Improved CDMA Performance Using Parallel interference Cancellation*

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Abstract

This paper considers a general parallel interference cancellation scheme that significantly reduces the degradation effect of user interference but with a lesser implementation complexity than the maximum-likelihood technique. The scheme operates on the fact that parallel processing simultaneously removes from *each* user the total interference produced by the remaining most *reliably* received users accessing the channel. The parallel processing can be done in multiple stages. The proposed scheme uses tentative decision devices with different optimum thresholds at the multiple stages to produce the most reliably received data for generation and cancellation of user interference. The l-stage interference cancellation was analyzed for three types of tentative decision devices, namely hard, null zone, and soft decision. Simulation results are given for 1- and 2-stage interference cancellation for equal as well as unequal power users.

Introduction

Multiuser communications systems that employ code division multiple access (CDMA) exhibit a user capacity limit in the sense that there exists a maximum number of users that can simultaneously communicate over the channel for a specified level of performance per user. This limitation is brought about by the ultimate domination of the other user interference over the additive thermal noise. Over the years researchers have sought ways to extend the user capacity of CDMA systems either by employing optimum (maximum-likelihood) detection [1] or interference cancellation methods [2-7]. In this paper, wc discuss a general parallel interference cancellation scheme that significantly reduces the degradation effect of user interference but with a lesser implementation complexity than the maximum-likelihood technique. The proposed scheme operates on the fact that parallel processing simultaneously removes from cacb user the total interference produced by the remaining reliable received users accessing the channel. In this way, *each* user in the system receives equal treatment in so far as the attempt is made to completely cancel his or her multiple user interference.

When compared with classical CDMA having no interference cancellation and also with the successive (serial) interference cancellation technique previously proposed by Viterbi [3] in which user interference is sequentially removed onc user at a time (the first user sees ail of the interference and last user sees none) , the parallel cancellation scheme discussed here achieves a significant improvement in performance. Aside from increasing the user capacity, the parallel cancellation scheme has a further advantage over the serial cancellation scheme with regard to the required delay necessary to fully accomplish the interference cancellation for all users in the system. Since in the latter, the interference cancellation proceeds serially, a delay on the order of M bit times (M denotes the number of simultaneous users in the CDMA system) is required whereas in the former, since the interference cancellation is performed in parallel for all users, the delay required is only one bit time for single stage cancellation.

1.0 Single Stage Interference Cancellation

1.1 *Tentative Hard Decisions- Equal Power, Synchronous Users*

We consider first the performance of the single stage parallel interference cancellation scheme illustrated in Fig. 1 where the tentative decision devices associated with each user are onc bit quantizers (hard decisions). This particular case corresponds to the scheme proposed in [2] and [4]. Wc assume that all users have the same power; thus, it is sufficient to characterize only the performance of any one user, say the first, he or she being typical of all the others. Furthermore, wc assume that all users have synchronous data streams and purely random PN codes[1]. While the assumption of synchronous users is perhaps unrealistic from a practical standpoint, it can be shown [11] that the synchronous user case results in worst case performance and thus serves as a lower bound on the user capacity achievable with this scheme. Alternately stated, any degree of data asynchronism among the users will yield a better performance, e.g., more users capable of being supported for a given amount of SNR degradation, than that arrived at in this section.

In general, the received signal in Fig. 1 is the sum of M direct sequence BPSK signals each with power S_i, bit time T_b, and PN chip time T_c, and additive white Gaussian noise with single-sided power spectral density (PSD) Now/Hz which at baseband can be written in the complex form[2]

$$r(t)=\sum_{i=1}^{M}\sqrt{S_i}\,m_i(t)PN_i(t)e^{j\phi_i}+n(t) \tag{1}$$

where for the ith user $PN_i(t)$ is the PN COdC, $m_i(t)=\sum_{k=-\infty}^{\infty}a_{ki}p(t-kT_b)$ is the modulation with kth bit a_{ki} taking on equiprobable values ±1 and unit power rectangular pulse shape $p(t)$ of duration T_b, and ϕ_i is the carrier phase. For our case of interest here, $S_i=S; i=1,2,...,M$. After despreading and demodulating[3] $r(t)$ with user 1's PN code and carrier reference signal (both of these operations arc assumed to be ideal), the normalized output of the I&D circuit is given by

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[1] For very long linear feedback shift registers, PN codes can be assumed to be purely random.

[2] For convenience, wc shall use complex notation to represent the various signals in the receiver.

[3] Since wc are working with a baseband model, the term "remodulation" nr "demodulation" refers to complex multiplication by the particular user's carrier phase or its complex conjugate, respectively.

$$x_{01} = a_{01}\sqrt{E_b} + n_1 e^{-j\phi_1} + \sum_{i=2}^{M} a_{0i} n'_{1i} e^{j(\phi_i - \phi_1)}$$

$$\triangleq a_{01}\sqrt{E_b} + n_1 e^{-j\phi_1} + \sqrt{E_b}\sum_{i=2}^{M} a_{0i}\gamma_{1i} e^{j(\phi_i - \phi_1)} \tag{2}$$

where $E_b = ST_b$ denotes the bit energy, a_{0i} is the polarity of user i's bit in the interval $0 \le t \le T_b$, $n_1 = \frac{1}{\sqrt{T_b}}\int_0^{T_b} n(t)PN_1(t)dt$ is a zero mean complex Gaussian random variable with variance $E\{|n_1|^2\} = N_0$ representing the thermal noise, and $n'_{1i} = \frac{\sqrt{S}}{T_b}\int_0^{T_b} PN_1(t)PN_i(t)dt \triangleq \sqrt{E_b}\gamma_{1i}$; $i = 2,3,\ldots,M$ are *the* interference noises contributed by the other M-1 users which are modeled as independent zero mean Gaussian random variables each with variance ST_c.[4] Also, the first subscript on x denotes the stage at which we are observing the I&D output while the second subscript denotes the particular user. This notation will be useful later on in our discussion of multiple stage cancellation schemes. The foregoing modeling of user interference as additive Gaussian noise follows from the assumptions made in similar analyses of CDMA systems [8,9], namely, a large spreading ratio $\eta = T_b/T_c$, and purely random PN codes.

