PRECESSION AND NUTATION FROM JOINT ANALYSIS 011' RADIO INTERFEROMETRIC AND LUNAR LASER RANGING OBSERVATIONS

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(to be submitted to *Astron. J.*)

Abstract

24 years of Lunar Laser Ranging (LLR) observations and 15 years of Very Long Baseline Interferometry (VLBI) observations are combined in a global analysis to yield improved estimates of the Earth precession and nutation. The correction to the IAU (1976) precession constant inferred from this joint VLBI/LLR analysis is −3.18 milliarcsecond/year (mas/yr). A significant obliquity-rate correction of –0.16 mas/yr is also found. In all, 32 forced nutation coefficients arc estimated. These coefficients confirm that the IAU (1980) nutation theory is in error by several milliarcseconds (mas). Forced circular nutations derived from this analysis agree with the ZMOA-1990-2 nutation theory at the 0.2 mas level for the 18.6 yr terms, and at the 0.05 mas level for the other terms (periods $\leq$ 1 yr). The estimated nutation coefficients are found to vary by as much as several tenths of mas, depending on the a *priori* nutation model used to analyze the VLBI and LLR data. A retrograde free core nutation with an amplitude of 0.27 mas is also detected. Its phase is found to be very sensitive to the precise value of the free core nutation period used in the solution, Separate analyses of four independent subsets of the VLBI data indicate no variations of the free core nutation since 1988. 'l'he pre-1988 estimates of the free core nutation are consistent with the post-1988 estimates but are not accurate enough to rule out possible variations of the free core nutation at these earlier epochs.

1. Introduction

The motion of the Earth's rotation axis with respect to an inertial reference frame is composed of a linear part (precession) and of small oscillations with periods between five days and 18.6 yr (notations). This motion is caused by luni-solar forces and can be theoretically predicted from the motions of the Sun and the Moon, depending on properties of the Earth. The present nutation series adopted by the International Astronomical Union (IAU 1980) for reduction of astrometric and geodetic observations (Seidelmann 1982) model the forced nutations for an elliptical, rotating, self-gravitating, elastic and oceanless Earth. It is based on Kinoshita's (1977) theory of a rigid Earth, which Wahr (1981) extended to a deformable Earth by using a normal mode expansion. This series was adequate for the older optical astrometric observations, but it is insufficient for observations acquired by modern astrometric and geodetic experiments such as the Very Long Baseline Interferometry (VLBI), Lunar Laser Ranging (LLR), Satellite Laser Ranging (SLR) and the GPS (Global Positioning System) techniques, whose measurement errors arc two orders of magnitude smaller than those of the optical observations. After only a few years, VLBI data have revealed errors at the milliarcsecond (mas) level in the IAU (1 980) nutation series, especially in the retrograde annual term (Herring *et al.* 1986). Based on a longer data span and increased precision, errors at the mas- or sub-mas level are now detectable in about ten IAU nutation terms (Herring *et* al. 1991, McCarthy & Luzum 1991). The present VLBI and LLR observations also indicate that the IAU precession constant (Lieske *et al. 1977)* may be in error by about - 3 mas/yr (Herring *et al.* 1991, Williams *et al.* 1991, McCarthy & Luzum 1991). It is useful to establish more accurate precession and nutation constants for improving the analyses of modern astrometric and geodetic observations. Such observations are also of prime importance to deepen our understanding of the Earth's interior.

Following the initial identification of discrepancies between the IAU (1980) nutation series and VLBI-observed nutations, several explanations have been considered. The rigid-Earth nutation theory has been reinvestigated by carrying out calculations to a higher order of precision (Zhu & Groten 1989, Kinoshita & Souchay 1990). A significant improvement over Kinoshita's (1977) theory was found, but the new rigid-Earth series remove only some of the discrepancies between observations and theory. Most probably, the remaining discrepancies are due to imprecisely modeled geophysical processes which cause errors in the the IAU (1980) nutation theory. Gwinn *et al.* (1986) showed that the discrepancy for the retrograde annual nutation can be eliminated by modifying the resonance frequency of the free core nutation (a notational normal mode of the Earth which produces relative motion between the core and the mantle) from 460 days to 430 days, implying that the Earth's core flattening is about 5% larger than the hydrostatic value. The retrograde annual nutation is most sensitive to such a modification because its amplitude is large and its frequency is close to that of the free core nutation. Mathews *et al. (1991a,b)* further studied the influence of the Earth's core on the nutations by explicitly including a solid inner core in their theory. They demonstrated the existence of a new notational normal mode related to the solid inner core with a period close to the diurnal period in an Earth-fixed reference frame. This new mode produces significant modifications of some nutation amplitudes (up to 0.36 mas for the 18.6 yr term) because its frequency is near resonance with some forced nutation frequencies. Additional effects like ocean tides (Wahr & Sasao 1981) and mantle inelasticity (Wahr & Bergen 1986) also produce corrections at the sub-mas level and need to be considered for improved accuracy (Herring *et al.* 1991).

In the past five years, three new nutation series, more accurate than the IAU (1980) theory, have been proposed by Zhu *et al.* (1990), Herring *et al.* (1991), and Souchay (1994). The approach used by Zhu *et al.* (1990) to build their theory is similar to that of Wahr (1981) which led to the IAU (1980) nutation series, but includes an improved rigid-Earth series (Zhu & Groten 1989), corrections for ocean tidal effects, and revised values of Wahr's numerical coefficients. These coefficients have been determined empirically from VLBI estimates of the amplitudes of the major nutation terms, based on 8.5 years of IRIS (International Radio Interferometric Surveying) data, whereas they were originally calculated from geophysical assumptions about the Earth's interior in Wahr's (1981) theory. This approach is purely observational, since it does not provide a geophysical basis for modifying Wahr's coefficients. By contrast, the series of Herring *et al.* (1991), referred to as ZMOA, is almost purely theoretical, having only two quantities derived empirically (the dynamical ellipticity of the Earth as a whole and the dynamical ellipticity of the core/mantle interface). The ZMOA theory is based on the rigid-Earth series of Zhu & Groten (1989) and the deformable-Earth normalized response of Mathews *et* al. (1991 a,b), also including corrections for ocean tides and mantle inelasticity. Roth the Zhu *et al.* (] 990) and the ZMOA series are found to reduce significantly the scatter of the nutation angle residuals measured by VLBI (see Zhu *et al.* 1990, Herring *et al.* 1991), The more recent series proposed by Souchay (1994), hereafter referred to as KSNRE, is the non-rigid version of the Kinoshita & Souchay (199o) rigid-Earth theory. It was derived by using Wahr's (1 981) approach and is purely theoretical. For this reason, it does not match the VLBI observed nutations as well as the Zhu *et al.* (1 980) and ZMOA series. However, it appears to be the most complete model available so far, since it contains the largest number of luni-solar terms and also include many planetary nutations unlike the Zhu *et al.* (1 990) and ZMOA series. Herring (1 991) proposed further an alternate version of the ZMOA theory which accounts for dissipation effects at the core-mantle boundary. Such effects were calculated from the VLBI data by estimating selected coefficients of the Earth normalized response. This alternate series is referred to as ZMOA-1990-2 (abbreviated as ZMOA-2 in the rest of the paper), and is likely more accurate than the initial theory as noted by the author. It agrees with the VLBI observations within their uncertainties (1 mas for the 18.6 yr terms, 0.04 mas for the other terms), except for two nutation terms (the prograde out-of-phase annual term and the prograde in-phase 13.66-day term). Herring (1991) also pointed out the detection in the observations of a retrograde free core nutation with an amplitude of about 0.3 mas, which was confirmed by McCarthy & Luzum (]99]). While satisfying for most of the nutation terms, the observational uncertainty for the principal term of nutation (period of 18.6 yr) was still large (1 mas), because the ~ 10 yr span of the VLBI data was not long enough to properly separate linear and]8.6 yr nutation terms.

