TARGETING AN OPTIMAL LUNAR TRANSFER TRAJECTORY USING BALLISTIC CAPTURE

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A new method is described for design of an Earth-Moon transfer trajectory with substantial savings of propellant over classical methods. This trajectory flies by the Moon to a region of space between the Earth and the Sun near the sphere of influence of the Earth-Moon system and returns to the Moon via ballistic capture. This method was successfully applied to the Japanese spacecraft Hiten and other potential applications are being considered.

INTRODUCTION

The classical method for design of lunar transfer trajectories is the Hohmann transfer orbit. A spacecraft is injected into an orbit about the Earth at perigee that intersects the Moon's orbit near apogee. An orbit transfer maneuver is performed at lunar periapsis that places the spacecraft in a closed orbit about the Moon. This type of trajectory was used by the Apollo mission and other missions to the Moon and includes many variations including free return trajectories.

In this paper, a new orbit transfer method is described that achieves lunar orbit with substantial savings of propellant. This method involves injection of a spacecraft on a trajectory that flies by the Moon and receives a gravity assist that takes the spacecraft to a region of space between the Earth and Sun near the sphere of influence of the Earth-Moon system. The sphere of influence as defined here is a region of space about 1.5 million km from Earth where the gravitational acceleration of the Earth, Moon, and Sun tend to cancel when combined with the inertial acceleration of the spacecraft. The resulting transfer orbit is approximately three months long and reencounters the Moon on return to the Earth-Moon system,

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The return trajectory is designed such that the spacecraft is ballistically captured by the Moon. The four body interaction of the Earth, Moon, Sun, and spacecraft is instrumental in the orbit design and contributes to the savings in propellant,

Spacecraft propellant consumption is related to the velocity change imparted to the spacecraft which is referred to as AV. The only spacecraft propulsive velocity change required for lunar capture once the spacecraft leaves the vicinity of the Earth is a small maneuver that is performed in lunar orbit to stabilize the orbit. By contrast, the Hohmann transfer type trajectory requires over 200 m/s AV to achieve lunar orbit.

The resulting ballistic capture orbit about the Moon has energy relative to the Moon near escape. The spacecraft will not stay captured for very many orbits unless an orbit stabilization maneuver is performed. For a periapsis altitude of 100 km, the capture orbit eccentricity is about 0.94 and a maneuver AV of about 30 m/s, performed at periapsis, should suffice for stabilization. A considerable expenditure of propellant would be required to achieve circular orbit; however, an orbit eccentricity of 0.90 should be adequate for lunar observation, Some missions require high inclination orbits with the periapsis point near the equator for photographic reconnaissance or gravity field determination. The accessibility of these orbits from Hohmann transfer trajectories is limited by the direction of the hyperbolic excess velocity vector. Since capture orbits approach the Moon with essentially zero relative velocity, a much wider range of orbit orientations are accessible.

The successful utilization of this trajectory by the Japanese spacecraft Hiten is described. It is shown that considerable savings in propellant amounting to 10% of the injected spacecraft mass may be achieved, This savings may be used for increased science pay load or a reduction in the required launch vehicle performance.

CLASSICAL METHODS OF EARTH-MOON TRANSFER

Travel between the Earth and the Moon involves the solution of a two point boundary value problem. In its simplest form, wc have a launch site on Earth and a landing site on the Moon and wc arc interested in finding the path that connects these two points with minimum fuel expenditure. The problem of travel to the Moon may be conveniently separated into three separate phases; these being launch into Earth orbit, transfer to lunar orbit, and descent from lunar orbit to the surface of the Moon. The problems of launch into Earth orbit and descent from lunar orbit have been considered elsewhere and in this paper wc are concerned with the optimum transfer between Earth orbit and lunar orbit.

An analytic solution of the optimum transfer trajectory between two circular orbits was obt ained by Walter Hohmann[1] in 1925 for the restricted two body problem. This solution is called the Hohmann transfer orbit which is an ellipse with the periapsis at the point of tangency with a circular Earth orbit and the apoapsis at the Moons orbit. Large thrusting propulsive maneuvers arc performed at both perigee and apogee to transfer a spacecraft from Earth orbit to orbit about the Moon,

