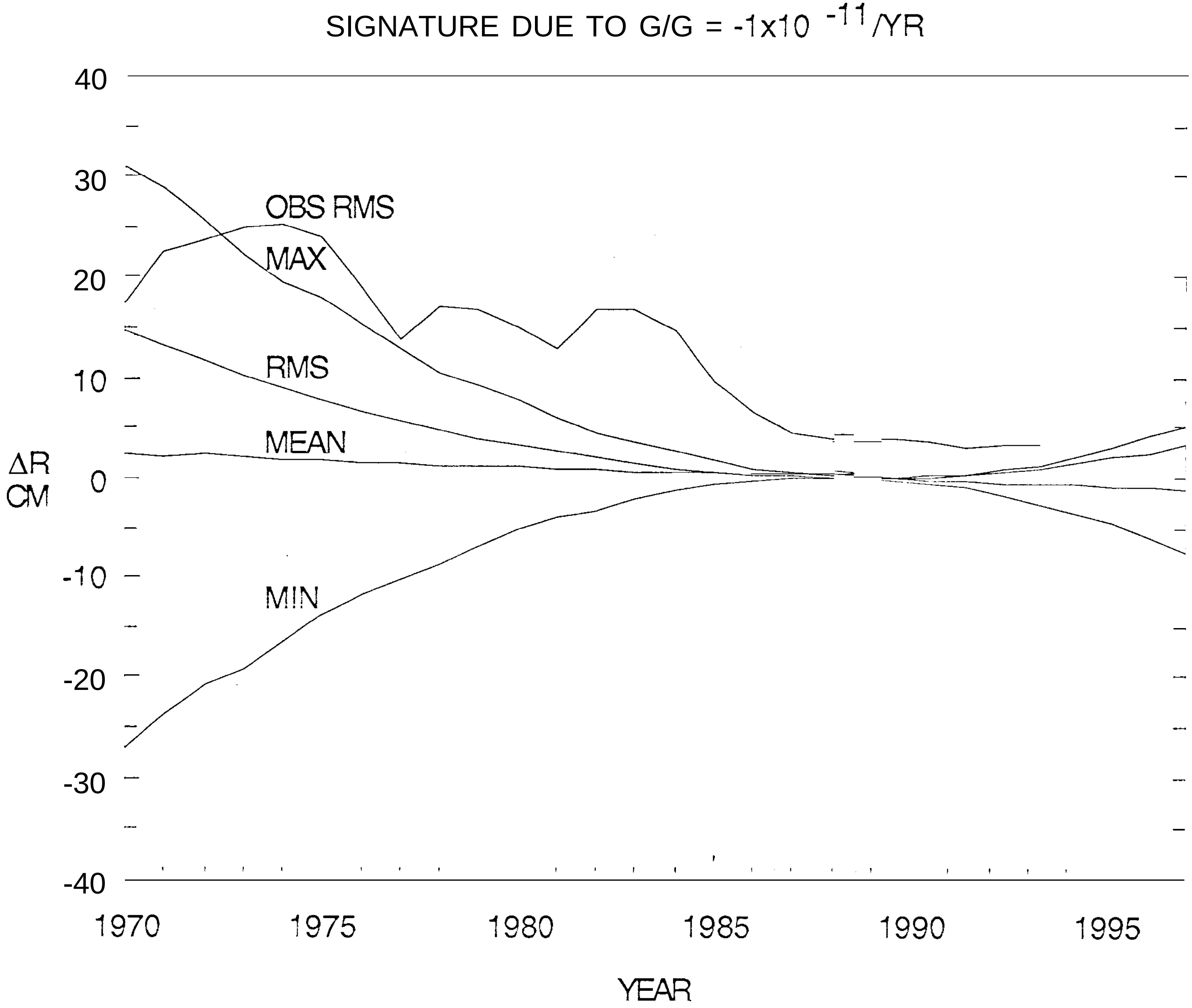
Figure Caption

Effect of $\dot{G}/G = -1 \times 10^{-11}$/yr on the radial coordinate of the Moon. The curves are annual samples of the observed weighted rms range residual and four curves based on the theoretical signature: the maximum, rms, average, and minimum. The reference time in Eq. (8) is 1989.



SIGNATURE DUE TO G/G = -1x10 -11/YR
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MEAN
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