The present paper reports estimates of the precession constant and the major nutation coefficients based on the longest series of data available to date, the Deep Space Network (DSN) VLBI data set which spans 15 years (1978--1993), and the LLR data set which spans 24 years (1970 -1994). To improve the precision of the individual VLBI and LLR determinations and to reduce technique-dependent systematic errors, the VLBI and LLR information matrices have been combined to produce joint VLBI/LLR estimates. Preliminary results of such a joint VLBI/LLR analysis were presented by Chariot *et al,* (1991). 'l'he estimates derived from this analysis arc totally independent of previous VLBI determinations based on data acquired by the IRIS network and the Crustal Dynamics Program (Zhu et al. 1990, Herring *et al.* 1991, McCarthy & Luzum 1991), which permits useful comparisons. This paper has two major goals: (1) to provide improved, estimates of the precession constant and the 18.6 yr nutation terms, taking advantage of the length of our data sets, (2) to confirm previous VLBI nutation amplitude estimates at short periods with an independent data set. Section 2 describes in more detail the VLBI and LLR data sets, the analysis method to combine VLBI and LLR observations, and the modeling used in the analysis, Our results are presented and discussed in Sections 3 and 4, which include comparisons of the VLBI/LLR estimates with the individual VLBI and LLR estimates, discussion of error sources such as those caused by the *a priori* nutation models, and comparisons with previous independent nutation estimates and with the ZMOA-2 nutation theory.

2. Data analysis

a) Observations

The VLBI data set used in our analysis consists of 20513 delay-delay rate pairs acquired with the Bandwidth Synthesis Technique during 136 dual-frequency (2.3 and 8.4 GHz) VLBI observing sessions carried out by the DSN between October 1978 and December 1993 on two intercontinental baselines: Goldstone-Madrid (length $\approx$ 8390 km) and Goldstone-Tidbinbilla (length $\approx$ 10590 km). More than half of these observations (starting in 1988) were recorded with the Mark HI data acquisition system (Rogers *et al.* 1983) with a precision of 1 cm. The older observations, recorded with the Mark 11 system (Clark 1973) have a precision of 10 cm.

The LLR data set consists of 8888 ranges (distance measurements) from telescopes on the Earth to reflectors on the Moon acquired between March 1970 and June 1994. The measurements of the first decade are all from the 2.7 m telescope at McDonald Observatory (Texas), which ceased LLR activity in 1985. Those of the past decade are from three telescopes at Grasse (France), Haleakala (island of Maui, Hawaii), and at McDonald Observatory (MLRS station), which are dedicated to ranging the Moon and Earth orbiting satellites. The current precision of LLR measurements is about 3 cm while the precision of the early data was 30 cm. Although less precise, these early data play a critical role in the separation of long-term effects like precession and 18,6 yr nutations.

b) The method used to combine VLBI and LLR data

Combining measurements acquired by two different techniques like VLBI and LLR can be achieved either by analyzing all the raw measurements together with a unique software capable of handling both data types, or by combining the normal equations calculated separately for each technique with independent software, as suggested by Archinal & Mueller (1989). Both methods should produce the same results as long as modeling is identical for parameters common to the two techniques. The latter method has been used to produce the joint VLBI/LLR estimates reported below, because normal equations for each technique were available from VLBI and LLR analyses developed at Jet Propulsion Laboratory (JPL) (Steppe *et al.* 1993, Newhall *et al.* 1993). The JPL VLBI and LLR software are both based on the Square Root Information Filter (SRIF) algorithm, which uses Householder transformations to triangularize the data equation matrix (see Bierman 1977). This algorithm allows new data equations to be added to a previously analyzed set of data equations and produce "updated" parameter estimates without reanalyzing all the data. In practice, this saves much computer time when large amounts of data are to be processed and updated. In our analysis, this capability has been used to combine observations of different types (VLBI and I, I, R).

The main steps of our joint VLBI/LLR analysis are shown in Figure 1. First theoretical values of LLR ranges and VLBI delays and delay rates arc calculated and differenced with observations to produce O-C's. This step is performed separately for each set of measurements by using the JPL VLBI software named MODEST (Severs & Jacobs 1994) and the J] 'I, LLR software. Then, the SRIF algorithm is applied to LLR O-C's and the R matrix is retained before the calculation of the LLR parameter estimates. This matrix is an upper triangular matrix obtained by applying Householder transformations to the data equations (see Bierman, 1977). It is further us-cd as an *a priori* information matrix in a way that mimics a VLBI R matrix, when the SRIF algorithm is applied to VLBI O-C's. This latter step requires identification of parameters common to VLBI and LLR and matching their names and units in order to produce consistent joint VLBI/LLR parameter estimates. In our analysis, these common parameters consist of periodic and linear nutation terms and UT1 tidal amplitudes (see below).

c) Modeling

VLBI and LLR computation was performed in Solar System Barycentric coordinates using the mean equator and equinox of J2000.0, and largely adhered to the International Earth Rotation Service (IERS) Standards (McCarthy 1992) for astronomical constants. VLBI-specific parameters include right ascension and declination of extragalactic radio sources, geocentric coordinates of radiotelescopes (new positions were estimated for each session), and session-specific clock and troposphere parameters at each station (one troposphere zenith delay for every 2- or 3-hour period). The position of Goldstone at epoch 1988.0 was fixed to its value in the ITRF92 frame (IERS 1993) to define the origin of the VLBI terrestrial frame, while its velocity was constrained to that predicted by the NNR-NUVEL-1 plate tectonic model (De Mets *et al.* 1990, Argus & Gordon 1991). The VLBI celestial frame was tied to the IERS celestial system by fixing the right ascension and declination of OJ287 (0851 + 202) and the declination of CTD20 (0234 + 285) to their values in the ICRF92 frame (IERS 1993), as devised by Steppe *et al.* (1993). An offset corresponding to the mean Celestial Ephemeris Pole position at J2000 was also estimated.

LLR-specific parameters include station locations, reflector locations, corrections to the orbit of the Earth-Moon barycenter about the Sun, as tabulated in the experimental JPL planetary and lunar ephemeris DE247/LE247, and lunar gravity, rotation and orbit parameters. The LLR stations were also assumed to move with the NNR-NUVEL-1 plate tectonic model. Earth orientation parameters (polar motion and UT1) for both VLBI and LLR were adopted from a combination-of-techniques series (Gross 1993) which has been aligned with the IERS system. In the LLR analysis, universal time and polar motion rates (measured over the total span of data) were also estimated to allow for possible inconsistencies between the rates of the *a priori*, series and the NNR-NUVEL-1 station velocities.

In addition to parameters described above, which are specific to VLBI or LLR, corrections to periodic nutation terms were estimated in common to both data types as well as linear corrections to nutation in longitude and obliquity (equivalent to corrections to the luni-solar precession and the obliquity rate). UT1 tidal coefficients at the nearly diurnal (K_1, P_1, O_1) and semi-diurnal (S_2, M_2, N_2) frequencies were also jointly estimated because the present theoretical models available are not sufficiently accurate to fit the data (e.g. Severs *et al.* 1993).

3. Joint VLBI/LLR precession and nutation estimates

The nutation of the Earth's rotation axis is described by two angles, the nutation in obliquity $\Delta\varepsilon(t)$ and the nutation in ecliptic longitude $\Delta\psi(t)$. $\Delta\psi(t)$ is defined in a left-handed sense. These quantities arc commonly expressed with the following series expansion:

$$\Delta\varepsilon(t) = \sum_{j=1}^{N} [\Delta\varepsilon_j^c \cos\theta_j(t) + \Delta\varepsilon_j^s \sin\theta_j(t)]$$

$$\Delta\psi(t) = \sum_{j=1}^{N} [\Delta\psi_j^s \sin\theta_j(t) + \Delta\psi_j^c \cos\theta_j(t)]$$

where $\Delta\varepsilon_j^c$ and $\Delta\psi_j^s$ are the in-phase nutation coefficients in obliquity and longitude for the j^{th} term of the nutation series, and $\Delta\varepsilon_j^s$ and $\Delta\psi_j^c$ are the. corresponding out-of-phase coefficients. θ_j (i!), the argument of the j^{th} term, is formed by a linear combination of five fundamental arguments related to the orbits of the Moon and the Sun at time *t* (see *e.g.* Explanatory Supplement 1992). The 1 AU (1980) nutation series contains in-phase nutation coefficients for N = 106 luni-solar periods. Among these coefficients, only those with the largest amplitudes (approximately ten) have measurable errors (see e.g. Herring *et al.* 1991). The other coefficients arc w c]] determined by the theory because they are smaller and consequently less sensitive to imprecise geophysical corrections. It is to be noted that the IAU (1980) nutation series is rounded to the nearest 0.1 rnas and truncates all nutation coefficients smaller than 0.05 mas. ‘l’his threshold is comparable to the precision of our VLBI and LLR estimates (see below) and is a source of systematic error. Out-of-phase coefficients are not present in the IAU (1980) nutation series but have been detected at some periods (especially 18.6 yr, 1 yr and OS yr) with VLBI observations (see Herring *et al.* 1991, McCarthy & Luzum 1991). They may be caused by dissipative geophysical processes such as mantle inelasticity and ocean tides. Kinoshita & Souchay (1990) pointed out that the planets also produce significant effects on the Earth's rotation axis. When added in phase, the effects of the planets can be as large as ±0.8 mas in longitude and ±0.3 mas in obliquity (Souchay 1993) and may affect precession and nutation estimation if not modeled. These planetary nutations are not included in the IAU (1980) theory.