When the masses of the Earth, Moon and Sun arc included, a minimum propellant consumption solution that is c1osc to the Hohmann transfer orbit may be obtained numerically. This numerical solution may be found on a computer using a trajectory search and optimization method.[2] In order to perform this search, wc must first formulate the problem in terms that arc amenable to a computer solution. For convenience, the orbit about the Earth is described in an inertial coordinate system with the x axis pointing toward the Moon at initial epoch, the z axis normal to the Moon's orbital plane, and the y axis completing the right hand system. The Earth departure trajectory initial conditions are defined by a modified set of osculating classical orbit elements consisting of periapsis radius (r_{p_e}), apoapsis radius (r_{a_e}), time from periapsis passage of injection (Δt_{p_e}), ascending node (Ω_e), inclination (i_e), and argument of periapsis (w_e). The actual initial conditions used for trajectory propagation arc Cartesian Earth mean equator and equinox of the year 2000 and are obtained by direct transforrnation of the local osculating orbit elements. The elements r_{p_e}, ω_e, and Δt_{p_e} relate to the launch site and constraints associated with the launch vehicle ascent trajectory. The elements Ω_e and i_e describe the direction of the Earth departure velocity vector. The element Ω_e is most directly associated with the time of launch and i_e is associated with the launch azimuth. When constraints are placed on the periapsis radius or altitude of the Earth injection point, the element $r_{a\,e}$ may bc used as a parameter to define the launch energy or propellant required to burn out of near Earth orbit, One other parameter is needed to define the Earth departure trajectory and this is the epoch of injection (T_e).

The orbit about the Moon may also be described by a modified set of classical orbit elements. These are periapsis radius (r_{p_m}), orbital eccentricity (e_m), time from periapsis passage of orbit insertion (Δt_{p_m}), ascending node (Ω_m), inclination (i_m), and argument of periapsis (ω_m). The epoch of orbit insertion (T_m) completes the parameter set necessary to define the orbit insertion point, The coordinate system is the same as was defined for Earth orbit only centered at the Moon. As an example, consider an orbit about the Moon with a periapsis radius of 100 km altitude and orbital eccentricity of 0.9 which is selected to give a stable orbit with a period of about 2.5 days. The inclination of the orbit is free to bc specified and is selected to place the orbit insertion point on the Earth side of the Moon resulting in a posigrade orbit, The angles Ω_m and ω_m that define the orientation of the orbit arc selected to be the same as the elements of the approach hyperbola and the orbit insertion maneuver is performed at pcriapsis. The accept ancc of the geometry associated with the approach hyperbola results in minimum propcllant expenditure to get into lunar orbit.

In order to completely define the transfer orbit from Earth injection to lunar orbit insertion, a total of eight parameters must bc specificd or dctcrmincd. Parametcrs that pertain to Earth injection and relate to the launch site and launch vchiclc constraints arc $r_{p_e}, \Delta t_{p_e}$ and ω_e. Additional Earth injection parameters that need to bc determined arc $r_{a\,e}, i_e, \Omega_e$, and T_e. Parameters that pertain to the orbit about the moon and place constraints on the transfer trajectory arc r_{p_m}, ω_m, i_m and T_m. Thc parameters that pertain to the Moon arc dcpcndcnt parameters and may bc determined as a function of Earth injection conditions by traj cctory propagation. Wc may thus define a trajectory search and optimization proccdurc where the target vector $\boldsymbol{\Psi}$ is given by

$$\boldsymbol{\Psi} = [\boldsymbol{\Psi}_\mathbf{i}, \boldsymbol{\Psi}_\mathbf{d}]$$

$$\boldsymbol{\Psi}_\mathbf{i} = [r_{p_e}, \Delta t_{p_e}, \omega_e]$$

$$\boldsymbol{\Psi}_\mathbf{d} = [r_{p_m}, i_m, T_m] \tag{1}$$

and the indcpcndcnt t argct paramct ers (Vi) are satisfied by specifying them and the dependent target parameters ($\boldsymbol{\Psi}_\mathbf{d}$) arc satisfied by targeting. The control vector **(U)** is given by

$$\mathbf{U} = [i_e, \Omega_e, T_e] \tag{2}$$

and is determined iteratively by targeting. The optimum trajectory may bc found by targeting to satisfy the dcpcndcnt target parameters ($\boldsymbol{\Psi}_\mathbf{d}$). Sincc there arc six indcpcndcnt target and control parameters specificd and eight parameters arc needed to completely specify the trajectory, wc select r_{a_e} and ω_m as two parameters that may be varied to minimize the orbit insertion AV subject to the constraint Ψ.

The resulting Hohrnann type transfer trajectory is shown in Fig. 1. The spacecraft closely follows the Hohmann ellipse until near the Moon. At this point, the Moon overtakes the spacecraft and the spacecraft is pulled into a trajectory that flies near the Moon. At lunar periapsis, a211 m/s propulsive maneuver is performed to insert the spacecraft into an orbit with a 100 km periapsis altitude and a 0.9 eccentricity. The Earth injection conditions result in an osculating elliptical transfer orbit with a pcriapsis radius of 6,544 ktn and apoapsis radius of 385,000 km. This corresponds to a launch energy (C_3) of $-2.04\,\mathrm{km}^2/\mathrm{s}^2$ and requires approximately 3.138 km/s of propulsive velocity increment (ΔV_e) to burn out of a circular Earth orbit.

