Intermittent Turbulence in Solar Wind from the South Polar Hole

A. A. Ruzmaikin[1], J. Feynman, B. E. Goldstein, and E. J. Smith
Jet Propulsion Laboratory, California Institute of Technology, Pasadena

A. Balogh
The Blackett Laboratory, Imperial College, London

Short title: INTERMITTENT TURBULENCE ...

---

[1]Also at Department of Physics and Astronomy, California State University Northridge

Abstract. The magnetic fields measured by the Ulysses spacecraft are used to study solar wind turbulence in the fast solar wind from the south polar hole. The spacecraft was at about 46 degrees south latitude and 3.9 AU. For a magnetic field with a Gaussian distribution the power spectrum (second order structure function) is sufficient to completely characterize the turbulence. However if the distribution is non-Gaussian the effects of intermittency must be taken into account. We show that our data sets are non-Gaussian. Thus the observed spectral exponents include effects of intermittency and can not be directly compared with the standard second- order spectral theories such as the Kolmogorov and Kraichnan theories.

To permit a better comparison of the observations with the theoretical models, we study the structure characteristics of the data. We find the exponents of the second order structure functions (power spectra) and the higher-order normalized structure functions for the components of the magnetic fields. We show that these sets of exponents can be approximately described by two basic numbers: the spectral exponent, and the intermittency exponent. The intermittency exponent characterizes correlation properties of the energy cascade from large to small scales. Before comparing the observations to the theoretically expected values a reduction must be made to the observed spectral exponent. The amount of the reduction depends on both the intermittency exponent and on the model of the energy cascade assumed in the turbulence theory.

We reduce the measured spectral indices according to a simple model for Alfvén turbulence that is developed here. We then compare our reduced spectral indices with second order spectral theory. The reduced spectral indices for the period range of 1 min. to about an hour are remarkably constant and in excellent agreement with the value of 3/2. Thus our treatment is self consistent. Our tentative conclusion is that the high frequency turbulence appears to agree with the model of random phased Alfv én waves. This tentative conclusion must be tested by further theoretical and observational work.

Introduction

The magnetic and velocity fields in the solar wind are composed of mean values and superposed fluctuations of a complicated nature. The fluctuations are essentially random i.e. they have a broad continuous power spectrum. These spectra can be characterized by their slopes and have been studied for many years [for reviews see *Barnes*, 1979; *Roberts and Goldstein*, 1991; Marsch, 1991]. These studies emphasized the differences in the properties of the solar wind depending on the wind velocity, and it was found that the variations in the slow solar wind were different from those in the fast wind from coronal holes. It was also found that the form of the spectrum varied with heliocentric distance.

If the turbulence is Gaussian the power spectrum (i.e. the spectrum of the second-order correlation function) would completely determine the statistics of the fluctuations. However, direct study of the distributions of observational data of the solar wind magnetic field shows that the statistics of the solar wind fluctuations is typically non-Gaussian and non-log-normal [see *Feynman and Ruzmaikin,* 1994 and references in that paper]. The velocity fluctuations have also been shown to be non-Gaussian [*Burlaga,* 1991,1993; *Marsch et al.,* 1992; *Marsch and Lui,* 1993]. Thus higher-order statistical moments are needed for a more complete characterization of solar wind turbulence.

In this paper we study the spectral slopes and high-order spectral slopes of magnetic fields observed by the Ulysses spacecraft as it was immersed in the wind from the south polar hole. We address the question of the relation of the measured spectral indices to the indices expected on the basis of the second-order statistical approach.

We use the approach of intermittent turbulence. This approach to solar wind turbulence has been previously used in several studies. *Burlaga,* [1991,1993] demonstrated the existence of intermittency in the solar wind velocity field at 1 AU in the range from 8 hours to 2.7 days and at 8.5 AU (Voyager data) in the range from

0.85 hour to 13.6 hours. *Marsch et al.*, [1992]; *Marsch and Lui*, [1993] used Helios data in the time domain between 40.5 seconds and 24 hours to provide evidence for the intermittent nature of temperature, flow velocity and Alfvén velocity in the inner solar wind between 0.3 AU and 0.9 AU. These authors compared their results with those expected for the fluid second order Kolmogorov turbulence. In this study we will compare our observational results with an MHD model developed below.

Two Number Approximation

In 1941 Kolmogorov conjectured that, in the inertial interval of scales (between the energy input scale and the energy dissipation scale), the spectrum of incompressible homogeneous hydrodynamic turbulence has a power-law form, i.e. can be characterized by one number, the power-law exponent α'. The value of α' depends on the mechanism of nonlinear interaction by which the energy cascades from larger structures to smaller structures. In classical fluid turbulence it is a resonant type of interaction which breaks whorls of fluid at every step of the cascade into pieces half the size (offspring), each receiving an equal fraction of the kinetic energy from the larger scales. The exponent found by Kolmogorov and Obukhov is $\alpha' = 5/3 \approx 1.67$ [*Monin and Yaglom,* 1975]. For incompressible magnetohydrodynamic turbulence the cascading process was described by *Kraichnan,* [1967] as being made up of random-phased Alfvén waves with different wavelengths propagating through the fluid at speeds determined by the magnetic field at the energy input scale. The energy cascade results from scattering between oppositely directed waves with almost equal wavelengths. The resultant spectral exponent was found to be $\alpha' = 3/2 = 1.5$.

This fascinating "one number approximation" is based on the assumption that the rate of energy transfer from the input to the dissipation scales, ε, is a fixed, scale-independent quantity. This means, in particular, that the energy is equally distributed between offspring before the next step of the cascade takes place. The time scale for this equilibration is, however, of the same order as the cascade time (the scale size divided by a typical velocity at that scale) and evidently only a certain degree of such equilibration is possible. Thus ε is, in fact, not a constant, but a random spatial variable. Its mean value can still be used as the rate of energy transfer in the inertial interval [*Monin and Yaglom,* 1975]. It has been discovered that in hydrodynamics this variable has long-range spatial correlations extending over the whole inertial interval [*Anselmet et al.,* 1984; *Meneveau and Sreenivasan,* 1991] . Physically, this suggests a picture in which structures in the form of ropes, sheets or more complicated fractal forms appear. It follows also that the dissipation of the turbulent energy is inhomogeneous and has the same type of structuring. Mathematically, this means that the correlation function of ε has the form of a power law, $\langle\varepsilon(\mathbf{x})\varepsilon(\mathbf{x}+\mathbf{r})\rangle \propto r^{-\mu}$. The new exponent μ was related to the dimension D of the structures in the limit of zero dissipation in that $\mu = 3 - D$ [*Mandelbrot,* 1975; *Frisch et al.,*1978; see also the original β-model in *Novikov and Stewart,* 1964]. For example, dimension 2 corresponds to sheet-like structures. The dimension can be fractal. In the framework of the cascade picture described above this exponent determines the degree of energy equilibration between the offspring.