To get an idea of the influence of the a *priori* model on the precession and nutation estimates, our VLBI and LLR data sets have been analyzed with three different *a priori* nutation series: (1) the IAU (1980) series, (2) the ZMOA-2 series, and (3) the KSNRE series. The ZMOA-2 series contains nutation coefficients for 197 luni-solar periods including some out-of-phase terms, and is truncated at the 0.005 mas level, which appears to be small enough for present VLBI and LLR analyses. The KSNRE series contains nutation coefficients for 263 luni-solar periods (without out-of-phase components) and 117 planetary periods (see the list of coefficients in Sovers & Jacobs 1994). It is also truncated to the 0.005 mas level. Tables 1 and 2 compare the nutation estimates derived from our VLBI and LLR data with these three *a priori* series. For ease of comparison, the results are reported in terms of corrections to the IAU (1980) theory for all three sets of estimates. Tables 1 and 2 show that our VLBI and LLR results are significantly affected by the a *priori* nutation model used to analyze the data. The differences between estimates based on the IAU (1980) and ZMOA-2 a *priori* models are at the level of 0.2-0.3 mas for the] 8,6 yr coefficients and at the' level of 0.05-0.1 mas (with a maximum value of 0.22 mas) for the short-period coefficients. ‘J’his level of systematic error is consistent with the magnitude of the truncation error of the IAU (1980) nutation series (0.05 mas). There are also some surprisingly large differences up to 0.75 mas at long period (ψ_1^s In Table 1) and up to 0.26 mas at short periods (ψ_{10}^s in ‘l’able 2) between the estimates based on the ZMOA-2 and KSNRE *a priori* models. Such differences are possibly caused by the planetary nutations, which are not included in the ZMOA-2 series. In terms of significance (numbers in parentheses in Tables 1 and 2 are ratios of change to formal uncertainty), the VLBI estimates are more affected than the LLR estimates (differences up to 4.5σ for VLBI and 2.1σ for LLR) because they have smaller uncertainties. The linear coefficients (obliquity rate and precession constant) vary at the level of 0.1 mas/yr, depending on which *a priori* nutation model is used in the analysis. The joint VLBI/LLR results presented below and discussed in the rest of the paper have been produced with the KSNRE *a priori* series. This model has been adopted for our analysis, because it is more complete than either the IAU (1 980) or ZMOA-2 series.

Table 3 shows our estimates of the largest corrections to the IAU (1980) nutation coefficients, based on the joint VLBI/LLR analysis described above, as well as corrections to the precession constant and the obliquity rate. In this analysis, the 9-yr coefficients were constrained to the values of the ZMOA-2 theory since they cannot be estimated reliably from our data set due to strong correlations with the 18.6 yr terms. The adopted ZMOA-2 9-yr values are believed to be accurate at the sub-mas level since they differ from the IAU values mainly because of improved rigid-Earth corrections. Because of software limitations, the IAU coefficients with indices 11, 32, 33 and 34 in Table 3 were estimated solely from VLBI data. This limitation should not affect our joint VLBI/LLR estimates for the other coefficients since the corrections for these terms are small (0.3 mas at most) and LLR data are not very sensitive to them. A free core nutation (FCN) was also estimated (see Table 3). Its period (~ 429.8 days) was adopted from Herring *et al.* (1991). Initial assessment of the significance of the free core nutation parameters was made by examining the weighted-root-mean-square (wrms) delay residuals and Chi-square in the VLBI fit. Estimating these additional parameters $(\Delta\varepsilon^c_F, \Delta\psi^s_F, \Delta\varepsilon^s_F, \Delta\psi^c_F)$ reduces the wrms delay residuals by approximately 2 mm in quadrature, relative to a fit including no free core nutation. Chi-square decreased by approximately 126 from its value of $\approx$ 16 000 for the additional 4 degrees of freedom, which indicates at a very high confidence level that the VLBI fit benefits from the introduction of the free core nutation parameters.

In Table 3, the individual VLBI and LLR estimates are also presented for comparison with our joint VLBI/LLR estimates, Comparing the formal uncertainties of these two individual solutions gives an indication of their relative strength in the combined solution. For the precession constant, VLBI and LLR uncertainties have approximately the same magnitude (see Table 3), but for the other terms, VLBI uncertainties are smaller by a factor of two or more than the corresponding LLR uncertainties. For this reason, the joint VLBI/LLR nutation estimates are generally closer to the estimates from VLBI than to those from LLR. The LLR solution, however, is less correlated at long periods (18.6 yr) than the VLBI solution because of its longer data span, For LLR, the largest correlation coefficient is 0.59 (between linear and 18.6 yr terms), whereas it is 0,92 for VLBI. It is also to be noted that the 1-yr term and the free core nutation term are significantly correlated in the VLBI solution (correlation coefficient of 0.75), as well as the two fortnightly terms (correlation coefficient of 0.88 between the IAU terms 31 and 33). In the LLR solution, the only significant correlation is between longitude and obliquity of the fortnightly term (correlation coefficient of 0.87).

indications on the accuracy of the independent VLBI and LLR estimates are given by their differences, also listed in Table 3. These differences often exceed the formal uncertainties, suggesting the presence of systematic errors. The largest discrepancies are for the precession constant (5σ) and for the 1 -yr terms in longitude (7σ for both in-phase and out-of-phase coefficients). The inconsistency between VLBI and LLR is apparently largest for highly correlated parameters, which can be expected because such parameters are more sensitive to imprecise modeling or systematic errors. Most of the systematic errors seen in Table 3 are likely to be technique-dependent. Those caused by the *a priori* nutation model used to analyze the data are assumed to be not significant because of our use of the improved KSNRE series. Such technique-dependent systematic errors should be reduced when combining data from different techniques like in the joint VLBI/LLR solution reported here, because it combines the strengths of the two techniques.

4. Comparison with independent results and theory

Table 4 shows a comparison of our joint VLBI/LLR estimates with the results of Herring *et al,* (1991), hereafter referred to as H91, and McCarthy & Luzum (1991), hereafter referred to as ML91. These results are based on analysis of VLBI data acquired by the IRIS and Crustal Dynamics Program (CDP) networks, and are independent of our estimates. The solution of H91 includes 8,5 years of IRIS and CDP data (1980-1989), while that of ML91 is slightly longer and includes 10.5 years of data (1980-1991). It should be noted that these data sets are denser than the VLBI and LLR data sets used in our joint VLBI/LLR analysis, but span a shorter period of time (only a decade against 15 years for our VLBI data set and 24 years for our LLR data set). For this reason, the formal uncertainties of our estimates for linear and 18.6 yr terms are smaller than those of H91 and ML91. The precession constant estimated in our analysis (- 3.18:1 0.05 mas/yr) is consistent at the 0.1 mas level with that determined by 1191 (- 3.2 ± 1.0 mas/yr) and at the 0.4 mas level (2σ) with that determined by ML91 (-- 2.74 ± 0.21 mas/yr). It also agrees with the value inferred from VLBI catalog comparisons (- 3.63. 1.1 mas/yr) by Walter & Ma (1994). All three determinations of the obliquity rate in Table 4 are consistent, but only our estimate is significantly different from zero (— 0.16 ± 0.02 mas/yr). This estimate is in reasonable agreement with the theoretical value (- 0.24 mas) calculated by Williams (1994).