Tentative hard decisions are made on the signals x_{0i}; $i = 1,2,\ldots,M$ and are used in an attempt to cancel the other user interference. If a correct tentative decision is made on a particular other user's bit, then the interference from that user can be completely cancelled. On the other hand, if an incorrect tentative decision is made, then the interference from that user will be enhanced rather than cancelled. A quantitative description of this will be given when we model the signal upon which *final* decisions are made. As we shall see, the performance analysis associated with this model is complicated by the, fact *the tentative **decisions** are not **independent of** one another.* More about this shortly.

After respreading/remodulation, interference cancellation, and despreading/demodulation, the normalized output of the I&D corresponding to the *final* decisions is given by

$$x_{11} = a_{01}\sqrt{E_b} + n_1 e^{-j\phi_1} + \overbrace{\sqrt{E_b}\sum_{i=2}^{M}\beta_i\gamma_{1i}e^{j(\phi_i - \phi_1)}}^{I_1} \tag{3}$$

where

$$\beta_i = a_{0i} - \mathrm{sgn}\left[\mathrm{Re}\left\{\sqrt{E_b}\left(a_{0i} + \sum_{\substack{m=1\\ m\ne i}}^{M} a_{0m}\gamma_{im}e^{j(\phi_m - \phi_i)}\right) + n_i e^{-j\phi_i}\right\}\right] \tag{4}$$

is a three-valued (0, ± 2) indicator random variable whose magnitude represents whether or not a correct tentative decision is made on the ith user's bit. It is tempting to model the β_i's as independent random variables. Unfortunately, this leads to optimistic results (when compared with the true performance results obtained from simulation). In addition to the fact that the β_i's are not themselves independent, they are. also dependent on the PN crosscorrelations, i.e., the γ_{1i}'s. Fortunately, however, the β_i's are not strongly dependent, i.e., the only terms that preclude *complete* independence of say β_i and β_j are $a_{0i}\gamma_{ij}$ in β_i and $a_{0j}\gamma_{ji} = a_{0i}\gamma_{ij}$ **in** β_j. Hence, for sufficiently large M, it is reasonable to assume a Gaussian model for the total residual (after cancellation) interference term I_1 in (3). The accuracy of this model will improve as M increases (actually as the number of nonzero terms in I_1 increases which implies a high tentative decision error rate). We shall be more detailed about this issue later on when comparing the performance results derived from this analytical model with those obtained from a true computer simulation of the receiver.

Assuming then a Gaussian model for I_1 (note that I_1 is not zero mean), then the average probability of error associated with the final decisions is given by

$$P_b(E) = \tfrac{1}{2}\Pr\left\{\mathbf{Re}\left\{x_{11} > 0\big|_{a_{01}=-1}\right\}\right\} + \tfrac{1}{2}\Pr\left\{\mathrm{Re}\left\{x_{11} < 0\big|_{a_{01}=1}\right\}\right\}$$

$$= \Pr\left\{\mathrm{Re}\left\{x_{11} > 0\big|_{a_{01}=-1}\right\}\right\} \tag{5}$$

$$= \Pr\left\{N_t > \sqrt{E_b} - \sqrt{E_b}\sum_{i=2}^{M}\overline{\beta_i\gamma_{1i}\cos(\phi_i - \phi_1)}\right\}$$

where[5]

$$N_t = \mathrm{Re}\left\{n_1 e^{-j\phi_1} + I_1 - \overline{I_1}\right\}$$

$$= N_1 + \sqrt{E_b}\sum_{i=2}^{M}\beta_i\gamma_{1i}\cos(\phi_i - \phi_1) - \sqrt{E_b}\sum_{i=2}^{M}\overline{\beta_i\gamma_{1i}\cos(\phi_i - \phi_1)}$$

is the effective noise seen by user 1 after cancellation which in view of the above is modeled as a real zero mean Gaussian noise random variable whose thermal noise component N_1 has variance $\sigma^2_{N_1} = N_0/2$. It is straightforward to compute the variance of N_t as

$$\sigma^2_{N_t} = E_b(M-1)\overline{\beta_i^2\gamma_{1i}^2\cos^2(\phi_i - \phi_1)} - E_b(M-1)^2\left(\overline{\beta_i\gamma_{1i}\cos(\phi_i - \phi_1)}\right)^2$$

$$+ E_b(M-1)(M-2)\overline{\beta_i\gamma_{1i}\beta_j\gamma_{1j}\cos(\phi_i - \phi_1)\cos(\phi_j - \phi_1)} \tag{7}$$

where i can take on any value from the set 2, 3,..., M. Hence, from (5), the average probability can be obtained as

$$P_b(E) = Q\left(\sqrt{\frac{2E_b}{N_0}\Lambda}\right) \tag{6}$$

where

$$\Lambda \triangleq \frac{\left(1 - (M-1)\overline{\xi_{1i}}\right)^2}{1 + 2\frac{E_b}{N_0}(M-1)\left[\overline{\xi_{1i}^2} - (M-1)\left(\overline{\xi_{1i}}\right)^2 + (M-2)\overline{\xi_{1i}\xi_{1j}}\right]}; \tag{9}$$

$$\xi_{1i} \triangleq \beta_i\gamma_{1i}\cos(\phi_i - \phi_1)$$

is an SNR degradation factor (relative to the performance of a single BPSK user transmitting alone) and $Q(x)$ is the Gaussian probability integral defined by

$$Q(x) \triangleq \frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp\left(-\frac{y^2}{2}\right)dy \tag{10}$$

[4] The normalized interference noises γ_{1i}; $i = 2,3,\ldots,M$ have variance equal to the reciprocal of the spreading ration, i.e., $\eta^{-1} = T_c/T_b$.