A NEW METHOD OF ORBIT TRANSFER

A new method of Earth-Moon transfer provides an alternate solution to the Hohmann transfer described above. This method involves construction of a trajectory that flies from the Earth to the Moon and receives a gravity assist that boosts the spacecraft orbit to a region of space about 1.5 million km from Earth where the gravitational acceleration of the Earth- Moon system and Sun tend to balance when combined with the inertia] acceleration of the spacecraft. Within this region, a small trajectory shaping maneuver may be performed to return the spacecraft on a trajectory that results in ballistic capture by the Moon,

A trajectory of this type has been studied by Tanbe et. al. for a lunar swingby mission to the Moon's L4 Lagrange point.[3] The trajectory, shown on Fig. 2, swings by the Moon and goes out to a region of space about 1 million km from the Earth where a 195 m/s maneuver(ΔV_2) is performed to bring the spacecraft back to the vicinity of the Moon, This trajectory was close to the starting point for a numerical search to find the optimum transfer. The act ual trajectory used for the starting point was modified, based on a suggestion by Edward Belbruno, to capture at the Moon. In astronomy, gravitational capture has received much attention and over 40 papers have been written on this subject.[4]

If the trajectory shown on Fig. 2 is expanded slightly, the spacecraft will escape into orbit around the Sun. However, if a carefully conducted search is performed to the very edge of the escape region, the maneuver at V_2 vanishes. The search must be performed very carefully, as will be discussed below, following the gradient of the performance criterion (ΔV_2) and it is very unlikely that this result could be found by accident. Furthermore, the existence of a ballistic trajectory that connects an Earth departure trajectory with a lunar capture trajectory was not predicted by any theory known to the author at the time of its discovery although speculation was made by Fesenkov, as discussed below, about its possible exist ence. The discovery of this trajectory[5] was made by the author on Memorial Day weekend 1990 working alone on the Navigation System computers at the Jet Propulsion Laboratory.

Construction of the transfer trajectory begins with design of the capture trajectory at the Moon. The spacecraft is placed in an orbit about the Moon with a periapsis altitude of 100 km and the periapsis point over the sub-Earth point. The spacecraft orbital eccentricity is initialized at about 0,94 which places the spacecraft in an orbit with nearly escape velocity relative to the Moon. A marginally stable orbit results and the eccentricity of the orbit is adjusted until the spacecraft acquires the right energy to escape from the Moon. Since we are interested in a ballistic capture orbit, the spacecraft trajectory is integrated backward in time. Some typical ballistic capture orbits for various values of the orbit eccentricity are shown on Fig. 3. These orbits were obtained by backward integration of the four-body equations of motion. Over the narrow range of eccentricities between 0.941 and 0.943, the spacecraft orbit transitions between a stable Earth bound orbit and an orbit that escapes the Earth-Moon system. At a critical eccentricity of about 0.94171, the spacecraft orbit lingers in the region of space defined here as the sphere of influence and then falls back toward the Earth. In order to complete the trajectory design, the backward integrated trajectory must be brought to the near vicinity of the Earth, This is partially accomplished by selecting an eccentricity that brings the spacecraft near the Moon on return. The backward integrated trajectory thus receives a gravity assist that brings the spacecraft back to the Earth as shown on Fig, 3. When we reverse the time integration, and integrate the equations of motion forward, we have a completely ballistic trajectory that goes from near Earth orbit to orbit about the Moon and the only propulsive maneuver required is the Earth injection maneuver. The existence of a lunar capture trajectory for a spacecraft launched from the Earth has been the subject of much speculation. It was demonstrated by V. G. Fesenkov[6] that, for the restricted three body problem, the Moon cannot capture a spacecraft launched from the Earth on the first circuit of the trajectory. However, Fesenkov's analysis ignores the perturbations caused by the Sun that can lead to closure as discussed by V. A, Egorov.[7]

The above trajectory which goes from the vicinity of Earth to lunar orbit numerically demonstrates a solution of the problem posed by Fesenkov and others. This trajectory is ballistic and includes all four bodies simultaneously and cannot be obtained by piecing together two body or three body problems. The existence of a ballistic trajectory that numerically demonstrates a solution to a four body problem suggests that a theory that includes four bodies may be readily found to explain this result. This theory was given the name "Weak Stability Theory" and the general approach to its development is described in References 8 and 9. It appears that this claim to a theory was premature and at this time we have no credible four body theory that can be used to predict or confirm the existence of trajectories of this type. Weak Stability Theory in its present form is essentially a three body theory and the Sun was introduced to the problem through the equations of motion used for the numerical search. A good description of the underlying orbital mechanics is given by Yamakawa[10] et al.