For comparing the three solutions in Table 4, the nutation series coefficients in longitude and obliquity may be decomposed into circular nutation terms. Each of the nutation series coefficients indeed results from the superposition of two circular nutations with frequencies equally spaced about one cycle per sidereal day when viewed from the rotating Earth. The relationship between circular nutation amplitudes and nutation series coefficients in longitude and obliquity was derived by Wahr (I 981) for the in-phase terms and extended by Herring *et al.* (1986) to include the out-of-phase terms. It is given as:

$$a_{rj}^{+} = - (\Delta\varepsilon_j^c + \Delta\psi_j^s \sin\varepsilon_0)/2$$

$$a_{ij}^{+} = - (\Delta\varepsilon_j^s - \Delta\psi_j^c \sin\varepsilon_0)/2$$

$$a_{rj}^{-} = - (\Delta\varepsilon_j^c - \Delta\psi_j^s \sin\varepsilon_0)/2$$

$$a_{ij}^{-} = (\Delta\varepsilon_j^s + \Delta\psi_j^c \sin\varepsilon_0)/2$$

where ε_0 is the mean obliquity of the ecliptic (IAU value: $\varepsilon_0 =$ 23 °26'21''.448), and the pairs (a_{rj}^{+}, a_{ij}^{+}) and (a_{rj}^{-}, a_{ij}^{-}) are real and imaginary components of the two circular nutation terms corresponding to the nutation series coefficients with index j. As noted above, the frequencies of (a_{rj}^{+}, a_{ij}^{+}) and (a_{rj}^{-}, a_{ij}^{-}) are equally spaced about one cycle per sidereal day in the frame of the rotating Earth. When the frequency of the nutation series coefficients is positive (case of most of the IAU coefficients estimated in 'Table 4), (a^+_{rj}, a^+_{ij}) corresponds to a retrograde motion and (a^-_{rj}, a^-_{ij}) corresponds to a prograde motion. When the frequency of the nutation series coefficients is negative (case of the 18.6 yr terms), the convention that defines prograde and retrograde motions is reversed. More details about the relationship between circular nutation terms and nutation series coefficients in longitude and obliquity are given in H91.

In 'Table 5, the circular nutation terms estimated in our joint VLBI/LLR analysis arc compared with those obtained by H91 and ML91, as well as with the theoretical ones given in the ZMOA-2 theory, Table 5 shows that our estimates for the 18.6 yr terms agree at the 0.2 mas level with the ZMOA-2 theory, They are also consistent with the results of H91 and ML91, but larger differences are found (up to 0.7 mas and 2.5σ). Our estimates of the short-period (≤ 1 yr) forced nutations arc consistent with the ZMOA-2 theory and with the previous results of H91 and ML91 at the level of 0.05 mas in avearge, with the exception of the in-phase prograde 13.66-day term (a^-_{r31}) *(see* Table 5). If this latter term is not considered, the average difference between our short-period estimates and those previous determinations is O.O5O mas for 119], 0.059 mas for ML91 and O.O45 mas for the ZMOA-2 theory, with maximum differences of 0.12 mas for H91 (a^-_{i9}), 0.13 mas for ML91 (a^+_{r10}) and O.O9 mas for the ZMOA-2 theory $(a^-_{r11}$ and $a^+_{r34})$. Our analysis shows that the in-phase prograde 13.66-day term (a^-_{r31}) *is* consistent with the ZMOA-2 theory (difference of 0.07 mas), but the results of H91 and ML91 indicate a discrepancy of about 0.2 mas. Herring (1991) discussed this discrepancy and noted that it disappears when diurnal and semi-diurnal UT1 variations are estimated in the VLBI analysis, which is the case in our joint VLBI/LLR reduction. Aliasing between nutation and tidally induced terms in the Earth rotation rate (polar motion and UT I) has been more generally discussed by Severs *et al.* (1993).

'l'able 5 also shows that a circular free core nutation with retrograde motion is detected in our joint VLBI/LLR analysis. The direction of this motion (retrograde) is consistent with that expected from theoretical considerations (Wahr 1981) and its amplitude (0.27 mas) is similar to that found by H91 (0.26 mas) and ML91 (0,32 mas). However, the in-phase and out-of-phase components of the free core nutation are different in the three solutions listed in Table 5 (VLBI/LLR, 1191, ML91). Our estimate of the out-of-phase component a_{iF}^{-} is consistent with that of H91 within 0,06 mas ($\sim 1\sigma$), but there is a discrepancy of 0.17 rnas ($\sim 4\sigma$) for the in-phase component a_{rF}^{-} *(see* Table 5), which is larger than the maximum difference between our results and those of H91 for the other short-period nutation terms (0.12 mas). When comparing our estimates with those of ML91, there is a discrepancy of $\sim$ 0.3 mas ($\sim 10\sigma$) for both the in-phase and out-of-phase components of the retrograde free core nutation.

We have investigated whether the free core nutation discrepancies could be caused by time variations of the phase of the free core nutation. Such possible variations would affect the overall free core nutation estimates, depending on the data span, which is different in the three solutions of Table 5. For this study, our VLBI data set has been separated into four independent subsets of data spanning 3 to 5 years (1978–1983, 1984-1987, 1988--1990, 1991–1993) with new free core nutation coefficients estimated for every subset. Forced nutations were estimated for the entire period (1978–1993) similarly to the VLBI analysis described above. The time span of the first two subsets of data was chosen to be longer because the data of the first decade (Mark H) are not as precise as those of the past six years (Mark ,111). 'I'his analysis shows that the last two sets of estimates (1988- 1990 and 1991- 1993) are consistent within 0.05 mas in average (1σ formal uncertainty), thus indicating that the free core nutation has not varied since 1988 (see Table 6). No significant prograde term is detected and only the out-of-phase part of the retrograde term with an amplitude of 0.3 mas is significantly different from zero. These results are also consistent with the joint VLBI/LLR circular terms estimated over the entire period of our data (see Table 5). The first two sets of free core nutation coefficients (1978-1983 and 1984-1987) arc consistent with the post-1 988 determinations, but their uncertainty (0.15 mas) is half the free core nutation amplitude (0.3 mas). Thus, it is not possible to draw conclusions about possible variations of the free core nutation at these earlier epochs. The data only indicate that the free core nutation amplitude between 1978 and 1987 was not several times larger than what it has been since 1988. Analysis of the Mark 111 VLBI data acquired by the CDP and IRIS networks since 1979 should solve this issue in the future.

We have also investigated whether the large discrepancies between our free core nutation estimates and those of ML91 (see Table 5) are due to the adoption of a different free core nutation period (- 429.8 days for our VLBI/LLR results and –418 days for the results of ML91). For this investigation, our VLBI fit has been repeated with five different free core nutation periods, ranging from - 415 days to --435 days. The free core nutation estimates derived from these five additional fits are listed in 'l'able 7. It is striking that these estimates depend strongly on the adopted free core nutation period, Differences up to 0.16 mas (5σ) are found when shifting the free core nutation period by only 5 days (see the variations of a^-_{rF} and a^-_{iF} in 'l'able 7). These differences reach $\sim$ 0.3 mas (1 0σ) when the free core nutation period is shifted by 10 days, Such variations are comparable to the magnitude of the discrepancies between our results and those of ML91. Indeed, an additional solution adopting the same free core nutation period as that used by ML91 (---418 days) reveals that our estimates of the retrograde free core nutation (a^-_{rF} = --0.293 0.03 mas and a^-_{iF} = –0.05 $\pm$ 0.03 mas) become in close agreement (1σ) with those of ML91 (a^-_{rF} = --0.3230.01 mas and a^-_{iF} = -0.02 $\pm$ 0.02 rnas) when the free core nutation period is matched. 'J'bus, the apparent discrepancy in Table 5 is only caused by a different choice of the free core nutation period. Among the five periods studied in Table 7, our VLBI data indicate a preference for the period of -425 days, based on total Chi-square comparisons between the five fits. Further geophysical studies will be important in the future to reconciliate the various values of the free core nutation period.