[5] To simplify the notation here and in what follows, it is understood that the statistical mean $\overline{\beta_i\gamma_{1i}\cos(\phi_i - \phi_1)}$ is computed under the hypothesis $a_{01} = -1$.

Thus, the evaluation of $P_b(E)$ reduces to the evaluation of the various statistical averages (moments) of ξ_{1i} required in (9). These statistical averages, which must be performed over the Gaussian noise and interference random variables as well as the uniformly distributed carrier phases, are not trivial to compute. Nevertheless, they can be obtained in the form of definite integrals of tabulated functions with the following results ($a_{01} = -1$):

$$\overline{\xi_{1i}} = \frac{1}{2\pi}\int_0^{2\pi} \sqrt{\frac{2\sigma^2\cos^2\phi}{\pi}\left(\frac{\alpha^2\sigma^2\cos^2\phi}{1+\alpha^2\sigma^2\cos^2\phi}\right)} \exp\left\{-\frac{\alpha^2}{2\left(1+\alpha^2\sigma^2\cos^2\phi\right)}\right\} d\phi \tag{11a}$$

$$\overline{\xi_{1i}^2} = \frac{1}{2\pi}\int_0^{2\pi} \sqrt{\frac{8}{\pi}}\sigma^4\cos^4\phi\left(\frac{\alpha^2}{1+\alpha^2\sigma^2\cos^2\phi}\right)^{3/2} \exp\left\{-\frac{\alpha^2}{2\left(1+\alpha^2\sigma^2\cos^2\phi\right)}\right\} d\phi$$

$$+\frac{1}{2\pi}\int_0^{2\pi}\left(4\sigma^2\cos^2\phi\right) Q\left(\frac{\alpha}{\sqrt{1+\alpha^2\sigma^2\cos^2\phi}}\right) d\phi \tag{11b}$$

$$\overline{\xi_{1i}\xi_{1j}} = \left(\frac{1}{2\pi}\right)^2 \int_0^{2\pi}\int_0^{2\pi} \frac{2}{\pi} \frac{\sigma^4\cos^2\phi_1\cos^2\phi_2\sqrt{B_1B_2}}{\sqrt{1+\sigma^2\cos^2(\phi_1-\phi_2)(B_1+B_2)}}$$

$$\times\exp\left\{-\frac{1}{2}\left[\frac{B_1+B_2}{1+\sigma^2\cos^2(\phi_1-\phi_2)(B_1+B_2)}\right]\right\} d\phi_1\, d\phi_2 \tag{11c}$$

$$B_i = \frac{\alpha'^2}{1+\alpha'^2\sigma^2\cos^2\phi_i}; \quad i=1,2$$

with

$$\alpha \triangleq \sqrt{\frac{2\left(E_b/N_0\right)_R}{1+\left(\frac{M-2}{\eta}\right)\left(E_b/N_0\right)_R}},$$

$$\alpha' \triangleq \sqrt{\frac{2\left(E_b/N_0\right)_R}{1+\left(\frac{M-3}{\eta}\right)\left(E_b/N_0\right)_R}}, \quad \sigma^2 = \frac{1}{\eta} = \frac{T_c}{T_b} \tag{12}$$

where $\left(E_b/N_0\right)_R$ denotes the ***required*** bit energy-to-noise spectral density ratio for M users communicating simultaneously, each of which operates at an average bit error rate $P_b(E)$.

It is common in analyses of CDMA systems [8] to define a *degradation factor* ***(loss),*** D as the ratio (in dB) of the $\left(E_b/N_0\right)$ required to achieve a given bit error rate in the presence of M users, namely, $\left(E_b/N_0\right)_R$ to that which would be required to achieve the same level of performance if only a single user was communicating, namely, $\left(E_b/N_0\right)_1$. By the definition of $\left(E_b/N_0\right)_1$, *we* have

$$P_b(E) = Q\left(\sqrt{2\left(E_b/N_0\right)_1}\right) \tag{13}$$

To obtain the degradation factor for a given value of $P_b(E)$, we substitute $D\left(E_b/N_0\right)_1 = D\left[\frac{1}{2}\left[Q^{-1}\left(P_b(E)\right)\right]^2\right]$ for $\left(E_b/N_0\right)_R$ in (12) which in turn is substituted in (11). Then using the given value of $P_b(E)$ one can solve for D. Unfortunately, a closed form expression for D cannot be obtained so the results will be obtained numerically. Before presenting these numerical results, however, we briefly review the analogous results for conventional CDMA and the successive (serial) interference cancellation scheme proposed by Viterbi [3] (later patented by Dent [10]) since we shall use these as a basis of comparison to demonstrate the increased effectiveness of parallel cancellation.