The exponent α' and the intermittency exponent μ define what may be called a "two number approximation" to the turbulence. An important effect of the intermittency is to change the spectral exponent α' expected from the second-order theories to some value α. This value is determined directly from the observation. The problem then is to find the "reduced' spectral exponent $\alpha' = \alpha - \delta\alpha$ which has to be compared with exponent given by the second-order theories such as the Kolmogorov or Kraichnan exponents. The value $\delta\alpha = \mu/3$ was found for the fluid turbulence [*Frisch et al.,*1978]. Other effects of intermittency can be found by using statistics of higher-order.

The simplest way to study high-order statistics is to use high-order structure functions, for example $\langle|B_i(\mathbf{x}+\mathbf{r}) - B_i(\mathbf{x})|^p\rangle$, $p = 1, 2, ...$; $i = x, y, z$. [*Monin and Yaglom,* 1975]. For a Gaussian distribution of turbulent fields, the structure functions of any order can be expressed through the second- order structure function: $\langle|B_i(t+\tau)-B_i(t)|^p\rangle = \langle|B_i(t+\tau)-B_i(t)|^2\rangle^{p/2}$. However, intermittent fields are essentially non-Gaussian so that the structure function of every order has its own exponent. In the "two number approximation" these exponents are functions of μ and the order of the structure function p. This gives an opportunity to find the intermittency exponent experimentally. In conjunction with the experimentally determined spectral exponent, one can then give a more complete description of the turbulence using this approximation than was possible using the original Kolmogorov approach.

In the present paper we present a new model for estimating the spectral index reduction $\delta\alpha$ and report on the results of analysis of several 1-min. averaged time series for the magnetic field. Three time series are taken from the data obtained by Ulysses in the fast solar wind from the south polar hole. We will find α , μ , $\delta\alpha$ and also exponents for normalized structure functions up to the order 10 from the observational data . Our results for the magnetic field data at large heliospheric latitudes, deep in the southern solar polar hole wind, show a remarkably good agreement with the Kraichnan model of Alfvén turbulence.

Measure of Intermittency

Spacecraft data are obtained as time series at a given spatial point. However they reflect the spatial distribution of the fields in so far as fluctuations can be considered as being frozen into the supersonic solar wind i.e. the Taylor hypothesis holds [*Matthaeus and Goldstein* 1982]. The Taylor hypothesis can be valid for time intervals that are small compared with a characteristic time of variation of the solar wind speed, $|V_{sw}/(\partial V_{sw}/\partial t)|$. In particular, in the sheared region between the fast and slow speed wind, the hypothesis will be satisfied only for time intervals shorter than $|\partial V_{sw}/\partial y|^{-1}$ where y is a coordinate across the shear. The Taylor hypothesis holds for the data used and the frequency interval considered in the present paper so that we can substitute time scales τ_n by spatial scales $l_n = V_{sw}\tau_n$.

The traditional approach to these time series is based on calculations of spectra and cross- spectra for the second-order correlations. Thus,

$$S_{ij}(\tau) = \langle [B_i(t+\tau) - B_i(t)][B_j(t+\tau) - B_j(t)] \rangle,$$

where the averaging is taken over all t of the data set, represent a second-order structure function (tensor) for the magnetic field $B_i, i = x, y, z$. This function is evidently related to the frequently used correlation function. Note that $S(0) = 0$ and $S(\infty) = 2\langle |B_i|^2 \rangle$ because $\langle B_i(t+\tau)B_i(t) \rangle \rightarrow 0$ as $\tau \rightarrow \infty$. For isotropic and homogeneous turbulence of a divergence free field, this tensor is fully defined by one scalar, the longitudinal structure function $S_L(\tau) = \langle |B_L(t+\tau) - B_L(t)|^2 \rangle$ [*Monin and Yaglom,* 1975]. In the inertial interval this function scales as $S_L(\tau) \propto \tau^{s(2)}$. The exponent is directly related to the spectral exponent $\alpha = 1 + s(2)$. In general, the p-order structure functions scale as $\tau^{s(p)}$.

For the velocity field in the solar wind the longitudinal direction can be naturally identified with the heliospheric radial direction, which we will denote as x-direction. The acquisition of data by spacecraft is also going along this direction. However the mean magnetic field has a spiral form which is close to a radial field only near the Sun. Thus for MHD turbulence in our study the longitude direction is different from the radial. In principle, we can always find this direction at every heliospheric distance but this would not help much because the data are taken along the radial direction. To avoid unnecessary assumptions we will study both the longitudinal and the latitudinal structure functions determined correspondingly by the x-, y- and z-components of the magnetic field. (Because structure functions are nonlinear functions of the field components there is not much meaning in calculating the structure functions for the absolute values of the field.) It is convenient to define the normalized structure function for the magnetic field as

$$I_i(\tau, p) = \frac{\langle |B_i(t+\tau) - B_i(t)|^p \rangle}{\langle |B_i(t+\tau) - B_i(t)|^2 \rangle^{p/2}} \quad (1)$$

similar to the definition used in hydrodynamics [*Frisch et al.*, 1978]. Here $i = x, y, z$. By definition $I_i(\tau, 2) = 1$. There are two advantages in using (1) instead of the standard high-order structure function. First (technical), we avoid very big or very small numbers appearing after taking the large powers of the fields. Second (physical), in the case of the Gaussian distribution the normalized structure function is independent of τ for every p, so that any change of I with τ indicates intermittency, which is why we denote this function by I. Note that in order to calculate I we first need to calculate the second-order structure function which will also be used to find the spectral exponent α.