ORBIT TRANSFER METHODOLOGY

A numerical solution of the restricted four-body lunar transfer problem may be found by a technique that is similar to that used for the Hohmann transfer orbit described above. The Earth departure trajectory initial conditions and the orbit about the Moon arc defined by the same osculating classical orbit elements as described above for the Hohmann transfer method, These arc $r_{p_e}, r_{a_e}, \Delta t_{p_e}, \Omega_e$, i_e, and ω_e for the Earth injection orbit and $r_{p_m}, e_m, \Delta t_{p_m}, \Omega_m, i_m$, and ω_m for the orbit about the Moon, The coordinate systems arc also the same as described above for the Hohmann transfer method. The epoch of Earth injection (T_e) and the epoch of orbit insertion (T_m) completes the parameter set necessary to define a transfer orbit.

The launch site and launch vehicle ascent trajectory place constraints on the parameters r_{p_e}, Δt_{p_e} and ω_e. Similarly, the orbit about the Moon places constraints on the parameters r_{p_m}, i_m and T_m. Wc thus have the same two point boundary value problem that was defined above for the Hohmann transfer method. A numerical solution could be attempted using as an initial guess the trajectory shown in Fig. 2. However, the extreme nonlinearity of the trajectory propagation along the path suggested by this trajectory results in problems with convergence and indeed the end points may be outside of the region of convergence. For these reasons, the trajectory search and optimization procedure is separated into two parts.

In order to improve the linearity and force a solution, a small trajectory shaping maneuver is introduced at the sphere of influence. This maneuver increases the number of parameters needed to define a solution from eight to twelve. The increased robustness will guarantee that a solution exists even though a small spacecraft propulsive ΔV penalty is incurred. We thus seek a solution that minimizes this trajectory shaping maneuver.

For the trajectory construction procedure, the spacecraft is first propagated backward in time from lunar capture to a point in the Earth-Sun sphere of influence at time T_s defining a position $\mathbf{X}_s^+$ and a velocity $\mathbf{V}_s^+$. An independent forward propagation of the spacecraft trajectory from Earth injection to the same time (T_s), that may include a lunar swingby on the way, is performed to define the position $\mathbf{X}_s^-$ and velocity $\mathbf{V}_s^-$. A trajectory search and optimization procedure is used to find the solution satisfying

$$\mathbf{X}_s^+ = \mathbf{X}_s^-$$

and minimizes the performance criterion J where

$$J = |\mathbf{V}_s^+ - \mathbf{V}_s^-|$$

The trajectory search and optimization procedure is similar to that used above for the Hohmann transfer method. The target parameters $\mathbf{\Psi}$ are partitioned into independent parameters ($\mathbf{\Psi_i}$) that are satisfied by constraining them and dependent parameters ($\mathbf{\Psi_d}$) that are satisfied by targeting.

$$\mathbf{\Psi} = [\Psi_i, \Psi_d]$$

$$\mathbf{\Psi_i} = [r_{p_e}, \Delta t_{p_e}, \omega_e, r_{p_m}]$$

$$\mathbf{\Psi_d} = [\mathbf{X}_s^+, \mathbf{X}_s^-] \tag{3}$$

The dependent constraints may be determined as functions of the independent constraints and control parameters by trajectory propagation. The control vector **(U)** is given by

$$\mathbf{U} = [T_e, i_e, \Omega_e] \tag{4}$$

The optimum trajectory may be found by first targeting the control parameters to make the position of the spacecraft at T_s determined by propagating the trajectory forward from Earth injection ($\mathbf{X}_s^-$) equal to the position of the spacecraft determined by propagating the trajectory backward from lunar capture ($\mathbf{X}_s^+$). Since the number of independent constraint parameters and control parameters total seven and twelve parameters are needed to completely specify the trajectory, we may select five additional independent parameters that may be varied to minimize J subject to the constraint Ψ. The parameters $r_{a\,e}, c_m, \Delta t_{p_m}, i_m, T_m, \Omega_m, \omega_m$ and T_s are selected. The parameter $r_{a\,e}$ may be varied to trade launch vehicle propellant for spacecraft propellant required for the maneuver. The parameters c,,,, T,,,, and T_s are most effective for trajectory shaping and minimization of J.