5. Conclusion

Estimates of precession, obliquity rate, forced nutations at eight frequencies, and free core nutation have been obtained from a joint analysis of 24 years of LLR data and 15 years of VLBI data. This global VLBI/LLR analysis was carried out by combining the information matrices derived separately from the VLBI and LLR data equations. Such an analysis is stronger than tithe.r the VLBI or the LLR analysis alone since it combines the strengths of the two techniques. Comparison of precession/nutation estimates derived from different a *priori* nutation series (IAU (1980), ZMOA-2, KSNRE) indicates that these estimates depend at some level on the *a priori* nutation model used to analyze the data. Our results for short period nutation terms (≤ 1 yr) are consistent at the 0.05 mas level with the previous forced circular nutation estimates of Herring *et al.* (1991) and McCarthy & Luzum (1991), both based on independent VLBI data. A retrograde free core nutation with amplitude 0.27 mas is detected, consistently with previous VLBI determinations. Our VLBI data shows that its phase depends strongly on the precise adopted free core nutation period. They also indicate that the amplitude and phase of the free core nutation have not varied significantly since 1988. At long period (18.6 yr), there are differences up to 0.7 mas between forced nutations estimated from independent data sets. Our estimates are based on the longest series of data available. The agreement between our estimates and the ZMOA-2 theory, which is presently the most accurate nutation model, is 0.2 mas for the 18.6 yr circular terms and 0.05 mas for the other terms. In the future, additional VLBI data acquired by various networks will lengthen the present VLBI data sets and will improve further the estimates of the 18.6 yr nutation. Such determinations will be important to deepen our understanding of the Earth's interior and to build even more accurate theories of the Earth's nutation.

Acknowledgements

This work was initiated during a stay of P. Chariot at Jet Propulsion Laboratory in 1990, supported in part by a Lavoisier fellowship from the French Foreign Affairs Ministry. This work would not have been possible without the efforts of many people involved in the acquisition and processing of DSN VLBI data and lunar laser ranging data. The research described in this paper was carried out in part at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

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Figure captions

Figure I : Main steps of the joint VLBI/LLR analysis

Table 1. VLBI estimated nutation corrections to the IAU (1980) theory derived with the IAU (1980), ZMOA-2 and KSNRE *a priori* series (mas).

Period (days)		IAU (1980) *a priori*	ZMOA-2 *a priori*	KSNRE *a priori*	Differences* ZMOA-2 - IAU	Differences* KSNRE - ZMOA-2
Linear**	$d\varepsilon/dt$	--0.14 ± 0.02	- 0.174 0.02	- 0.233 0.02	0.03 (1 .5)	-0.06 (3.0)
	$d\psi/dt$	- 3.054 0.06	-3.11:1 0.06	-2.993 0,06	· 0.06 (1 .0)	0,12 (2.0)
6798.38	$\Delta\varepsilon_1^c$	2.263 0.07	2,32 ± 0.07	2.593 0.07	0,06 (0.9)	0.27 (3.9)
	$\Delta\psi_1^s$	-7.50:1 0,33	-7.79 ± 0.33	-7.043 0.33	-0,29 (0.9)	0.75 (2.3)
	$\Delta\varepsilon_1^s$	2.30 ± 0.13	2.09:1 0.13	1.92 ± 0.13	-0.21 (1.6)	-0.17 (1.3)
	$\Delta\psi_1^c$	3.913'0.17	4,213 0.17	3.983 0.17	0.30 (1.8)	-0.23 (1,4)
365,26	$\Delta\varepsilon_{10}^c$	2.053.0.04	1.90 ± 0,04	1.97 ± 0.04	-0.15 (3.8)	0.07 (1.8)
	$\Delta\psi_{10}^s$	4.42 ± 0.10	4.353:0.10	4,51:1 0.10	-0.07 (0.7)	0.16 (1,6)
	$\Delta\varepsilon_{10}^s$	−0.40 ± 0.04	-0.28 ± 0.04	· 0.27 ± 0.04	0.12 (3.0)	0.01 (0.2)
	$\Delta\psi_{10}^c$	1.31 ± 0.10	1,26 ± 0.10	1.42 ± 0.10	-0,05 (0.5)	0.16 (1.6)
182,62	$\Delta\varepsilon_9^c$	−0.57 ± 0.03	- 0,57 ± 0.03	--0,583 0,03	0.00 (0.0)	-0,01 (0.3)
	$\Delta\psi_9^s$	1.52 ± 0.07	1,524 0.07	1.602:0.07	0.00 (0.0)	0,08 (1.1)
	$\Delta\varepsilon_9^s$	"0.54 ± 0.03	-0.48 ± 0.03	- 0.483 0.03	0.06 (2,0)	0,00 (0.0)
	$\Delta\psi_9^c$	-1.533 0.08	--1,443.0.08	--1.42 ± 0.08	0.09 (1.1)	0,02 (0,2)
121.75	$\Delta\varepsilon_{11}^c$	0.082 0,02	0.13 ± 0.02	0.15 ± 0.02	0,05 (2,5)	0.02 (1.0)
	$\Delta\psi_{11}^s$	--0.14 ± 0,06	--0,09 ± 0.06	· 0.17 ± 0.06	0.05 (0,8)	-0.08 (1 ,3)
	$\Delta\varepsilon_{11}^s$	--0.05 ± 0.03	- 0.074 0.03	- 0.084-0,03	-0.02 (0.7)	-0.01 (0.3)
	$\Delta\psi_{11}^c$	0.01 ± 0.06	0.043.0.06	0,05 ± 0.06	0.03 (0.5)	0.01 (0.2)
13,66	$\Delta\varepsilon_{31}^c$	--0.01 ± 0,05	-0.01 ± 0.05	0,01 ± 0.05	0.00 (0.0)	0,02 (0.4)
	$\Delta\psi_{31}^s$	--0.213:0.14	−0.16 ± 0.14	---0.153:0.14	0.05 (0.4)	0.01 (0.1)
	$\Delta\varepsilon_{31}^s$	0.04 ± 0.06	0.04 ± 0.06	0.063.0.06	0.00 (0.0)	0.02 (0.3)
	$\Delta\psi_{31}^c$	0.24 ± 0.13	0.30 ± 0.13	0.20 ± 0.13	0,06 (0.5)	-0.10 (0.8)
27'.55	$\Delta\varepsilon_{32}^c$	-0.10:1 0.03	0.01 ± 0.03	0.003 0.03	0.11 (3.7)	-0.01 (0.3)
	$\Delta\psi_{32}^s$	-0.22 ± 0,08	-0.333 0.08	-0.313.0.08	--0.11 (1.4)	0.02 (0.2)
	$\Delta\varepsilon_{32}^s$	0.06 ± 0,03	0.10 ± 0.03	0.063 0.03	0.04 (1.3)	-0.04 (1.3)
	$\Delta\psi_{32}^c$	- 0.15 ± 0.07	-0,04 ± 0.07	-0.033 0.07	0.11 (1.6)	0.01 (0.1)
13.63	$\Delta\varepsilon_{33}^c$	0.003 0.05	0.053 0.05	0.03 ± 0,05	0,05 (1.0)	-0.02 (0,4)
	$\Delta\psi_{33}^s$	0.00 ± 0.13	--0.043 0.13	0.023 0.13	-0.04 (0.3)	0.06 (0,5)
	$\Delta\varepsilon_{33}^s$	0.044 0.05	0.044-0.05	0:04 ± 0.05	0.00 (0.0)	0.00 (0,0)
	$\Delta\psi_{33}^c$	--0.22 ± 0,14	- 0.01 ± 0.14	0.034 0.14	0.21 (1 .5)	0.04 (0.3)
9.13	$\Delta\varepsilon_{34}^c$	0.18 ± 0.04	0.11 ± 0.04	0.113.0,04	-0.07 (1.8)	0.00 (0.0)
	$\Delta\psi_{34}^s$	--0.203:0,09	-0.02 ± 0.09	0.00:10,09	0.18 (2,0)	0.02 (0.2)
	$\Delta\varepsilon_{34}^s$	0.12 ± 0.03	0.044-0.03	0.02 ± 0.03	-0.08 (2.7)	-0.02 (0,7)
	$\Delta\psi_{34}^c$	0.11 ± 0.09	0.07 ± 0.09	0.013 0.09	-0.04 (0.4)	--0,06 (0,7)
-429.8 (FCN)	$\Delta\varepsilon_F^c$	0.162:0.04	-0.02 ± 0.04	0.023 0.04	--0.18 (4,5)	0.04 (1 .0)
	$\Delta\psi_F^s$	-0.03 ± 0,09	- 0.15 ± 0.09	-0.083 0.09	-0.12 (1.3)	0.07 (0.8)
	$\Delta\varepsilon_F^s$	--0.20 ± 0.03	-0.19 ± 0.03	-0.184-0.03	0.01 (0.3)	0.0) (0.3)
	$\Delta\psi_F^c$	-0.963 0.10	- 0.92 ± 0.10	-0.944-0,10	0.04 (0.4)	-0.02 (0.2)

*Numbers in parentheses are ratios of difference to forma] uncertainty.