For τ small compared to the time scale of variations of the global parameters, the intermittency measure is expected to be a self-similar (power-law) function

$$I(\tau, p) \propto \tau^{\zeta(p)}. \tag{2}$$

The form of the function $\zeta(p) = s(p) - s(2)p/2$ depends on the distribution function of the turbulence. This dependence is known for two theoretical models of fluid turbulence [see for example *Frisch et al.*,1978]:

$$s(p) = \mu + (1 - \mu)\frac{p}{3}, \qquad \zeta = (1 - p/2)\mu \tag{3}$$

for the so-called β-model with a fixed share of space occupied by offspring having the energy $\beta = 2^{-\mu}$ at every step of cascade, and

$$s(p) = (2 + \mu)\frac{p}{6} - \mu\frac{p^2}{18}, \qquad \zeta = -p(3 - p)\mu/18 \tag{4}$$

for the log-normal distribution of ε. From $s(p)$ one can find that the intermittency correction to the second-order spectral exponent of a turbulent fluid is $\mu/3$ for the β-model and $\mu/9$ for the log-normal model. The fit to either of these models gives an estimate for the intermittency exponent μ in a turbulent non-magnetic fluid.

Reduced Spectral Exponent in Random-Phased Alfvén Turbulence

The models described above were developed for fluid turbulence. Here we develop a model for Alfvén turbulence.

Consider a discrete sequence of scales or wavelengths $l_n = l_0 2^{-n}, k_n = l^{-1}, n = 0, 1, 2, \ldots$. The magnetic energy, equal in this model to the kinetic energy per unit mass in the scale l_n, is defined as

$$M_n = E_n = \int_{k_n}^{k_{n+1}} E(k) dk.$$

In stationary turbulence the energy is input at the scale l_0 and cascades through the scales $l_1, l_2, \ldots$ until it reaches the dissipation scale. The characteristic magnetic and velocity field fluctuations are defined by

$$M_n \propto \beta^n b_n^2 \propto \beta^n v_n^2$$

where the factor β defines how much space is filled by the waves. It is convenient to present this filling factor as $\beta = 2^{-\mu}$ and to use instead of β the exponent μ. The characteristic time for transfer of energy from the large to small scales is $\tau_n = l_n / v_A \propto l_n / B_0$, where B_0 is the magnetic field in large scales ($\geq l_0$). The rate of magnetic (and kinetic) energy transfer is

$$\varepsilon \propto \beta^n \frac{(\delta b_n)^2}{\tau_n} \propto \beta^n \frac{(b_n^2)^2}{l_n}$$

where we substitute $\delta b_n \approx (b_n \nabla) v_n \cdot \tau_n \propto (b_n^2 / l_n) \tau_n$ according to the induction equation. When there is no intermittency, i.e. $\beta = 1$, the condition that ε is independent of n gives the Kraichnan spectrum $b_n^2 \propto v_n^2 \propto l_n^{1/2}$, or $M(k) \propto k^{-3/2}$ in Fourier space. In the intermittent case, when $\beta^n = 2^{-n\mu} = (l_n / l_0)^\mu$, we have

$$b_n^2 \propto v_n^2 \propto \beta^{-n/2} l_n^{1/2} \propto l_n^{1/2} (l_n / l_0)^{-\mu/2},$$

or

$$M(k) = E(k) \propto k_n^{-1} \beta^n b_n^2 \propto k_n^{-3/2} (k_n/k_0)^{-\mu/2}. \quad (5)$$

It follows that the reduced spectral index in this MHD model of random Alfvén waves is

$$\alpha' = \alpha - \mu/2. \quad (6)$$

Let us find now the relationship between the exponents of the intermittency function and μ in this model. Since the turbulence in this model occupies a fraction β of the volume, for the sake of estimation it is enough to substitute every averaging $\langle\rangle$ by β. Thus we have from (1)

$$I = \frac{\langle|\delta B_i|^p\rangle}{\langle|\delta B_i|^2\rangle^{p/2}} \propto \frac{\beta b_n^p}{\beta^{p/2} b_n^p} \propto (\frac{l_n}{l_0})^{\mu(1-p/2)}.$$

Hence

$$\zeta(p) = \mu - \mu\frac{p}{2}. \quad (7)$$

It follows from here for the structure function exponents

$$s(p) = \zeta(p) + s(2)\frac{p}{2}\mu + (1-\mu)\frac{p}{4}. \quad (8)$$

It is interesting to note that the exponents $s(p)$'s for the fluid and MHD model, see Eqs. (3) and (9), are different while ζ's are the same. We will calculate μ as a fit to the linear dependence (8) corresponding to random-phased Alfvén turbulence:

$$\zeta(p) = \mu(1 - \frac{p}{2}) + \delta, \quad (9)$$

where δ stands for the error of this fit. For comparison let us point out that *Burlaga,* [1991, 1993] calculated an intermittency exponent in the framework of fluid (non-magnetic) turbulence using $\mu = 2 - s(6)$. For the model of random-phased Alfvén turbulence the corresponding relation has the form $\mu = 2 - s(8)$. Unfortunately, μ calculated in such a way from our data has a large error since s(8) has a large error. In this paper we use the expression (9) to calculate the intermittency exponent since it is less subject to error.

Note that the exponents $\zeta(p)$ and $\alpha = 1 + s(2)$, calculated from the observational data, are model independent.