A typical example of a lunar transfer trajectory using this method is illustrated in Fig. 4. The Earth is at the center of an inertial coordinate system with the $x - y$ plane coincident with the Moon's orbit plane. The spacecraft leaves the Earth and flies by the Moon where it receives a gravity assist. It continues on to the sphere of influence where a 31 m/s maneuver is performed that returns the spacecraft to the Moon via a capture trajectory, Also shown on Fig, 4 is the position of the Sun relative to the Earth and spacecraft trajectory. For this example, the Sun is on the opposite side of the Earth from the spacecraft during the time that the spacecraft is in the sphere of influence. Other examples have been generated with the Sun on the same side as the spacecraft. Further analysis has indicated that the Sun is primarily instrumental in removing angular momentum from the spacecraft orbit. Thus, the orbit dynamics associated with this method arc related to the solar tide and launch opportunities occur twice within the lunar month. A launch period of two or three days is expected for each launch opportunity. The launch period may be extended by techniques such as parking the spacecraft in Earth orbit a number of revolutions before the initial fly by of the Moon.

COMPARISON OF ORBIT TRANSFER METHODS

A comparison of various methods of lunar transfer is shown in Table 1. The basis for comparison is the Hohmann transfer orbit described above that requires a launch energy of –2. 04 km^2/s^2 corresponding to an Earth departure velocit y magnitude of 3.138 km/s, a three day flight time, and lunar orbit insertion maneuver of 211 m/s to insert into an orbit of 100 km periapsis altitude and 0.9 orbital eccentricity. The total AV for the Hohmann transfer is thus 3.349 km/s. The lunar swingby trajectory designed by Tanabc et al to go to the Moon's L4 may be

Table 1
COMPARISON OF LUNAR TRANSFER METHODS FOR MISSION TO LUNAR ORBIT

Maneuver	Hohmarm Transfer (km/s)	Tanabc et al Transfer (km/s)	Ncw Method Ballistic (km/s)	Ncw Method Typical (km/s)
Launch Energy	3.138	3.130	3.191	3.180
Midcourse	0	.0.195	0	0.032
Lunar Orbit Insertion	0.211	0.028	0.028	0.028
Total	*3.349*	3.353	3.219	3.240

modified slightly to achicvc lunar orbit and and the AV required for this method is about 3.349 km/s, slightly more than the Hohmann transfer. The completely ballistic new method requires even lcss propellant expenditure, Only 28 m/s is needed to stabilize the orbit at the Moon and no propulsive maneuvers arc needed cnroutc except for navigational purposes. However, this method requires launch energy augmentation of 53 m/s which may bc obtained by requiring more launch vehicle performance or using spacecraft propcllant to supplcmcnt the launch vehicle. Thc total AV for this method is 3.219 km^2/s^2 which is more than 100 m/s lcss than required for a Hohmann transfer although a thrcc month flight time is required to realize this gain, The last column of Table 1 shows the propulsive AV for a typical trajectory obtained by targeting as dcscribcd above. The total AV is 3.240 km^2/s^2 which compares favorably with the ballistic case,

The attainment of a loosely bound capture orbit enables onc to dcorbit with a small amount of AV and rcndczvous with the Moon's L4 point. The paper by Tanabc[3] ct al deals directly with this problcm and providcs another point of comparison. The AV required for a mission to the Moon's L4 point is summarized in Table 2. For the case "of Hohmann transfer, a motor burn is performed at a lunar altitude of 100km. The results shown by Tanabc ct al assume a direct mission to L4 and requires substantially more AV since they did not take advantage of the Moon for gravity assist during capture, The ballistic new method can achicvc rendezvous wit h practically zero deterministic spacecraft A *V* after launch vehicle injection,

Other points of comparison relate to spacecraft rocket engine thrust and accessibilit y of orbits at the Moon. The Hohmann transfer method requires a large motor burn to bc performed in a short time at lunar orbit insertion to achicvc capture. Therefore, a relatively high thrust rocket engine is nccdcd. The ncw method may be implemented with a relatively low thrust rocket engine. , The orbit stabilization maneuver at the Moon may bc performed over many revolutions by spiraling into successively tighter orbits and the maneuver at the sphere of influcncc may bc performed over several weeks.

Table 2
COMPARISON OF LUNAR TRANSFER METHODS FOR MISSION TO THE MOON% L4

Maneuver	Hohmann Transfer (km/s)	Tanabc et al Transfer (km/s)	Ncw Methoı Ballistic (km/s)	New Method Typical (km/s)
Launch Energy	3.138	3.130	3.191	3.180
Midcourse	0	0.195	0	0.032
Near Moon Maneuver	0.183	0.167†	0	0
Total	3.321	3.491	3.191	3.212

† This ΔV could be reduced to about 28 m/s by performing the maneuver near the Moon rather than at L4.