1.1.1 *Comparison with Conventional CDMA and Successive Interference Cancellation*

In a conventional CDMA system, there is no attempt made to cancel the other User interference. Hence, $\left(E_b/N_0\right)_1$ is given by

$$\left(\frac{E_b}{N_0}\right)_1 = \frac{E_b}{N_0+(M-1)ST_c} = \frac{E_b/N_0}{1+(M-1)\eta^{-1}E_b/N_0} = \frac{\left(E_b/N_0\right)_R}{1+(M-1)\eta^{-1}\left(E_b/N_0\right)_R} \tag{14}$$

Thus, the degradation factor, D, is [8]

$$D = \frac{\left(E_b/N_0\right)_R}{\left(E_b/N_0\right)_1} = \frac{1}{1-(M-1)\eta^{-1}\left(E_b/N_0\right)_1} \tag{15}$$

For the successive cancellation scheme [3], Viterbi showed that to guarantee that each user in the system sees the same amount of interference from the other users, the user powers should be assigned as

$$S_k = S_1\left(1+\frac{S_1T_b}{N_0}\eta^{-1}\right)^{k-1}, \quad k = M, M-1, \ldots, 2 \tag{16}$$

where S_1 is the power of the user to processed last (the weakest one) and S_M is the power of the user to be processed first (the strongest one). Distributing the powers as in (16) ideally guarantees that all users see the same ratio of signal power to effective noise spectral density and thus the user to be processed first (the one that sees all the user interference) is not any SNR disadvantage relative to the user to be processed last (the one for which all interference has been removed). In view of the above, the degradation factor for the kth user is given by

$$D_k = \frac{\left(E_b/N_0\right)_{R_k}}{\left(E_b/N_0\right)_1} = \frac{S_k}{S_1} = \left(1+\eta^{-1}\left(E_b/N_0\right)_1\right)^{k-1} \tag{17}$$

where $\left(E_b/N_0\right)_{R_k}$ denotes the required bit energy-to-noise spectral density ratio for the kth user. The *average* degradation factor, D for the M user system is obtained by averaging (17) over k which yields

$$D = \frac{1}{M}\sum_{k=1}^{M} D_k = \frac{\left(1+\eta^{-1}\left(E_b/N_0\right)_1\right)^M - 1}{M\eta^{-1}\left(E_b/N_0\right)_1} \tag{18}$$

It should be emphasized that the result in (18) ignores the effect of decision errors made at the various successive interference cancellation stages, that is, the interference cancellation is assumed to take perfectly. As a result, numerical results derived from (18) will be optimistic when compared to the actual performance of the scheme.

1.1.2 *Numerical Results*

10 illustrate the significant performance advantage of the parallel interference cancellation scheme in Fig. 1, we consider a plot of D (loss) versus M for an average bit error probability[6] $P_b(E) = 10^{-2}$ and a spreading ratio $\eta = 100$. Figure 2 shows the analytical performance of the three schemes (conventional, successive interference cancellation, parallel interference cancellation) as well as computer simulation results for the latter. We see that for the conventional and parallel interference cancellation schemes there exists a user capacity limit in that regardless of how much one is willing to increase $(E_b/N_0)_{PC}$ (for a given $(E_b/N_0)_1$, or equivalently, a given $P_b(E)$), the required bit error rate cannot be achieved if more than $M_{\max}$ users simultaneously access the system. For conventional CDMA

$$M_{\max} = 1 + \frac{\eta}{\left(\frac{E_b}{N_0}\right)_1} = 1 + \frac{\eta}{\frac{1}{2}\left[Q^{-1}\left(P_b(E)\right)\right]^2} \tag{19}$$

whereas *for the* parallel interference cancellation scheme the solution is determined from

$$10^{-2} = Q\left(\frac{1-(M_{\max}-1)\overline{\xi_{1i}}}{\sqrt{(M_{\max}-1)\left[\overline{\xi_{1i}^2}-(M_{\max}-1)\left(\overline{\xi_{1i}}\right)^2+(M_{\max}-2)\overline{\xi_{1i}\xi_{1j}}\right]}}\right) \tag{20}$$

together with the moments in (11) where now

$$\alpha \triangleq \sqrt{\frac{2}{\eta^{-1}(M_{\max}-2)}}, \quad \alpha' \triangleq \sqrt{\frac{2}{\eta^{-1}(M_{\max}-3)}} \tag{21}$$

It is emphasized that the user capacity limit for the parallel interference cancellation scheme comes about entirely because of the finite probability of error associated with the tentative decisions. From Fig. 2 it appears that the successive interference cancellation does not have a user capacity limit. This is because in [3], it was assumed for this scheme that the interference cancellation is perfect, i.e., the effects of decision errors at the various interference cancellation stages were not accounted for.

Comparing the analytical and simulation results for the parallel interference cancellation scheme, we observe that the analytical results are somewhat optimistic. The discrepancy between the two stems from the assumption of an analytical Gaussian model for the total residual user interference in (3) whereas the computer simulation makes no such assumption and thus predicts the exact performance.

1.2 *Tentative Hard Decisions - Unequal Power, Synchronous Users*

The results of the previous section can be generalized to the case where the users have unequal powers, i.e., S_i, $i = 1,2,\ldots,M$. Let $\alpha_{ij} = S_i/S_j$ denote the ratio of the power of the ith user to that of the jth user who is arbitrarily considered to be the desired user. After interference cancellation, the normalized output of the I&D corresponding to the *final* decision of user j is by analogy with (3)

$$x_{1j} = a_{0j}\sqrt{E_{bj}} + n_j e^{-j\phi_j} + \underbrace{\sum_{\substack{i=1\\ i\neq j}}^{M}\sqrt{E_{bi}}\,\beta_i\gamma_{ji}e^{j(\phi_i-\phi_j)}}_{I_j}$$