The Structure Index of Second Order; Comparison to Spectral Analysis

Ulysses was in the ecliptic plane measuring particles and magnetic fields from October 1990 till February 1992 as it traveled from 1 AU to 5.4 AU [*Balogh et al.*, 1992; *Bame et al.*, 1992]. This was during the solar maximum and decline of solar cycle 22 and more transient events were seen in the beginning of the period than towards the end [*Burton et al.*, 1992; *Balogh et al.*, 1993]. The simplest way to characterize the magnetic field time series is to find structure indices of first order using a simple "length of curve" technique that had been applied earlier to solar wind data [*Burlaga and Klein*,1986; *Ruzmaikin et al.*, 1993]. The length of the curve of the data time series is defined similarly to the first-order structure function

$$L(\tau) = \sum_{j=1}^{n} |B_i(t_j + \tau) - B_i(t_j)|. \tag{10}$$

Note that the sum here is taken over $n = T/\tau$ points, where T is the whole time interval under the consideration, so that $L(\tau) \propto \tau^{-1}|\delta B_i|$. In the definition of $S(\tau, 1)$, instead of the sum, the operation of averaging $\langle\rangle$ is used which includes the normalizing factor n so that $S(\tau, 1) \propto |\delta B_i| \propto L(\tau)\tau$. In addition, $S(\tau, p)$ can be averaged (and we actually will do this) over $T - \tau$ points, not just n points.

The "length of curve" technique or the calculation of the second order structure function is actually a simple way to find a slope of the spectrum. Theoretically, the exponent α defining the slope of the spectrum is linearly related to the exponent of the length of curve $s(L)$ [*Berry*, 1979], and to the exponent of the second-order structure function [*Monin and Yaglom*, 1975]:

$$\alpha = 3 - 2s(L) = 1 + s(2) \tag{11}$$

Practically, because of somewhat different approximations involved in the calculations of the standard power spectrum, the length of curve, and the structure functions, these methods give somewhat different spectral indices. As an example we calculated the spectral exponent for 5–minute averaged magnetic data obtained by Ulysses in the interval from November 27 to December 27, 1991. The total number of experimental points is 8352, i.e. quite large from the statistical point of view. Table 1 lists the results which give a rough estimate of the accuracy inherit in these types of calculations. The indices calculated by the three methods are equal to an accuracy of about ±0.05. These calculated spectral exponents include the effect of intermittency.

The Structure Indices of Solar Wind from the South Polar Hole.

In late 1993 the spacecraft was well within the solar wind from the south polar hole [*Balogh et al.*, 1992; *Bame et al.*, 1992]. The solar wind conditions have been described by *Phillips et al.*, 1994. The solar wind speed was consistently in the 700 to 800 km/s

Table 1. The spectral index α calculated by three different methods

	power spectrum	$L(\tau)$ 3-2s(L)	$S(\tau, 2)$ 1+s(2)
B_x	1.66	1.66	1.73
B_y	1.86	1.80	1.90
B_z	1.85	1.75	1.85
B	2.15	2.00	2.00

range. Compressions, rarefactions and shock waves have weakened or disappeared. There are few coronal mass ejections. In brief we have a steady, fast solar wind with few disturbances. These data present a unique opportunity to study the waves and turbulence in an undisturbed wind. In this study we use 3 data sets from the Ulysses magnetometer (Balogh et al. 1992). The Ulysses magnetometer measures the magnetic field vector every 2 seconds. Here we use 1-min. averaged series. The duration of each time series is approximately 3 days (=4320 data points, see Figure 3). These data will be used to characterize the high-frequency MHD regime.

We selected for our analysis three undisturbed time periods when the spacecraft was at large heliocentric latitudes. The first time interval starts December 6 (00h), 1993. The spacecraft was at a latitude of -45.9 degrees south and a heliocentric distance of 3.95 AU. The second interval began 13 days later at December 19 (00h), 1993 when the spacecraft was at 47.1 degrees south and 3.89 AU. The third interval starts January 20, 1994 with the spacecraft at 50.17 degrees and 3.73 AU.

Figure 1 shows the distribution of magnetic field intensities for the x component of the field for the first data set. (In the coordinate system used, the x component was radial from the sun, the y component was parallel to the direction of planetary motion and the z component completed the orthogonal set.) The bars show the observed distribution. The line is a Gaussian with the same mean and standard deviation. The number of data points in the set and the first 4 moments of the distribution are given. It is clear that the distribution is non-Gaussian. The distribution functions for the other two intervals have the similar form and the third and fourth statistical moments (skewness and kurtosis). This indicates that the effects of intermittency must be taken into account in comparing the observed second order spectral index to that expected on the basis of theory.

The magnetic field data used in constructing figure 1 is shown in Figure 2, in the top panel. The lower panel shows the second order structure function of this data set.

Note that at small values of the lag τ the values increase linearly. At larger values of τ the curve flattens. That is, the index of the power spectrum (slope of the structure function) depends on the frequency considered. In this study we are concerned with only the high frequency, self similar regime as determined from the 2nd order structure functions. In making this frequency selection we were very conservative and used a frequency cut off at high enough frequencies so that the 2nd order structure functions was clearly self similar. It was found to be self-similar in the time domain between 1 min. and about an hour. For the spacing in Eq. (1) we used $\tau = 1, 2, 4, 8, \ldots, 32$ in the units 1 min.

The measures of intermittency (normalized structure functions) for this data set is shown in the top panel of figure three. Intermittency measures from 2 to 10 are such that lower orders are below higher orders. Thus order 2 are the points along the abscissa and order 10 is the top line. For the intermittency measures up to order $p = 10$ the dependence $\zeta(p)$ found from the Ulysses observational data can quite satisfactorily be approximated by a straight line. For larger orders the intermittency measures constructed from real data become more and more inaccurate and deviate more and more from the linear dependence. The appearance of the growing uncertainties is related with the well known fact that high- order statistical moments are sensitive to the tail of the distribution function, especially when the distribution function is non-Gaussian. The intermittancy exponent is constructed from the slopes of the intermittency measures shown in the lower panel of figure 3. Thus for p=2 the slope of the intermittency measure is zero and the slopes of the intermittency measures increase with p as shown. The intermittency measure μ is found using expression (9) above. The points in the lower panel of figure 3 lie on a straight line to a good approximation, suggesting that the two number approximation is a good one.