For the Hohmann transfer method, the accessibility of orbits about the Moon is restricted by the direction of the approach asymptote at the Moon. Two im-port ant orbit parameters are the inclination and argument of pcriapsis relative to the Moon's equator. High inclination orbits arc generally desirable for orbital rc-connaissancc and the argument of pcriapsis determines the latitude of the pcriapsis point and thus the regions of the Moon where close observations may bc obtained, The inclination and argument of pcriapsis also relate directly to accessibility of landing sites on the Moon, A wide range of orbital inclinations may be obtained by targeting the approach asymptote to the appropriate aim point with respect to the target plane which is defined perpendicular to the approach asymptote. However, once an inclination is selected, only a small range of the parameter argument of periapsis is accessible without significant expenditure of additional AV, Thus, for the H ohmann transfer method, the inclination and argument of periapsis arc coupled and the trajectory designer dots not have complete freedom in controlling the latitude of the periapsis point relative to the Moons equator for a given inclination. The longitude of the ascending node on the Moon's equator is also coupled with the inclination and direction of the approach asymptote. This orbit parameter relates to Sun and Earth occultations and the longitude of the spacecraft ground track, Since this parameter varies periodically with the lunar month and Earth year, it is most easily controlled by selection of launch date,

For the ncw method of orbit transfer, the spacecraft approaches the Moon with essentially zero excess hyperbolic velocity and falls into a loosely bound capture orbit, Thus, a wide range of orbit orientations may be selected with little additional expenditure of AV. The inclination may be varied from equatorial to polar and the latitude of periapsis placed within plus or minus 40 degrees of the equator, independent of onc another, for about 1 m/s in additional overall AV penalty. The longitude of the ascending node is near the sub-Earth point for the orbits studied and further study is required to define the range of this parameter.

APPL1CATIONS

The first application of the method of lunar transfer was the Japanese spacecraft Hiten. A transfer trajectory similar to that shown on Fig. 4 numerically demonstrated a way to get Hiten into orbit about the Moon via ballistic capture for a total of 44 m/s AV. This is accomplished by modifying the Hiten ellipse to fly by the Moon and phase into the orbit transfer trajectory.

Other missions that may benefit from this ncw method are those missions involving transport of large amounts of freight to the Moon or at the other end of the spectrum those missions involving low thrust rocket engines. A fleet of space tugs could bc launched sequentially on the approximately three month transfer trajectory and then collcctcd in lunar orbit for latcr descent to the” surface at savings approaching 10% in cost of mass dclivcrcd to the Moon. On the other hand, spacecraft employing low thrust rocket engines could transfer to the Moon on a trajectory that would not bc possible with the Hohmann transfer method bccausc of the need for high thrust for lunar orbit insertion.

CONCLUSION

This paper has described a ncw method for design and optimization of lunar transfer trajectories. Considerable savings of propellant arc shown over classical methods of orbit transfer such as the Hohmann transfer method. In order to realize the savings, a threc month flight time is required compared with three or four days for the Hohmann transfer. The savings in AV required to bc performed by the spacecraft ranges from 100 to 200 m/s. This translates into a 5 to 10% reduction in spacecraft propellant that may bc used for sciencc payload.

Another advantage of this method is in utilization of low thrust propulsion systems. Since the transfer trajectory is nearly completely ballistic, thrusting propulsivc maneuvers may bc performed over a long time duration.