**Units for linear coefficients are mas/yr.

'1'able 2. LLR estimated nutation corrections to the IAU (1980) theory derived with the IAU (1 980), ZMOA-2 and KSNRE *a priori* series (mas).

l'cried (days)		IAU (1980)" " *a* priori	ZMOA-2 *a priori*	KSNRE *a priori*	Differences*	
					ZMOA-2- IAU	KSNRE- ZMOA-2
Linear**	$d\varepsilon/dt$	0.01 ± 0.05	0.0) ± 0.05	- 0.023 0.05	0.00 (0.0)	-0,03 (0.6)
	$d\psi/dt$	--3,54 ± 0,09	-3.564 0.0'3	- 3.523 0.09	-0.02 (0.2)	0.04 (0.4)
- 6798,38	$\Delta\varepsilon_1^c$	2.9.5 ± 0.34	2.983 0.34.	3,21 ± 0.34	0.03 (0.1)	0.23 (0.7)
	$\Delta\psi_1^s$	--7.82 ± 0.99	- 7.90 ± 0.99	- 7.55 ± 0,99	--0.08 (0.1)	0.35 (0.4)
	$\Delta\varepsilon_1^s$	0.79 ± 0,42	0.754 0.42	0.60 ± 0,42	-0,04 (O.l)	-0.15 (0.4)
	$\Delta\psi_1^c$	5.83 ± 0,66	6.21 ± 0.66	5.89 ± 0,66	0.38 (0.6)	-0.32 (0.5)
365.26	$\Delta\varepsilon_{10}^c$	2.002 0.14	1.85 ± 0.14	1.88 ± 0.14	–0.15 (1.1)	0.03 (0.2)
	$\Delta\psi_{10}^s$	6.54 ± 0.27	6.323-0.27	6.58 ± 0.27	--0,22 (0,8)	0.26 (I .0)
	$\Delta\varepsilon_{10}^s$	--0.50 ± 0.10	--0.29 ± 0.10	--0.34 ± 0.10	0.21 (2.1)	-0.05 (0,5)
	$\Delta\psi_{10}^c$	--0.31 ± 0.23	--0.34 ± 0.23	--0.254,0.23	-0.03 (0.1)	0.09 (0.4)
182.62	$\Delta\varepsilon_9^c$	---0,752:0.12	–0.69 3:0.12	--0,66 ± 0.12	0.06 (0.5)	0.03 (0.2)
	$\Delta\psi_9^s$	1.24 ± 0.23	1.29 ± 0.23	1,29 ± 0.23	0.05 (0.2)	0.00 (0.0)
	$\Delta\varepsilon_9^s$	–0.32 4:0.11	-0.284 0.11	-0,394-0.11	0.04 (0.4)	-0.11 (1.0)'
	$\Delta\psi_9^c$	--1.34 ± 0.25	-1.234.0.25	---1.464.0.24	0.11 (0.4)	-0.23 (1.0)
13.66	$\Delta\varepsilon_{31}^c$	0.253 0.23	0.33 ± 0.23	0.303 0.23	0.08 (0,3)	-0,03 (0.1)
	$\Delta\psi_{31}^s$	–1.08 ± 0.48	---1.1040.48	--1.12 ± 0.48	–0.02 (0.0)	-0.02 (0.0)
	$\Delta\varepsilon_{31}^s$	0,593 0.23	0.633.0.23	0.62 ± 0.23	0.04 (0.2)	--0.01 (O.O)
	$\Delta\psi_{31}^c$	0.70 ± 0.48	0.734-0.48	0,594,0,48	0.03 (0.1)	-0.14 (0.3)
-429.8	$\Delta\varepsilon_F^c$	--0.423:0.11	--0.43 ± 0.11	--0.49 ± 0.11	--0.01 (0.1)	-0.06 (0.5)
(FCN)	$\Delta\psi_F^s$	0.123-0.24	0.14 ± 0,24	0.054,0.24	0.02 (0.1)	--0,09 (0.4)
	$\Delta\varepsilon_F^s$	--0.52 i 0.12	--0.50 ± 0.12	--0,53 ± 0.12	0.02 (0.2)	-0.03 (0.2)
	$\Delta\psi_F^c$	---0.82 ± 0.19	-0.67 ± 0.19	--0.84 ± 0.19	0.15 (0.8)	-0.17 (0.9)

* Numbers in parentheses arc ratios of difference to formal uncertainty.

** Units for linear coefficients are mas/yr.

Table 3. Comparison of joint VLBI/LLR and individual VLBI and LLR nutation corrections to the IAU (1 980) theory derived with the KSNRE *a priori* series (mas).

Period (days)		VLBI/LLR	VLBI	LLR	v],]]]- LLR*
Linear**	$d\varepsilon/dt$	---0.16 ± 0,02	-0.23:1 0,02	- 0.02 ± 0.05	-0.21 (3.9)
	$d\psi/dt$	--3.183 0.05	- 2.99 ± 0.06	--3.523-0.09	0.53 (4 .9)
6798.38	$\Delta\varepsilon_1^c$	2.65 ± 0.06	2.594 0.07	3.213,0.34	-0.62 (1.8)
	$\Delta\psi_1^s$	- 7.86 ± 0.27	-7,044 0.33	- 7.554 0.99	0.51 (0.5)
	$\Delta\varepsilon_1^s$	2.18 ± 0.12	1.92 ± 0.13	0,603.0,42	1.32 (3,0)
	$\Delta\psi_1^c$	3.74 ± 0.16	3,984 0.17	5.89 ± 0.66	1 . 9 1 (2.8)
365.26	$\Delta\varepsilon_{10}^c$	1.94 ± 0.03	1.974 0.04	1.88 ± 0.14	0,09 (0.6)
	$\Delta\psi_{10}^s$	4,75 ± ().08	4.513 0.10	6.583 0.27	2,07 (7.2)
	$\Delta\varepsilon_{10}^s$	--0,3040.03	-0.274 0.04	--0.344 0.10	0.07 (0,6)
	$\Delta\psi_{10}^c$	1.11:1 0.09	1.423 0.10	--0.25 ± 0.23	1,67 (6,7)
182,62	$\Delta\varepsilon_9^c$	-0.62:1 0.03	-0.584 0.03	-0.66 ± 0,12	0,08 (0.6)
	$\Delta\psi_9^s$	1.63 ± 0,07	1.60 i 0.07	1.294 0.23	0.31 (1.3)
	$\Delta\varepsilon_9^s$	--0.484 0,03	--0.484 0.03	-0.39 ± 0.11	-0.09 (008)
	$\Delta\psi_9^c$	--1.54 ± 0.07	-- 1.423 0.08	--1.463:0.24	0.04 (0.2)
121,75	$\Delta\varepsilon_{11}^c$	0,17 ± 0.02	0.15 ± 0.02	...	...
	$\Delta\psi_{11}^s$	--0.113.0.06	--0.174 0.06	...	...
	$\Delta\varepsilon_{11}^s$	-0.084:0.03	--0.08 ± 0.03	...	...
	$\Delta\psi_{11}^c$	0,14 ± 0.06	0.05 ± 0.06	...	...
13.66	$\Delta\varepsilon_{31}^c$	--0.04 ± 0.05	0.01 ± 0.05	0.304.0.23	-0,29 (1.2)
	$\Delta\psi_{31}^s$	- 0.143.0.12	--0.153 0.14	--1.12 ± 0.48	0.97 (1 .9)
	$\Delta\varepsilon_{31}^s$	0,10 ± 0.05	0.0630.06	0.62 ± 0.23	-0.56 (2,4)
	$\Delta\psi_{31}^c$	0.023 0.12	0,204 0.13	0.59 ± 0.48	-0.39 (0.8)
27.55	$\Delta\varepsilon_{32}^c$	0,013 0,03	0.00:1 0.03	...	...
	$\Delta\psi_{32}^s$	--0,36 ± 0.07	--0.31 ± 0.08	...	...
	$\Delta\varepsilon_{32}^s$	0.053 0.03	0.06:1 0.03	...	...
	$\Delta\psi_{32}^c$	-0,02 ± 0,06	-0.03 ± 0.07	...	...
13.63	$\Delta\varepsilon_{33}^c$	0,06:4.0.05	0.034 0.05	...	...
	$\Delta\psi_{33}^s$	0.21 ± 0.12	0.024:0.13	...	..
	$\Delta\varepsilon_{33}^s$	0.09:1.0.05	0,04 ± 0.05	...	..
	$\Delta\psi_{33}^c$	0,10 ± 0.12	0.03 ± 0.14	...	...
9.13	$\Delta\varepsilon_{34}^c$	0,124:0.04	0.114:0.04	...	...
	$\Delta\psi_{34}^s$	0.12 ± 0.08	0.00 ± 0.09	...	...
	$\Delta\varepsilon_{34}^s$	0,04 ± 0.03	0.02 ± 0.03	...	...
	$\Delta\psi_{34}^c$	0.03:1"0.09	0.01 ± 0.09	...	...
- 429.8	$\Delta\varepsilon_F^c$	-0,04 i 0.03	0.02 ± 0.04	-0.49 ± 0.11	0.51 (4.4)
(FCN)	$\Delta\psi_F^s$	-0,16 ± 0.08	-0.084.0.09	0.05 ± 0.24	-0.13 (0.5)
	$\Delta\varepsilon_F^s$	-0,21 ± 0.03	-0.184 0.03	--0.53 ± 0.12	, 0.35 (2,8)
	$\Delta\psi_F^c$	-0.81 ± 0.08	-0.944.0.10	-0.84 ± 0.19	-0.10 (0.5)