The results of calculations in the two number approximation are presented in Table 2 for all data sets. In this Table α is the observed slope of the 2nd order structure function (power spectrum), μ is the intermittency exponent determined from the data, δ is the error in the straight line fit, and α' is the reduced spectral index, to be compared to the theoretical spectral index. The results for the reduced spectral exponent are remarkably constant. In figure 4 we have plotted the reduced spectral indices and compared them with the values of 3/2 (appropriate to a Kraichnan spectrum) and 5/3 (appropriate to a Kolmogorov spectrum). Our results are clearly in agreement with 3/2 and clearly not in agreement with 5/3, indicating agreement with a Kraichnan spectrum. These results justify the model used to derive the correction to the spectral exponent and thus strongly indicate that the turbulence in the undisturbed solar wind is consistent with the spectral index of 1.50 predicted for Alfv én turbulence.

Discussion

We have studied the solar wind as observed by the Ulysses spacecraft using a "two number approximation" for the description of turbulence. In this approximation the turbulence is characterized by the spectral index and the intermittency exponent. For all our data sets we found substantial values for the intermittency exponent in the high frequency region in which the spectra were self similar. As discussed above, in order to compare measured values of the spectral indices with expectations based on models of turbulent cascades, it is necessary to reduce the measured spectral indices by an amount which depends on the intermittency exponent. Unfortunately, the amount of reduction is model dependent. Three models have been described above and the corrections required for any of them are significant with the values of the intermittency measure found in this study. Two of the models were developed for to describe fluid turbulence. We have introduced a third model developed here for MHD turbulence. Using this model we have determined the reduced spectral exponent for the three time periods when the spacecraft was in a nearly constant speed undisturbed solar wind from the south polar hole. We have analyzed the high-frequency self-similar regime.

The reduced spectral exponents for the magnetic field components are found to be remarkably constant. Even more remarkable is their very close agreement with the value of 3/2. Based on these results we can tentatively conclude that the reduced spectral exponents in the frequency range corresponding the time range between 1 min. and half an hour agree with the Kraichnan model of random phased Alfv én waves. To say this differently, the reduced spectral exponents are consistent with the model used to estimate the reduction. However a firm conclusion requires more determinations of the spectral indices and intermittency measures as well as further theoretical considerations.

We explored here only a small part of the Ulysses data. Further and much more extended studies are needed to obtain the best estimates of the spectral indices and intermittency exponents in the framework of the two number approximation, and to find from observations all high-order exponents needed for the complete characterization of the solar wind MHD turbulence.

It is clear that in the relatively undisturbed high speed wind from the polar hole we have a unique tool to study intermittent MHD turbulence by using space data. These studies have to be compared with laboratory studies of hydrodynamic turbulence which are currently undergoing intensive development. In some sense the experimental conditions in the solar wind are similar to those in the fluid mechanics laboratory, in which the flow passes the observer who measures the longitude and transverse velocities. Space studies, however, are of special importance because there is no high magnetic Reynolds number MHD turbulence on Earth. The Ulysses experiment is supplying us with excellent data on an important and fundamental physical problem.

Acknowledgments. We are grateful to J. Wolf for the help in computations. A. R. was supported by grant No. NAS 7-918 from the National Aeronautic and Space Administration. The work was performed in part for the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautic and Space Administration.

Table 2. The spectral exponents, intermittency exponents, and reduced spectral exponents of the components of magnetic field for three time intervals: 6 December, 20 December 1993, and 20 January 1994 correspondingly

	α	μ	δ	α'	α	μ	δ	α'	α	μ	δ	α'
B_x	1.79	0.54	-0.12	1.52	1.81	0.58	-0.13	1.52	1.81	0.62	-0.11	1.50
B_y	1.86	0.66	-0.15	1.53	1.87	0.57	-0.11	1.59	1.84	0.76	-0.16	1.46
B_z	1.83	0.60	-0.11	1.53	1.86	0.68	-0.15	1.52	1.84	0.57	-0.11	1.56

References

Anselmet, F., Y. Gagne, and E. J. Hopfinger, High-order velocity structure functions in turbulent shear flows, J. Fluid Mech., *140*, 63, 1984.

Balogh, A., T. J. Beek, R. J. Forsyth, P. C. Hedgecock, R. J. Marquedant, and E. J. Smith, The magnetic field investigation on the Ulysses mission: Instrumentation and preliminary results, Astron. Astrophys., Suppl. Ser., *92*, 221, 1992.

Balogh, A., G. Erdos, R. J. Forsyth, and E. J. Smith, The evolution of the interplanetary sector structure in 1992, *Geophys. Res. Lett.20*, 2331, 1993.

Bame, S. J., D. J. McComas, B. L. Barraclough, J. L. Phillips, K. J. Sofaly, J. C. Chavez, B. E. Goldstein, and R. K. Sakurai, The Ulysses solar wind plasma experiment, Astron. & Astrophys., Suppl. Ser., *92*, 237, 1992.

Barnes, A., Hydromagnetic waves and turbulence in the solar wind, in *Solar System Plasma Physics*, edited by E. N. Parker, C. F. Kennel, and L. J. Lanzerotti, p.249, North-Holland, 1979.

Berry, M. W., Diffractals, J. Phys. A: Math. Gen. , *12,* 781, 1979.

Burlaga, L. F., and L. W. Klein, Fractal structure of the interplanetary magnetic field, *J. Geophys. Res., 91,* 347, 1986.

Burlaga, L. F., Intermittent turbulence in the solar wind, *J. Geophys. Res., 96,* 5847, 1991.

Burlaga, L. F., Intermittent turbulence in large-scale velocity fluctuations at 1 AU near solar maximum, *J. Geophys. Res., 98,* 17,467, 1993.

Burton, M. E., E. J. Smith, B. E. Goldstein, A. Balogh, R. J. Forsyth, and S. J. Bame, Ulysses: Interplanetary shocks between 1 and 4 AU, *Geophys. Res. Lett.19*, 1287, 1992.

Feynman, J., and A. Ruzmaikin, Distributions of the interplanetary magnetic field revisited, to appear in *J. Geophys. Res.*, 1994 .

Frisch, U., P. L. Sulem, and M. Nelkin, A simple dynamical model of intermittent fully developed turbulence, J. Fluid Mech., *87,* 719, 1978.

Kraichnan, R., Inertial-range spectrum of hydromagnetic turbulence, The Physics of Fluids *8,* 1385, 1965.

Mandelbrot, B., *Fractals: Form, Chance and Dimension*, W. H. Freeman, San Francisco, 1975.