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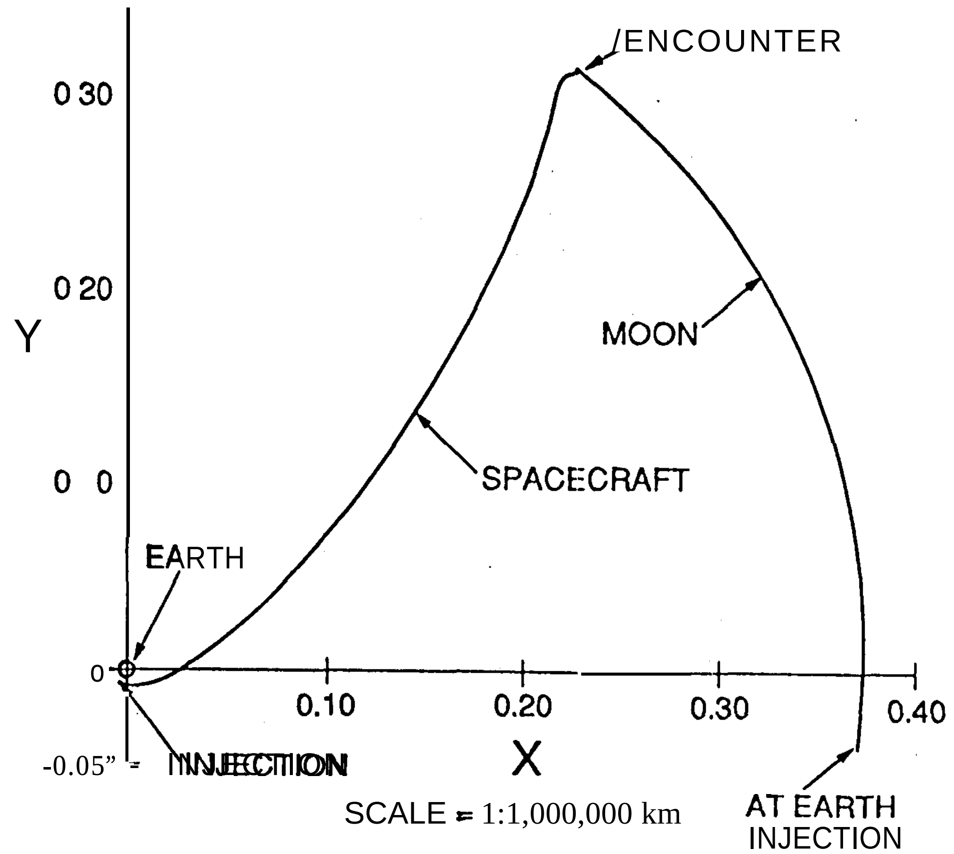
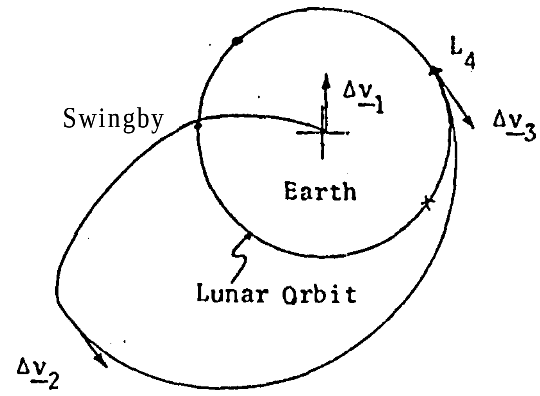


Fig. 1 Hohmann Method of Lunar Transfer Trajectory



o : Moon position at launch
x : Moon position at arrival

Encounter Conditions
λ = S.691 (optimal value)
M = l
Total Impulse
A v_T = 3.491 (minimum value)