$$= a_{0j}\sqrt{E_{bj}} + n_j e^{-j\phi_j} + \sqrt{E_{bj}}\sum_{\substack{i=1\\ i\neq j}}^{M}\sqrt{\alpha_{ij}}\,\beta_i\gamma_{ji}e^{j(\phi_i-\phi_j)} \tag{22}$$

where $n_j = \frac{1}{\sqrt{T_b}}\int_0^{T_b} n(t)PN_j(t)dt,\ j = 1,2,\ldots,M$ is a zero mean complex Gaussian random variable with variance N_0 representing the thermal noise of the jth user, $\gamma_{ji} \triangleq \frac{1}{T_b}\int_0^{T_b} PN_j(t)PN_i(t)dt;\ i \neq j$ are the normalized interference noises of the other $M-1$ users as seen by user j (γ_{ji} has variance η^{-1}—see footnote 1) and $E_{bi} \triangleq S_i T_b$ is the bit energy of the ith user. Also, analogous to (4), β_i is now defined by

$$\beta_i = a_{0i} - \mathrm{sgn}\left[\mathrm{Re}\left\{\sqrt{E_{bi}}\left(a_{0i} + \sum_{\substack{m=1\\ m\neq i}}^{M}\sqrt{\alpha_{mi}}\,a_{0m}\gamma_{im}e^{j(\phi_m-\phi_i)}\right) + n_i e^{-j\phi_i}\right\}\right] \tag{23}$$

Following steps analogous to (5) - (7) we arrive at the desired result for the bit error probability of the desired (the jth) user, namely,

$$P_{bj}(E) = Q\left(\sqrt{\frac{2E_{bj}}{N_0}\Lambda_j}\right) \tag{24}$$

where $(a_{0j} = -1)$

$$\Lambda_j \triangleq \frac{\left(1-\sum_{\substack{i=1\\ i\neq j}}^{M}\sqrt{\alpha_{ij}}\,\overline{\xi_{ji}}\right)^2}{1+2\frac{E_{bj}}{N_0}\left[\sum_{\substack{i=1\\ i\neq j}}^{M}\alpha_{ij}\overline{\xi_{ji}^2}-\left(\sum_{\substack{i=1\\ i\neq j}}^{M}\sqrt{\alpha_{ij}}\,\overline{\xi_{ji}}\right)^2+\sum_{\substack{i=1\\ i\neq j}}^{M}\sum_{\substack{m=1\\ m\neq j,i}}^{M}\sqrt{\alpha_{ij}\alpha_{mj}}\,\overline{\xi_{ji}\xi_{jm}}\right]};$$

$$\xi_{ji} \triangleq \beta_i\gamma_{ji}\cos\left(\phi_i-\phi_j\right) \tag{25}$$

As an example, consider a group of M users with powers exponentially distributed (linearly distributed on a dB scale) over a range of 10 dB between the minimum and the maximum. This model might correspond to a distribution of users that are exponentially distant from the base station within a cell. Assume that we fix the error probability of the *lowest power user* (assumed to be user 1 for convenience of notation) equal to 10^{-2} (all others would then obviously have a lower error probability). Then, Fig. 3 illustrates the degradation factor, D_1, of user 1 versus M. For comparison, the results corresponding to conventional CDMA with the same user power distribution are also shown in this figure. By comparison with Fig. 2, we observe that in the unequal power case, parallel interference cancellation offers more of an advantage over conventional CDMA. The reason behind this observation is that the larger power of the other users (which are producing the user interference to user 1) produces tentative decisions with a smaller error probability which in turn results in a better degree of cancellation with regard to the final decisions.

2.0 Parallel Interference Cancellation Using Null Zone Tentative Decisions

Much like the idea of including erasures in conventional data detection to eliminate the need for making decisions when the SNR

[6] The value of $P_b(E) = 10^{-2}$ is chosen to allow for obtaining computer simulation results in a reasonable amount of time.

is low, one can employ a null zone hard decision device (see eq. 27) for the tentative decisions to further improve the fidelity of the interference cancellation process. The idea here is that when a given user's signal to interference ratio is low, it is better not to attempt to cancel the interference from that user than to erroneously detect his data bit and thus enhance his interference. Following the development in Section 1,1 for a single stage scheme with equal power synchronous users, then the normalized output of the I&D corresponding to the final decision on user 1's bit a_{01} is still given by (3) with β_i now defined by

$$\beta_i = a_{0i} - \operatorname{nsgn}\left[\operatorname{Re}\left\{\sqrt{E_b}\left(a_{0i}\gamma_{0i} + \sum_{\substack{m=1\\ m\neq i}}^{M} a_{0m}\gamma_{im}e^{j(\phi_m-\phi_i)}\right) + n_i e^{-j\phi_i}\right\}\right] \tag{26}$$

where "nsgn" denotes the null zone signum function defined by

$$\operatorname{nsgn} x = \begin{cases} 1, & x > \zeta \\ 0, & -\zeta \le x \le \zeta \\ -1, & x < -\zeta \end{cases} \tag{27}$$

Here β_i takes on possible values (0, ±1, ±2) and its magnitude is an indicator of whether a correct decision is made (*ith* user's interference is perfectly cancelled), no decision is made (*ith* user's interference is unaltered), or an incorrect decision is made (ith user's interference is enhanced). Once again making a Gaussian assumption on the total residual inteference, then since the final decisions are still made as hard decisions, the average bit error probability is still given by (8) together with (9) with the statistical moments of ξ_{1i} now given by

where $\zeta' = \zeta/\sqrt{E_b}$ is the normalized decision threshold which should be chosen to minimize D for a given $P_b(E)$ and $(E_b/N_0)_1$ determined from (13). Superimposed on the performance results for the hard limiter previously given in Fig. 2 are the results for the null zone limiter. For the specified processing gain and average bit error probability, we see that using a null zone limiter allows the maximum number of users that can be supported to be increased by about 1070. For convenience, the normalized threshold has been fixed at $\zeta' = 0.2$. Here again, we see a modest improvement in performance. For an unequal (exponentially distributed) power distribution among the users, the corresponding results using null zone tentative decisions are superimposed on those previously discussed in Fig. 3. For convenience, the normalized threshold has been fixed at $\zeta' = 0.4$. Here again we see a modest improvement in performance.

