RELATIVITY PARAMETERS DETERMINED FROM LUNAR LASER RANGING

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Abstract

Laser ranging to retroreflectors placed on the Moon by lunar space missions have been collected over the past 24 years. A comprehensive set of parameters relating to the Earth and Moon can be estimated from these data. This paper presents current lunar laser ranging results for five relativity parameters: the Principle of Equivalence, geodetic precession, the Parametrized Post-Newtonian parameters \( \beta \) and \( \gamma \), and a possible time rate of change of the gravitational constant \( G \). Neither a violation of general relativity nor a change in \( G \) is found.

1. Introduction

Beginning in 1969, lunar space missions placed four corner-cube retroreflectors on the lunar surface. A "range" is the elapsed round-trip traveltime of a pulse of light between a terrestrial observatory and a lunar retroreflector. As of January, 1994, more than 8400 lunar laser ranging (1,1,1) normal points have been collected from three observatories, with accuracies over the past five years being \( \pm 2.5 \text{ cm} \). Among parameters estimated are five related to relativity.

2. The Principle of Equivalence

A consequence of the Principle of Equivalence is that the gravitational mass \( M_G \) of any object is equal to its inertial mass \( M_I \). (The Weak Equivalence Principle (WEP) holds if independent of composition; the Strong Equivalence Principle (SEP) holds if true for gravitational self-energy.) A failure of this principle would lead to the Nordtvedt effect, in which the geocentric lunar orbit is displaced in the direction of the Sun, and would exhibit a signature in the LlR data ([1], [2], [3]).

The quantity measured by LlR is \( (M_G/M_I)_{\text{Earth}} - (M_G/M_I)_{\text{Moon}} \). A correction for solar radiation pressure [4] is applied to the estimated value. With this correction,

\[
\frac{M_G}{M_I}_{\text{Earth}} - \frac{M_G}{M_I}_{\text{Moon}} = (3 \pm 5) \times 10^{-13}
\]

The dependence of the ratio \( M_G/M_I \) on the Strong Equivalence Principle can be expressed as \( M_G/M_I = 1 - \eta (U_G/Mc^2) \) ([1], [2], [3]), where \( \eta \) is a dimensionless parameter and \( U_G \) is the gravitational self-energy of the celestial body. For the Earth and Moon, \( (U_G/Mc^2)_{\text{Earth}} = (U_G/Mc^2)_{\text{Moon}} - 4.45 \times 10^{-10} \). The LlR result for \( \eta \) is

\[
\eta = \begin{cases} 
0.0043 \pm 0.0051 & \text{if WEP uncertainty is included} \\
0.0007 \pm 0.0000 & \text{if WEP is assumed}
\end{cases}
\]

where the results of Su et al. [5] are used for the WEP.
3. Geodetic Precession

General relativity predicts that the lunar node, longitude of perigee, and mean longitude should precess at 19.2 milliarcseconds/year. This rate $P_\theta$ is implicit in the equations of motion used to model the Moon and planets. The JLR result is

$$\Delta P_\theta = 0.3\% \pm 0.7\%$$

4. Parametrized Post-Newtonian $\beta$ and $\gamma$

Two parametrized Post-Newtonian ($'N$) parameters can be estimated from JLR data: $\beta$, measuring non-linearity of superposition, and $\gamma$, measuring space curvature produced by unit mass. Both $\beta$ and $\gamma$ appear in the equations of motion; $\gamma$ also appears in the light-time equation. Using only these formulations,

$$|\beta - 1| < 0.005, \quad |\gamma - 1| < 0.005, \quad |\beta + \gamma - 2| < 0.003$$

Nordtvedt ([11], [2], [3]) showed that $\beta = (\eta + \gamma - 3)/4$. Using $\eta$ from the Equivalence Principle above and $|\gamma - 1| < 0.002$ from interplanetary ranging [6].

$$|\beta - 1| < 0.0014 \text{ if WEP uncertainty is included}$$

$$< 0.0006 \text{ if WEP is assumed}$$

5. Rate of Change of the Gravitation Constant

The present JLR data span gives sensitivity to any time rate of change in the gravitation constant $G$, primarily through the solar perturbation on the lunar orbit. The change in $G$ is expressed as $\dot{G}/G$. The result from JLR is

$$\dot{G}/G = (0.1 \pm 0.9) \times 10^{-11} \text{ /year}$$

6. summary

No significant change from general relativity is found, and no change in $G$ is evident. A more extensive discussion of this work is in [7].

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References