Fig. 2 Lunar Swingby Trajectory

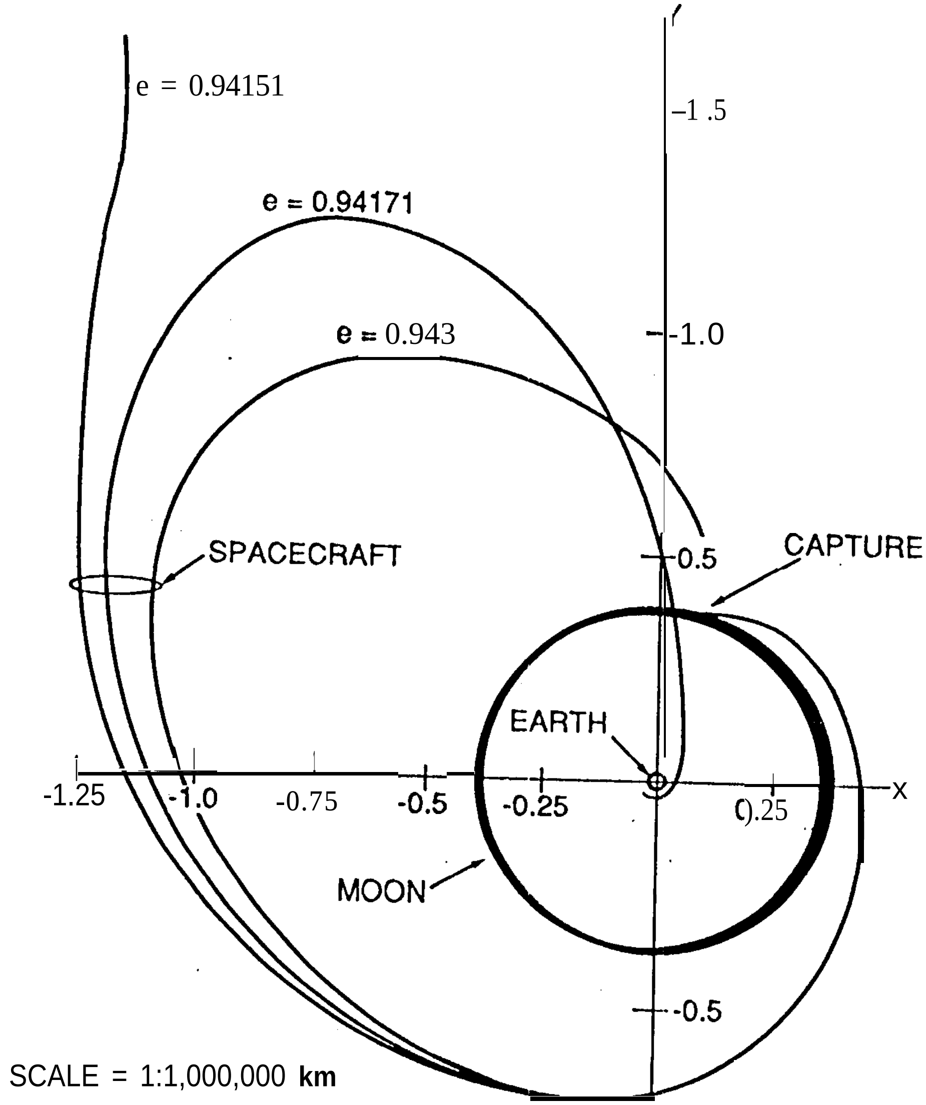
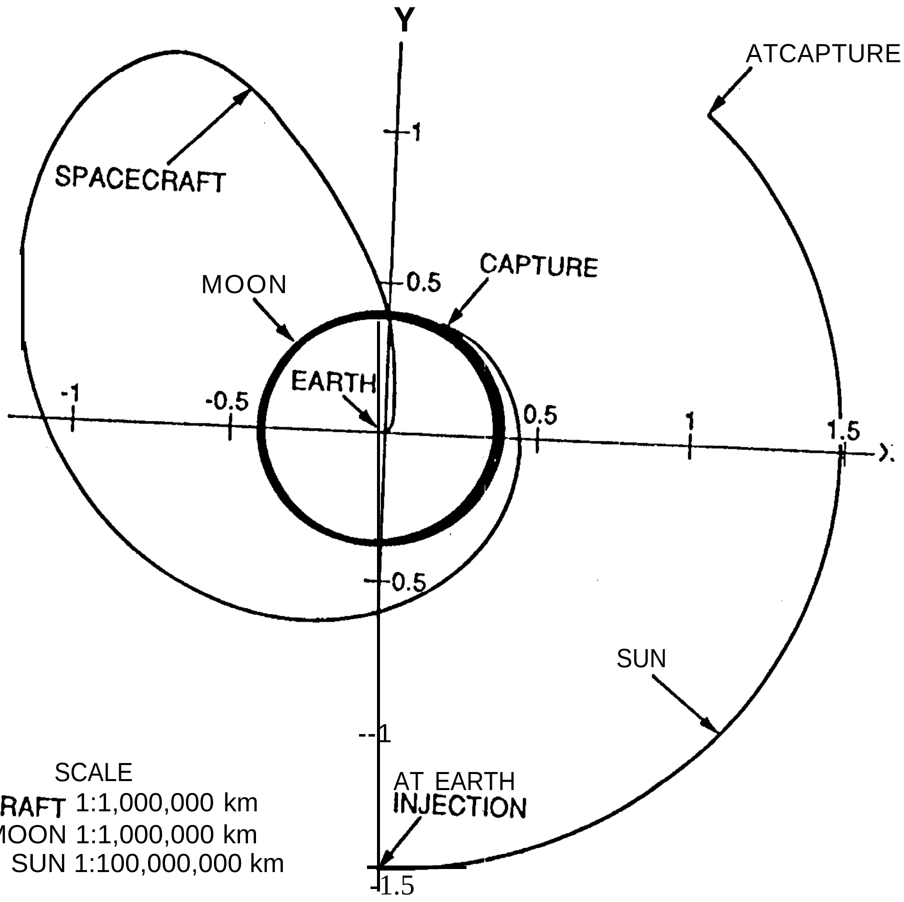


Fig. 3 Examples of Lunar Capture Orbits



SCALE
SPACECRAFT 1:1,000,000 km
MOON 1:1,000,000 km
SUN 1:100,000,000 km

Fig. 4 **Example of the New Lunar Transfer Trajectory**