3.0 Multiple Stage Interference Cancellation

The single stage scheme of Fig. 1 can be improved upon by cascading multiple stages of parallel interference cancellation.[7] The idea here is to repeatedly improve the fidelity of the M tentative decisions since each successive stage sees less and less interference. Note that in principle this idea is similar to what Viterbi accomplishes in the serial interference cancellation scheme except that here at each stage we simultaneously act on the interference from the most reliable users rather than one user at a time. An analysis of the performance of such a multistage scheme is difficult if not impossible to obtain due to the fact that the tentative decisions at the ith interference cancellation stage depend on the tentative decisions at the *(i–1)*st stage.

[7] For the case where the tentative decisions associated with each user are one bit quantizers (hard decisions) the scheme again reduces to that proposed in **[4-6]**.

$$\overline{\xi_{1i}} = \frac{1}{2\pi}\int_0^{2\pi}\sqrt{\frac{\sigma^2\cos^2\phi}{2\pi}\left(\frac{\alpha^2\sigma^2\cos^2\phi}{1+\alpha^2\sigma^2\cos^2\phi}\right)}\left[\exp\left\{-\frac{\alpha^2(1+\zeta')^2}{2(1+\alpha^2\sigma^2\cos^2\phi)}\right\} + \exp\left\{-\frac{\alpha^2(1-\zeta')^2}{2(1+\alpha^2\sigma^2\cos^2\phi)}\right\}\right]d\phi \tag{28a}$$

$$\begin{aligned}\overline{\xi_{1i}^2} = {} & \frac{1}{2\pi}\int_0^{2\pi}\sqrt{\frac{1}{2\pi}}\sigma^4\cos^4\phi\left(\frac{\alpha^2}{1+\alpha^2\sigma^2\cos^2\phi}\right)^{3/2}\left[3(1+\zeta')\exp\left\{-\frac{\alpha^2(1+\zeta')^2}{2(1+\alpha^2\sigma^2\cos^2\phi)}\right\} + (1-\zeta')\exp\left\{-\frac{\alpha^2(1-\zeta')^2}{2(1+\alpha^2\sigma^2\cos^2\phi)}\right\}\right]d\phi \\ & + \frac{1}{2\pi}\int_0^{2\pi}(\sigma^2\cos^2\phi)\left[3Q\left(\frac{\alpha(1+\zeta')}{\sqrt{1+\alpha^2\sigma^2\cos^2\phi}}\right) + Q\left(\frac{\alpha(1-\zeta')}{\sqrt{1+\alpha^2\sigma^2\cos^2\phi}}\right)\right]d\phi\end{aligned} \tag{28b}$$

$$\begin{aligned}\overline{\xi_{1i}\xi_{1j}} = {} & \left(\frac{1}{2\pi}\right)^2\int_0^{2\pi}\int_0^{2\pi}\frac{1}{2\pi}\frac{\sigma^4\cos^2\phi_1\cos^2\phi_2\sqrt{B_1B_2}}{\sqrt{1+\sigma^2\cos^2(\phi_1-\phi_2)(B_1+B_2)}}\left[\exp\left\{-\frac{1}{2}\left[\frac{(B_1+B_2)(1+\zeta')^2}{1+\sigma^2\cos^2(\phi_1-\phi_2)(B_1+B_2)}\right]\right\}\right. \\ & + \exp\left\{-\frac{1}{2}\left[\frac{4\zeta'^2B_1B_2\cos^2(\phi_1-\phi_2)+(1+\zeta')^2B_1+(1-\zeta')^2B_2}{1+\sigma^2\cos^2(\phi_1-\phi_2)(B_1+B_2)}\right]\right\} \\ & \left. + \exp\left\{-\frac{1}{2}\left[\frac{4\zeta'^2B_1B_2\cos^2(\phi_1-\phi_2)+(1-\zeta')^2B_1+(1+\zeta')^2B_2}{1+\sigma^2\cos^2(\phi_1-\phi_2)(B_1+B_2)}\right]\right\}\right]d\phi_1d\phi_2\end{aligned}$$

$$B_i = \frac{[illegible]}{1+\alpha'^2\sigma^2\cos^2\phi_i}; \quad i = 1,2 \tag{28c}$$

Because of this difficultly, numerical results for the performance of the multi-stage parallel interference scheme will be obtained from computer simulation. Illustrated in Fig. 2 are performance results for a 2 stage parallel interference canceller with hard and null zone[8] tentative decisions, respectively. We observe that there is significant gain to be achieved by going to more than one stage. Numerical results for a two stage parallel cancellation scheme with an unequal power distribution are superimposed on those previously discussed in Fig. 3. Here the normalized threshold for the first stage is fixed at $\zeta' = 0.4$ and for the second stage it is fixed at zero.