* Numbers in parentheses are ratios of difference to formal uncertainty.

**' Units for linear coefficients are mas/yr.

Table 4. Comparison of corrections to nutation coefficients in longitude and obliquity (mas).

Period (days)		VLBI/LLR (this paper)	Herring *et al.* (1991)	McCarthy & Luzum (1'391)
Linear*	$d\varepsilon/dt$	-0.16 ± 0.02	- 0.40:1 0.50	0.053 0.07
	$d\psi/dt$	- 3,18:1 0.05	- 3.20:1 1.00	2.74 ± 0.21
- 6798.38	$\Delta\varepsilon_1^c$	2.65 ± 0.06	3.39:1 1.40	2.68 ± 0.14
	$\Delta\psi_1^s$	-7.86 ± 0,27	--7.113 3.50	- 5.803 0,72
	$\Delta\varepsilon_1^s$	2.183 0.12	0.984 1.40	2.16 ± 0.26
	$\Delta\psi_1^c$	3.74 ± 0.16	4.22 ± 3.50	3,34 ± 0.36
365,26	$\Delta\varepsilon_{10}^c$	1.94 ± 0.03	1.90 ± 0,06	2.01 ± 0.02
	$\Delta\psi_{10}^s$	4,75 ± 0.08	4.98 ± 0.14	5,213 0.13
	$\Delta\varepsilon_{10}^s$	-0.304 0,03	--0.17 ± 0.06	--0.273 0.02
	$\Delta\psi_{10}^c$	1,11:1 0,09	1.18 ± 0.14	1,06 ± 0.07
182.62	$\Delta\varepsilon_9^c$	-0,62 ± 0.03	-0.47:1 0.06	-0.48 ± 0.02
	$\Delta\psi_9^s$	1.633 0,07	1.43 ± 0.14	1.48 ± 0.05
	$\Delta\varepsilon_9^s$	-0,48:1 0.03	-0,403 0.06	- 0.443 0,02
	$\Delta\psi_9^c$	-1.543 0.07	--1.3640.14	---1,22 ± 0.05
121.75	$\Delta\varepsilon_{11}^c$	0.173 0,02	0.08 ± 0.06	0.045 0.02
	$\Delta\psi_{11}^s$	-0.11 ± 0.06	--0.054 0.14	- 0,043 0.08
	$\Delta\varepsilon_{11}^s$	-0.08:1 0.03	--0.074 0.06	- 0.05:1 0.04
	$\Delta\psi_{11}^c$	0.144.0.06	--0.033 0,14	0,014 0.06
13.66	$\Delta\varepsilon_{31}^c$	–0.04 ± 0,05	0,33 ± 0,06	0.303-0.06
	$\Delta\psi_{31}^s$	–0.14 ± 0.12	--0.783 0.14	-0.543 0.12
	$\Delta\varepsilon_{31}^s$	0.10 ± 0.05	-0.083.0.06	-0.10 ± 0.04
	$\Delta\psi_{31}^c$	0.02 ± 0.12	0.10 ± 0.14	-0.07 + 0.11
27.55	$\Delta\varepsilon_{32}^c$	0.01 ± 0.03	--0.035.0.06	-0.04 ± 0.02
	$\Delta\psi_{32}^s$	-0.363 0.07	-0.13 ± 0.14	-0.113 0.05
	$\Delta\varepsilon_{32}^s$	0.052 0.03	0.053 0.06	0.033 0.02
	$\Delta\psi_{32}^c$	--0.023 0.06	- 0,083 0.14	-0.10 ± 0.04
13.63	$\Delta\varepsilon_{33}^c$	0.065 0.05		0 . 0 5 3 0 . 0 4
	$\Delta\psi_{33}^s$	0,214 0.12		--0.312 0.12
	$\Delta\varepsilon_{33}^s$	0.09 ± 0.05	...	-0.013 0.04
	$\Delta\psi_{33}^c$	0.10 ± 0.12	...	- 0.095 0.08
9.13	$\Delta\varepsilon_{34}^c$	0.12 ± 0.04	0.07 ± 0.06	0.063 0.02
	$\Delta\psi_{34}^s$	0.123 0.08	–0.23 ± 0.14	- 0.234-0.05
	$\Delta\varepsilon_{34}^s$	0.04 ± 0.03	0.072 0.06	-0.01:1 0.03
	$\Delta\psi_{34}^c$	0.03 ± 0.09	0.033 0.06	-0.09 ± 0.09
–429.8** (FCN)	$\Delta\varepsilon_F^c$	--0,044.0.03	--0,094 0,06	0.3230,02
	$\Delta\psi_F^s$	–0.16 ± 0.08	0.583 0.14	- 0.823 0.05
	$\Delta\varepsilon_F^s$	- 0.213 0.03	-0.114 0.06	- 0.093 0.04
	$\Delta\psi_F^c$	-0.81 ± 0,08	--0.783 0.14	0.12 ± 0.07

* Units for linear coefficients are mas/yr.

** The free core nutation period used by McCarthy & Luzum (199]) is - 418. days.

'l'able 5. Comparison] of corrections to circular nutation amplitudes (mas).