Marsch, E., MHD turbulence in the solar wind, in *Physics of the Inner Heliosphere*, edited by R. Schwenn and E. Marsch, p.159, Springer-Verlag, 1991.

Marsch, E., S. Lui, and C. -Y. Tu, Spectra and structure functions of temperature fluctuations in the inner solar wind, *Proceedings of the 26th ESLAB Symposium- Study of the Solar-Terrestrial System*, Killarney, ESA SP-346, 1992.

Marsch, E., and S. Lui, Structure functions and intermittency of velocity fluctuations in the inner solar wind, Ann. Geophysicae, *11*, 227, 1993.

Matthaeus, W. H., and M. L. Goldstein, Measurements of the rugged invariants of magnetohydrodynamic turbulence in the solar wind, J. Geophys. Res., *87*, 6011, 1982.

Meneveau, C., and K. R. Sreenivasan, The multifractal nature of turbulent energy dissipation, J. Fluid Mech., *224*, 429, 1991.

Monin, A. S., and A. M. Yaglom, *Statistical Fluid Mechanics: Mechanics of Turbulence*, vol. 2, The MIT Press, Cambridge, Massachusetts, and London, 1975.

Novikov, E. A, and R. W. Stewart , Intermittency of turbulence and the spectrum of fluctuations of energy dissipation, Izv. Acad. Nauk USSR Geophys. Ser., *3*, 408, 1964.

Phillips, J. L. , A. Balogh, S. J. Bame, B. E. Goldstein, J. T. Gosling, J. T. Hoeksema, D. J. McComas, M. Neugebauer, N. R. Sheeley, Jr. and Y.-M. Wang, Ulysses at 50 degrees South: Constant immersion in the high-speed solar wind. Preprint, 1994.

Roberts, D. A., and M. L. Goldstein, Turbulence and waves in the solar wind, in *US National Report to International Union of Geodesy and Geophysics*, edited by M. A. Shea, p.932, American Geophysical Union, 1991.

Ruzmaikin, A. A., I. P. Lyannaya, V. A. Styashkin, and E. G. Eroshenko, The spectrum of the interplanetary magnetic field, *J. Geophys. Res., 98,* 13,303, 1993.

This manuscript was prepared with the AGU LaTeX macros v3.0.

Figure Captions

Fig. 1. The distribution function of x-component of the interplanetary magnetic field at -46 degree beyond the ecliptic plane (20 December, 1993). The first four statistical moments are shown in the right corner of the panel.

Fig. 2. The data and the second-order structure function of B_x at - 46 degree beyond the ecliptic plane (December, 1993).

Fig. 3. The measure of intermittency $I(p)$ and exponents of this function for p=2,3,...,10 in the same time interval as in Figure 1.

Fig. 4. The reduced spectral exponents of the magnetic field for three time intervals of the solar wind from the south polar hole.