4.0 Parallel Interference Cancellation Using Infinitely Soft Quantized Tentative Decisions

One disadvantage of the parallel cancellation scheme with hard or null zone limiter tentative decisions is that, in order to perform the respreading and remodulating operations, the receiver must ideally have complete knowledge of each user's power, carrier phase and frequency, and PN code chip timing epoch. Since in practice the receiver does not have knowledge of these parameters, it must estimate them. One simple way of circumventing some of these problems is to use *linear* (infinitely soft quantization) tentative decisions. Since the signal component of the output of the tentative decision devices is now linearly proportional to the user powers, it is no longer necessary for the receiver to estimate these powers prior to the cancellation operation and thus the $\sqrt{S_i}$ gains following these devices in Fig. 1 may be eliminated. Another simplification that is now possible is that the final decisions can be performed with a differential (rather than coherent) detector thus eliminating the need for carrier synchronization at all stages.

The primary disadvantage to using linear tentative decisions is that additive thermal noise from each user is now introduced into the interference cancellation process. This will result in a performance that is inferior to the hard and null zone tentative decision schemes but still better than conventional CDMA which employs no interference cancellation at all. Furthermore, in principle, one can now *analytically* compute the performance of a multiple stage parallel interference scheme (using linear tentative decisions) although the analysis becomes quickly complex as the number of stages increases beyond two or three [11].

It is shown in [11] that for a single stage parallel cancellation scheme using linear tentative decision devices (soft decision) and with equal power, synchronous users the degradation factor is given by

$$D = \frac{\left(E_b/N_0\right)_R}{\left(E_b/N_0\right)_1} = \frac{1+\frac{M-1}{\eta}}{\left(1-\frac{M-1}{\eta}\right)^2 - \left(\frac{E_b}{N_0}\right)_1 \left(\frac{M(M-1)}{\eta^2}\right)} \qquad (29)$$

resulting in a maximum number of users given by

$$M_{max} = 1 + \frac{-2\eta - \left(\frac{E_b}{N_0}\right)_1 + \sqrt{\left(\left(\frac{E_b}{N_0}\right)_1\right)^2 + 4\eta(1+\eta)\left(\frac{E_b}{N_0}\right)_1}}{2\left(\left(\frac{E_b}{N_0}\right)_1 - 1\right)} \qquad (30)$$

[8] Ink null zone results of Fig. 2, the normalized thershold in the first stage has been fixed at <' -0.3 and in the second stage it has been scr equal to zero, i.e., hard-limited tentative decisions.

Table 1 tabulates the values of M_{max} as given by (30) and for comparison the values of M_{max} for conventional CDMA as given by (19) for several values of P_b. We observe that while an improvement relative to conventional CDMA exists, the infinitely soft tentative decision cancellation method is still quite inferior to the hard tentative decision case. Also the amount of improvement relative to CDMA increases as P_b decreases.

Conclusion

In this paper a parallel inter-fc.rcncc cancellation scheme was proposed that uses tentative decision devices with different optimum thresholds at the multiple stages to produce the most reliably received data for generation and cancellation of user interference. The 1-stage interference cancellation was analyzed for three types of tentative decision devices, namely hard, null zone, anti soft decision. Simulation results are given for 1- and 2-stage interference cancellation for equal as well as unequal power users. The performance results indicate that by using maltiple stages with optimum thresholds at each stage performance can significantly improve relative to the conventional CDMA.

TABLE 1.

Bit Error Rate	M_{max} Conventional CDMA	M_{max} l-stage Interference Soft Decision Cancellation
10^{-2}	37	38
10^{-3}	21	32
10^{-5}	11	25

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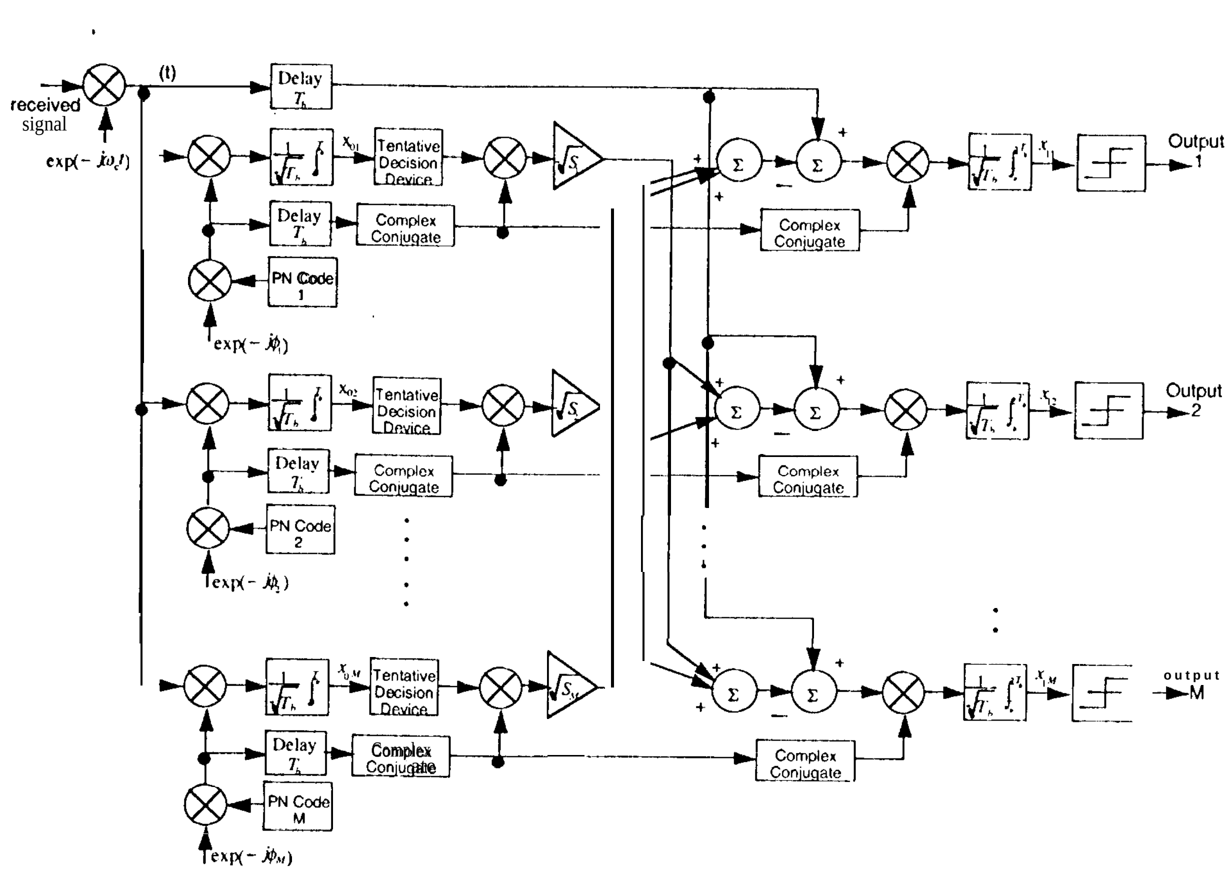

