

**The Gravitational Redshift in a Local Inertial Frame:  
A New Null Test of the Equivalence Principle**

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## ABSTRACT

We consider the gravitational redshift effect measured by an observer in a local freely-falling frame (LFFF) in the gravitational field of a massive body. For purely metric theories of gravity, the metric in a LFFF is expected to differ from that of flat spacetime by only "tidal" terms of order  $(GM/c^2 R)(r'/R)^2$ , where  $R$  is the distance of the observer from the massive body, and  $r'$  is the coordinate separation relative to the origin of the LFFF. A derivation is presented which shows that a violation of the Equivalence Principle for certain types of "clocks" could lead to a larger apparent redshift effect of order  $(1 - \alpha)(GM/c^2 R)(r'/R)$ , where  $\alpha$  parametrizes the violation ( $\alpha = 1$  for purely metric theories, such as general relativity). Therefore, redshift experiments in a LFFF with separated clocks can provide a new null test of the Equivalence Principle. Possible opportunities to perform this null test are discussed. With presently available technology, it is possible to reach an accuracy of 0.01 % in the gravitational field of the Sun using an atomic clock orbiting the Earth. A 1 % test in the gravitational field of the galaxy would be possible if an atomic frequency standard were flown on a space mission to the outer solar system.

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Gravitational redshift experiments provide an interesting test of the Equivalence Principle,<sup>1</sup> and in particular whether there exist certain nonmetric couplings of new fundamental interactions.<sup>2</sup> For purely metric theories of gravity, the gravitational redshift is expected to be independent of the particular type of frequency standard used in the measurement. This has been called the universality of the gravitational redshift (UGR).<sup>3</sup> The principle of the universality of free fall (UFF) is also a fundamental feature of metric theories, and it continues to be verified in highly precise experiments.<sup>4</sup> Constraints on UFF violations are expected to apply to related violations of the UGR.<sup>5</sup> However, certain types of nonmetric couplings can preserve the UFF, but still lead to violations of the UGR (discussed in Ref. [2]).

in addition to using different types of clocks, it is also important to perform experiments in the gravitational field of bodies of different composition. Interactions violating the UGR could be dependent upon the type of matter comprising the body and its physical state. The conventional approach to testing the redshift is to probe as near to the body as physically possible, and then compare the results to measurements made far away. Although the redshift has been tested to 0.02% in the Earth's gravitational field,<sup>6</sup> the results are presently less precise (by roughly two orders of magnitude) for direct probes of other bodies in the solar system. Observations of solar spectral lines,<sup>7</sup> and a recent probe with the Galileo spacecraft,<sup>8</sup> have provided a 1 % test for the Sun. Similar accuracies can be obtained from the Voyager spacecraft flybys of the outer planets.<sup>9</sup> A spacecraft flyby of the Sun to within 4 solar radii is feasible, and with an atomic clock could test the redshift to an accuracy of 1 part in  $10^9$  (see [10] and references therein).

A possible way to avoid the difficulty of sending a clock to a distant, massive body, such as the Sun, is to perform a "null" redshift test. This approach involves

comparing two different energy- state transitions at the same location in a gravitational field, while the field varies from natural causes. For example, a test in the gravitational field of the Sun can be performed as the Earth rotates or orbits between perihelion and aphelion. Although the Earth orbital motion can produce a larger effect than its daily rotation, it is necessary to wait a sufficient amount of time for a significant variation of the solar Newtonian potential. This can limit the accuracy of the test, because of systematic errors. Both types of frequency standards must be highly stable if they are to provide a successful test. A 1.7% test was obtained with a hydrogen maser oscillator and a superconducting- cavity stabilized oscillator (SCSO).<sup>11</sup> More recently, the comparison of nuclear energy- state transitions in a differential Mössbauer experiment has provided a null test at the level of 0.01 %.<sup>12</sup> In the future, a 0.01 % test has been proposed involving the planned Stanford- NASA gyroscope experiment. A particularly interesting possibility is a comparison of the 1 S- 2S transition in hydrogen and antihydrogen, which could provide a null test approaching 1 part in  $10^9$ .<sup>14</sup> In this paper, we wish to point out that there exists a second “null redshift” effect which could also provide a useful test.