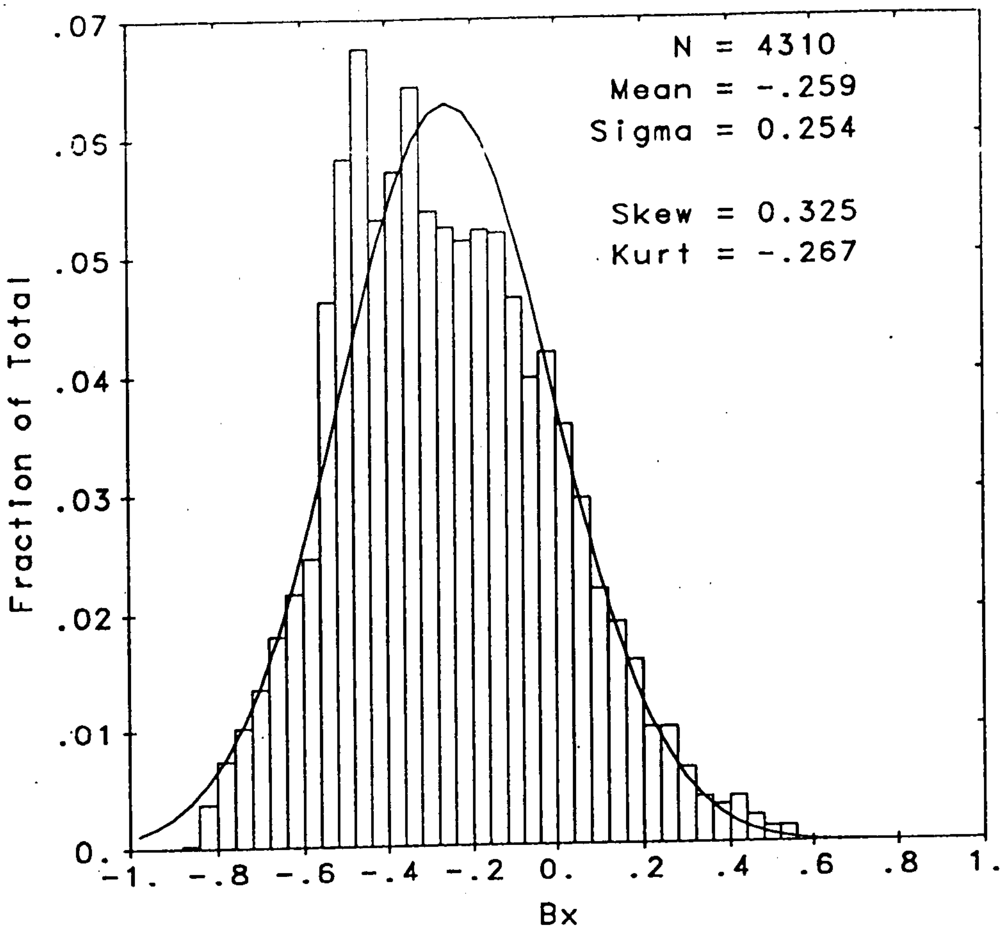
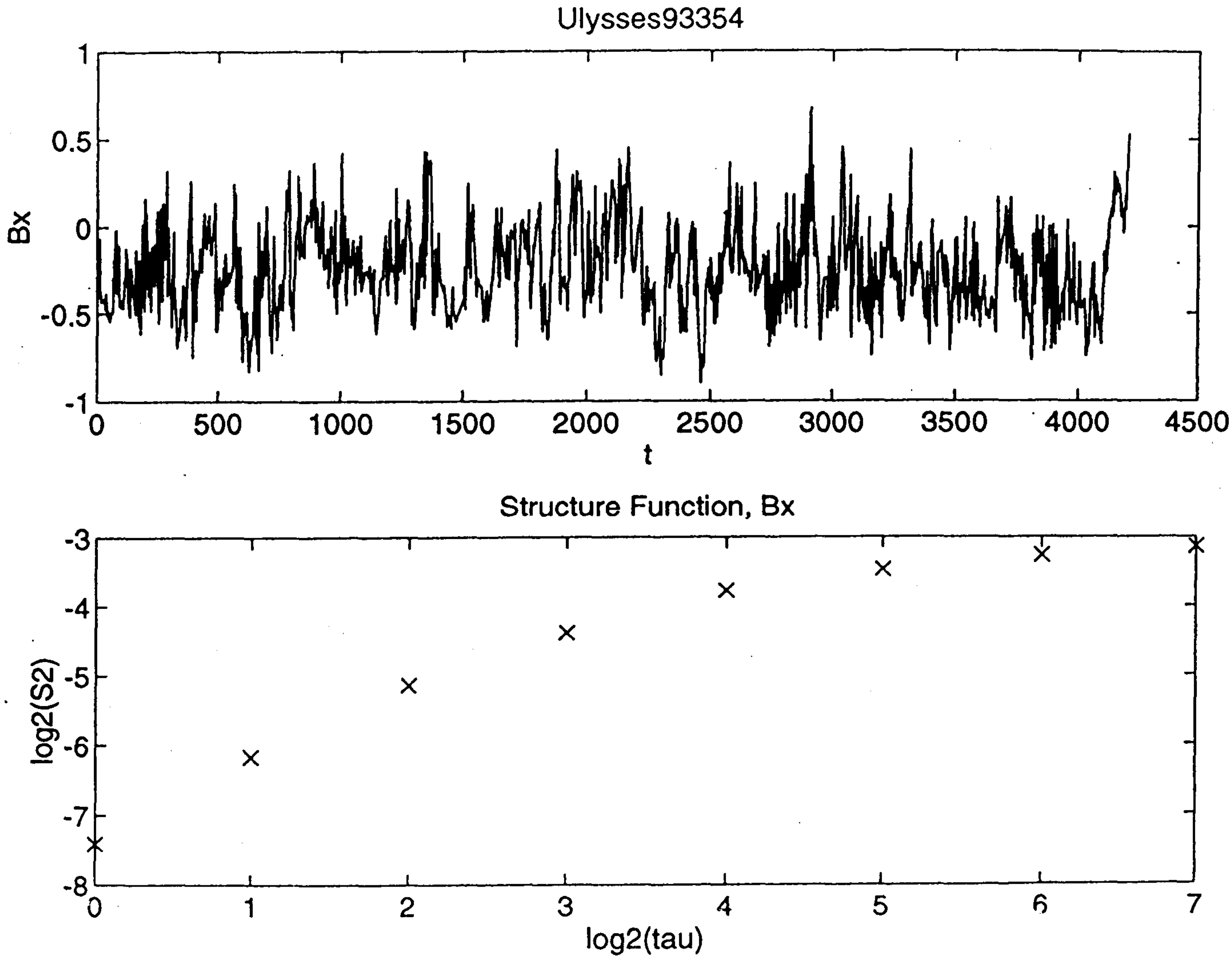
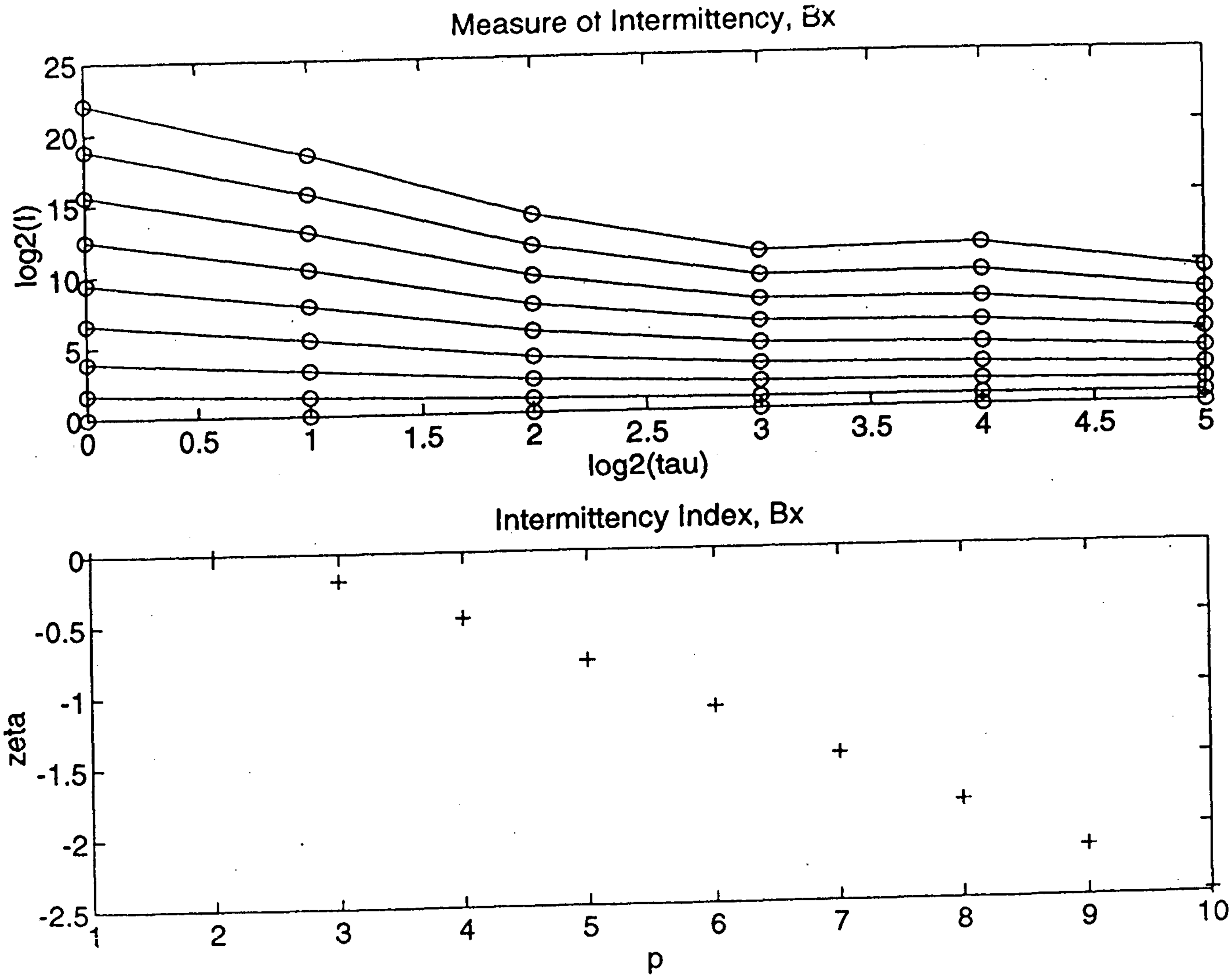