Figure 1. A single stage interference cancellation scheme with parallel processing for CDMA (complex baseband model)

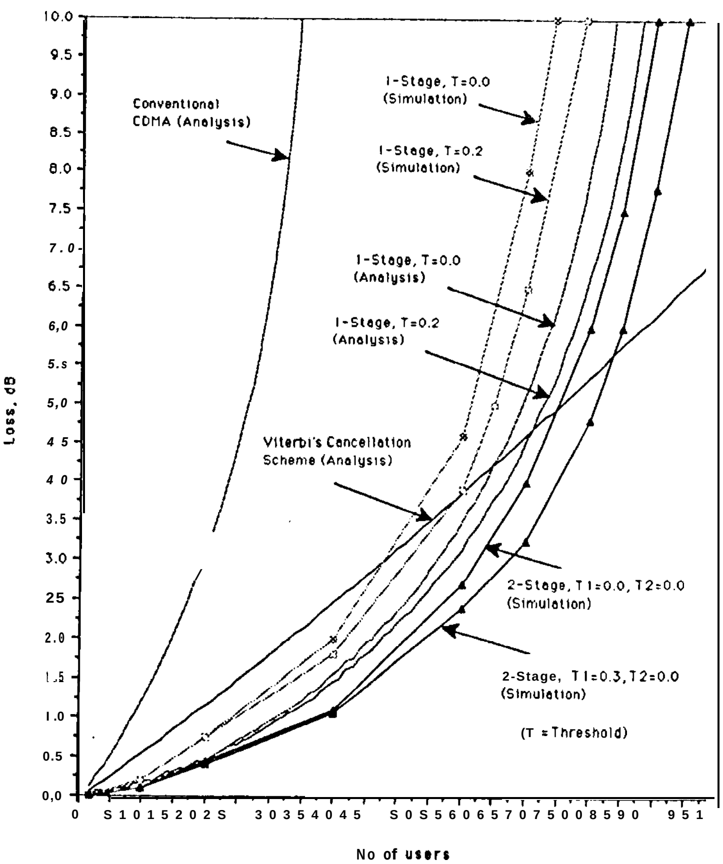

Figure 2. Performance of **interference** cancellation schemes **with equal** power users

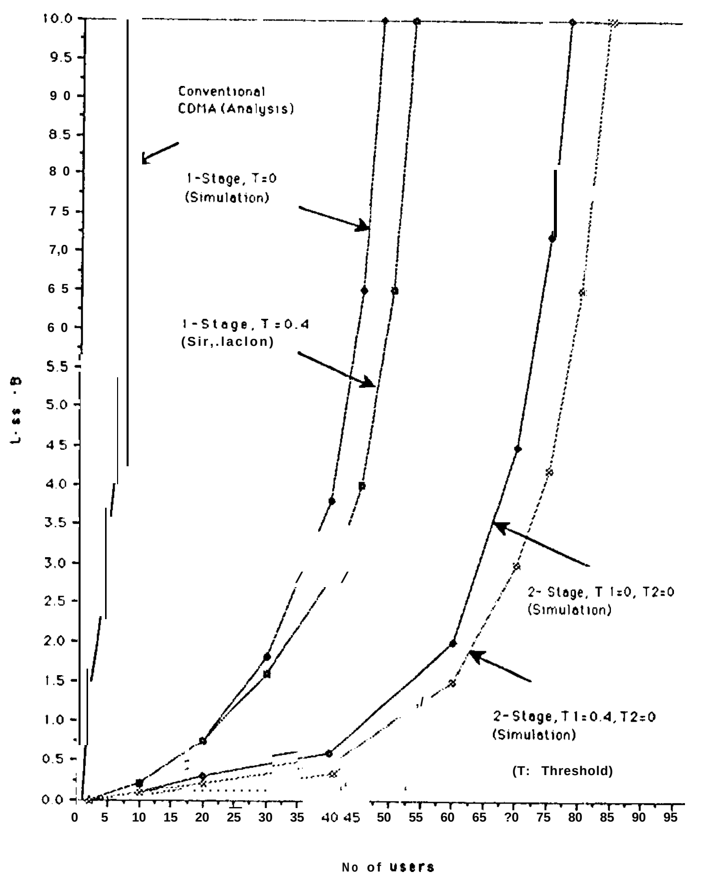

Figure 3. Performance of interference **cancellation** schemes **with unequal power users**