Period (days)		VLBI/LLR (this paper)	Herring *et al.* (1991)	McCarthy & Luzum (1991)	ZMOA-2 theory
- 6798.38	a^+_{r1}	0.243 0.06	0.283 1.00	0.19 ± 0.16	0.04
	a^+_{i1}	- 0,353 0.07	0.353 1.00	-0.42,1 0,15	- 0.09
	a^-_{r1}	- 2.89 ± 0.06	-3.113 1.00	-2.49 ± 0.16	- 2.79
	a^-_{i1}	1.83:1 0.07	1,33:1 1.00	1,74:1 0.15	1.59
365,26	a^+_{r10}	- I .91 ± 0.02	1.94:1 0.04	-2.043 0,03	- 1.96
	a^+_{i10}	0.37:1 0.02	0.323 0.04	0.35:1 0.02	0.30
	a^-_{r10}	-0.023 0.02	0.04 ± 0,04	0.034 0.03	0,04
	a^-_{i10}	0,07:1 0.02	0.15 ± 0.04	0,083 0.02	0.02
182.62	a^+_{r9}	- 0.01 ± 0.02	- 0.054 0,04	-0.053 0,01	- 0.04
	a^+_{i9}	-0,07 ± 0.02	- 0.073 0,04	-0.024 0.01	--0.04
	a^-_{r9}	0.643 0.02	0,52 ± 0,04	0.53 ± 0.01	0.59
	a^-_{i9}	--0.55 ± 0.02	- 0.47 ± 0,04	-0.463.0.01	-0.56
121,75	a^+_{r11}	-0.063 0,02	- 0.03:1 0.04	-0.013 0.02	-0.02
	a^+_{i11}	0,07 ± 0.02	0.033:0.04	0.03 ± 0.02	0.00
	a^-_{r11}	- 0.11 ± 0.02	- 0.054.0.04	- 0.034:0.02	- 0.02
	a^-_{i11}	-0.01:1 0.02	- 0.044 0.04	-0,02 ± 0.02	- 0.02
13.66	a^+_{r31}	0.05:1 0.03	- 0.01:1 0.04	-0.044 0.04	-0.01
	a^+_{i31}	-0.05:1 0.03	0.063 0.04	0.04 ± 0.03	- 0.01
	a^-_{r31}	-0.01 ± 0.03	-0.323 0.04	--0.264.0.04	-0.08
	a^-_{i31}	0.05 ± 0.03	-0.024 0,04	-0.063 0.03	0.00
27.55	a^+_{r32}	0.07:1-0.02	0.043 0.04	0.044 "0.01	0.00
	a^+_{i32}	-0.03 ± 0.02	-0.04 ± 0.04	-0.03 ± 0,01	-0.02
	a^-_{r32}	-0.07 ± 0.02	–0.01 ± 0.04	0.00 ± 0.01	-0.03
	a^-_{i32}	0.02:1 0.02	0.01.4:0.04	0.003 0.01	0.01
13.63	a^+_{r33}	-0,07 ± 0.03	...	0.044 0.03	--0.01
	a^+_{i33}	-0.033 0.03	...	--0,014.0.03	0,00
	a^-_{r33}	0.01:1 0.03	...	-0.093 0.03	-0.05
	a^-_{i33}	0,07 ± 0.03	...	- 0.02 ± 0.03	0.00
9.13	a^+_{r34}	–0.08 ± 0.02	0,0140.04	0.02 ± 0,01	0.01
	a^+_{i34}	–0.01 ± 0.02	-0.033 0,04	--0,013-0.02	0.00
	a^-_{r34}	-0.043:0.02	-0.08 ± 0.04	- 0.084 0.01	0.00
	a^-_{i34}	0.02 ± 0.02	0.04 ± 0.04	--0.02 ± 0.02	0.01
–429.8* (FCN)	a^+_{rF}	0.05 ± 0.02	--0.07 ± 0.04	0.00.1 0.01	...
	a^+_{iF}	--0.0630.02	'0.10 ± 0.04	0.073 0.02	...
	a^-_{rF}	– 0.014:0.02	0.164.0.04	-0.32 ± 0.01	...
	a^-_{iF}	-0.27 ± 0.02	-0,214 0.04	–0.02 ± 0.02	...

* The free core nutation period used by McCarthy & Luzum (1991) is - 418 days,

Table 6. VLBI estimates of the free core nutation (in mas) over four time spans.

Component*	1978-1983	1984-1987	1988-1990	1991-1993
a^{+}_{rF}	-0,30 ± 0,15	-0,06 ± 0.13	0,04 ± 0.06	-0.01 ± 0,04
a^{+}_{iF}	-0.18 ± 0.15	-0,27 ± 0.14	-0.03 ± 0.06	-0.09 ± 0.04
a^{-}_{rF}	0.24 ± 0.15	0.11 ± 0.13	0.04 ± 0.06	-0.04 ± 0.04
a^{-}_{iF}	0.07 ± 0.15	-0.41 ± 0.14	-0.29 ± 0.06	-0.31 ± 0.04

*The period of the free core nutation is - 429,8 days.

Table 7. VLBI estimates of the free core nutation (in mas) for several values of the adopted period.

Component	-415 d	420 d	-425 d	430 d	435 d
a^+_{rF}	0.10 ± 0.03	0.10 ± 0.03	0.06 ± 0,03	0.00 ± 0.03	-0.04 ± 0.03
a^+_{iF}	0.05 ± 0,03	-0.03 ± 0.03	0.08 ± 0,03	-0.10 ± 0.02	-0,08 ± 0.02
a^-_{rF}	-0.30 ± 0.03	0.27 ± 0.03	0.17 ± 0.03	0,02 ± 0.03	0.12 ± 0.03
a^-_{iF}	0.05 ± 0,03	-0.11 ± 0.03	0.23 ± 0.03	0.28 ± 0.02	-0.24 ± 0.02

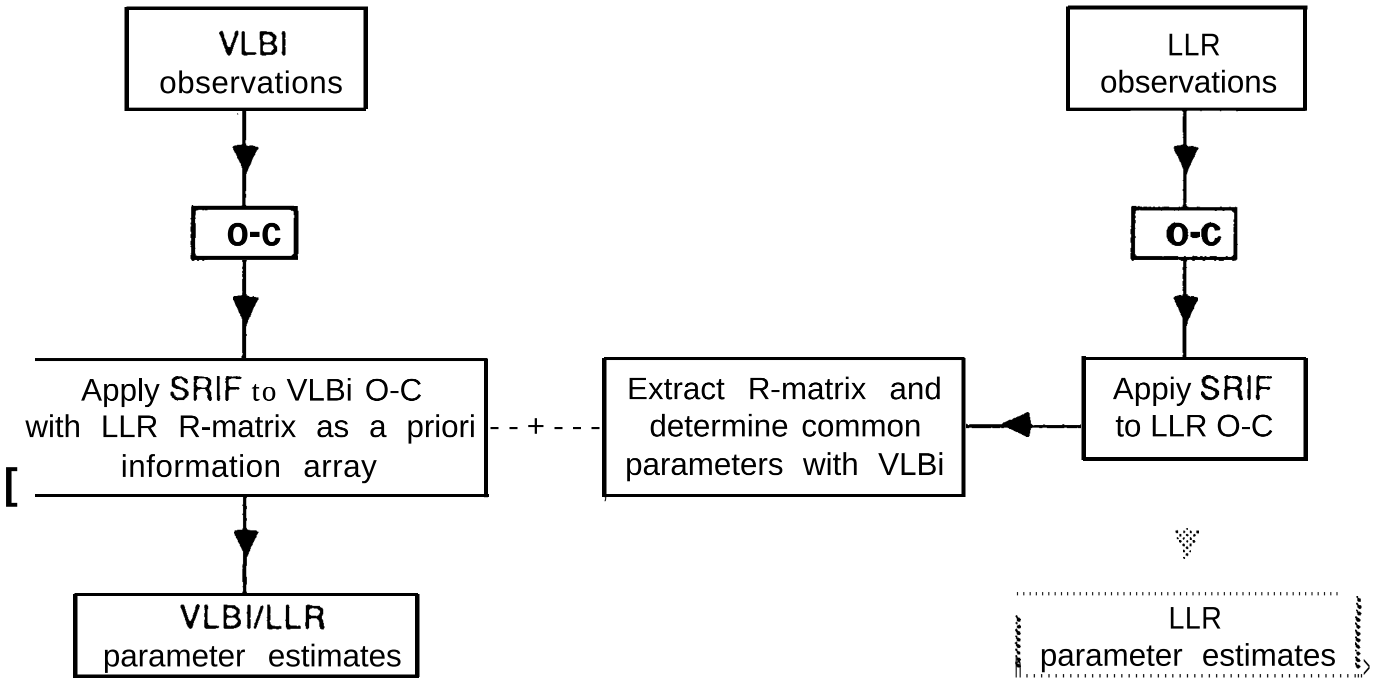
VLBI observations
O-C
Apply SRIF to VLBi O-C with LLR R-matrix as a priori information array
VLBI/LLR parameter estimates
Extract R-matrix and determine common parameters with VLBi
LLR observations
O-C
Appiy SRIF to LLR O-C
LLR parameter estimates

Figure 1