N = 4310
Mean = -.259
Sigma = 0.254
Skew = 0.325
Kurt = -.267
Fraction of Total
.07
.06
.05
.04
.03
.02
.01
0.
-1.
-.8
-.6
-.4
-.2
0.
.2
.4
.6
.8
1.
Bx

Fig 1



Ulysses93354
Bx
1
0.5
0
-0.5
-1
0
500
1000
1500
2000
2500
3000
3500
4000
4500
t
Structure Function, Bx
log2(S2)
-3
-4
-5
-6
-7
-8
0
1
2
3
4
5
6
7
log2(tau)

Fig. 2



Measure of Intermittency, Bx
25
20
15
10
5
0
log2(I)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
log2(tau)
Intermittency Index, Bx
0
-0.5
-1
-1.5
-2
-2.5
zeta
1
2
3
4
5
6
7
8
9
10
p

Reduced Spectral Exponent
Magnetic Field in Polar Hole

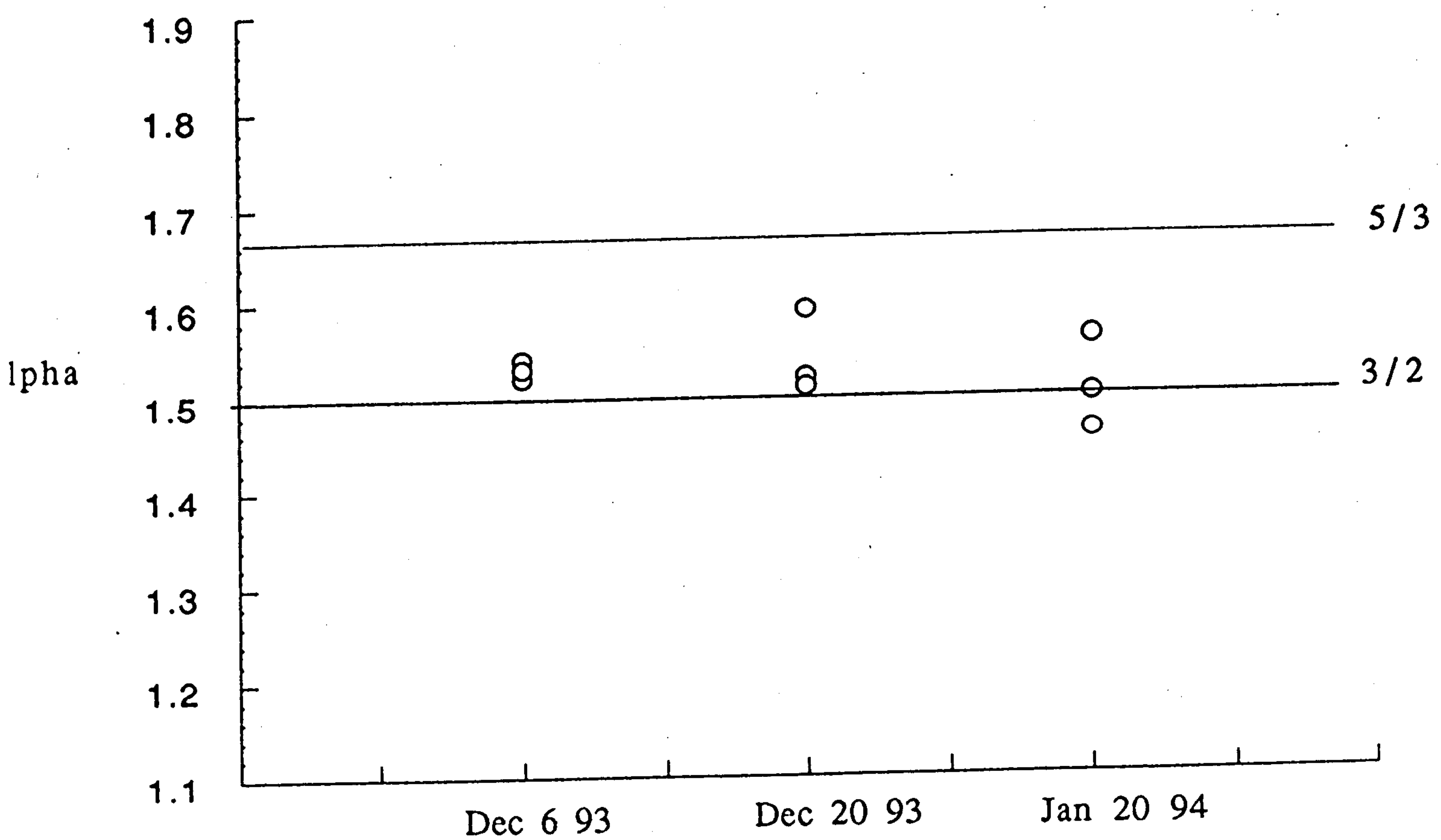


Fig. 4