A Parallel Incompressible Navier-Stokes Solver with Multigrid iterations

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Abstract

Wc developed a parallel, numericall y accurate and stable, and computationally efficient finite-diffcrcncc incompressible Navicr-Stokes (N-S) fluid flow solver. The solver runs on both sequential and massively parallel computers. The numerical method used here is a second-order projection method (Bell ct. al [1]) on a staggered grid, A multigrid scheme is used to speed up the solutions of velocity and pressure equations at each time step. A domain-decomposition strategy is used for parallelizing both the projection method and the multigrid scheme on fine and coarse grids. The solver runs on any (logical) rectangular processor meshes. The parallel solver was implemented in C with Intel NX and M PI interfaces for message-passing The code is highly modular and it can be used either as a stand-alone flow solver and or a template code which can be adapted or expanded to a specific application, Numerical results and parallel performances of our code on Intel Delta and Paragon arc reported.

1. The Projection Method and Ifs Parallel Implementation

The idea of projection method for solving incompressible Navicr-Stokes equations was first described in a paper by Chorin [2]. Bell ct. al extended the method to second-order in time and used a Godunov-type scheme in discretizing the convection term for numerical stability. Projection method is a type of operator-splitting method, which, in solving the N-S equal ions, separates the solutions of velocit y and pressure fields with an iterative procedure. In particular, al each time step, the momentum equations arc solved first for an intermediate velocity field without the correct pressure field and therefore no incompressibility condition is enforced; The intermediate velocity field is then "corrected" by a projection step in which wc solve a pressure equation and then usc the computed pressure to produce a (more) divergence-free velocity field, Our projection step, which makes usc of the highl y efficient multigrid solver, is mathematically equivalent to but different from what described in [1]. This prediction-correction type procedure is usually repeated a fcw times until reasonably good velocity and pressure fields have been reached for that time step. In each time step of computing an N-dimensional ($N = 2$ or 3) viscous flow problem using the second-order projection method, wc need to solve at least N Halmholtz equations for velocity and onc Poisson equation for pressure. A fast multigrid elliptic solver can thus be used to greatly improve the computational performance of the flow solver.

Our code was designed to be a general-purpose incompressible flow solver on a rectangular grid. The computations in tbc flow solver is parallelized with a domain-decomposition approach. For processing on the physical (finest) grid, explicit schemes can be naturally computed in parallel with some exchange of information at grid partition boundaries. What takes

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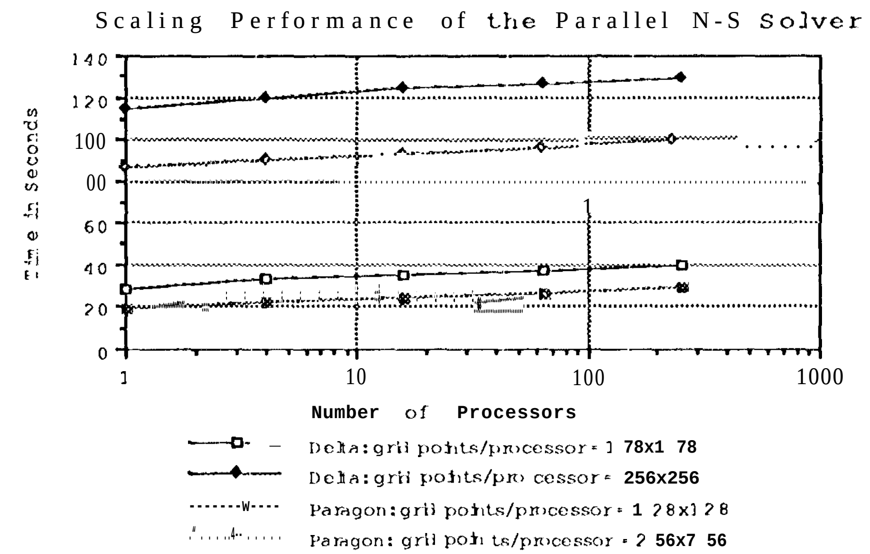
more work in parallel implementation is the solutions of Halmholtz and Poisson equations with a multigrid scheme. Wc usc a full V-cycle multi grid scheme (Briggs [3]) with red-black relaxation in our parallel multigrid solver, Since some of the coarse grids arc distributed in subsets of' working processors which contain the physical finest grid, some extra work is needed for processing on those coarse grids. Our strategy is to set up an hierarchy of (logical) processor meshes corresponding to the hierarchy of multigrid. The first fcw finer grids usuall y arc mapped to the original processor mesh. This approach avoids global communications for processing on coarse grids and the processing pattern remains the same on all grids.

2, Numerical and Parallel Performances

The parallel 2D N-S solver has been tested on a smooth model problem and it shows a sccmd-order convergence rate, as expected. The parallel solver has also been tested on a number of flow problems, including a 2D viscous driven-cavily flow and an inviscid jet flow in a doubly periodic box. The scaling performance of the solver on Intel Delta and Paragon is shown below, using up to 256 processors with the largest global grid of 4096x4096. It can bc seen that the flow solver scales very well on these machines for moderate sizes of grid in each processor. For future work, wc plan to extend the parallel 2D flow solver to 3D cases and generalize i! to deal with problems with variable density and temperature variations.



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2. A. J. Chorin, "Numerical Solution of the Navicr-Stokes Equations," Math, Comp., vol. 22, pp. 745-762, Oct. 1968.
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