For metric theories of gravity, the “Local Flatness Theorem” assures us that at any point in a gravitational field the curved- spacetime metric can be reduced to the Minkowski tensor, modulo “tidal” corrections due to spacetime curvature.<sup>15</sup> In the local reference frame of an observer who is freely- falling in the gravitational field of a static body of mass  $M$ , these corrections are of order  $(GM/c^2R)(r'/R)^2$  at a coordinate separation  $r'$  from the origin, where  $R$  is the distance between the observer and the body. Because of the  $1/R^3$  fall- off, the gravitational redshift observed in such a local freely- falling frame (LFFF) is expected to be negligible for measurements made between small coordinate separations. For example, the scalar redshift in a geocentric frame is only of order  $4.4 \times 10^{-25}(r'/\text{km})^2$ . The galactic redshift in a

heliocentric frame is exceedingly small: roughly  $2 \times 10^{-25} (r'/\text{AU})^2$

A violation of the UGR by a nonmetric coupling can lead to two additional corrections for observers in a LFFF. This can be easily demonstrated by considering a simple derivation of the metric result. We will start with the expression, valid to order  $c^{-2}$ , for the frequency shift of a signal propagated between two clocks at the points  $\vec{x}_1$  and  $\vec{x}_2$  in an inertial reference frame (see Ref. [10]):

$$f_2 = f_1 \left[ 1 - \hat{n} \cdot (\vec{v}_2 - \vec{v}_1)/c - (1/2)(v_1^2 - v_2^2)/c^2 - (\hat{n} \cdot \vec{v}_1)(\hat{n} \cdot \vec{v}_2)/c^2 + (\hat{n} \cdot \vec{v}_1)^2/c^2 - (U_1 - U_2)/c^2 \right], \quad (1)$$

where  $\vec{v}_1$  is the velocity of the clock at  $\vec{x}_1$ ,  $\vec{v}_2$  is the velocity of the clock at  $\vec{x}_2$ ,  $\hat{n}$  is a unit vector pointing from  $\vec{x}_1$  to  $\vec{x}_2$ , and  $U_1$  and  $U_2$  are the Newtonian gravitational potentials at each point (defined positively). Both clocks are allowed to fall freely in the gravitational field of a single static, spherically symmetric body of mass  $M$  at the origin.

Without loss of generality, we may specialize to the case in which both clocks are initially at rest when a signal is sent from the clock at  $\vec{x}_1$ . Equation (1) then reduces to

$$(f_2 - f_1)/f_1 = -\vec{v}_2 \cdot \hat{n}/c + (1/2)v_2^2/c^2 - (\mathbf{u} \cdot \hat{n} - U_2)/c^2, \quad (2)$$

where  $\hat{n} = (\vec{x}_2 - \vec{x}_1)/|\vec{x}_2 - \vec{x}_1|$ ,  $U_1 = GM/|\vec{x}_1|$ , and  $U_2 = GM/|\vec{x}_2|$ . To sufficient accuracy, the propagation time of the photon is simply  $\Delta t = |\vec{x}_1 - \vec{x}_2|/c$ . During

this time, the second clock reaches a velocity of  $\vec{v}_2 = -GM\vec{x}_2 \Delta t / |\vec{x}_2|^3$ . Defining the separation vector  $\vec{x}' = \vec{x}_1 - \vec{x}_2$ , and expanding  $U_1$  about  $\vec{x}_2$  in terms of  $\vec{x}'$ , gives

$$U_1(\vec{x}_1) = \frac{GM}{r_2} \left[ -\frac{\vec{x}_2 \cdot \vec{x}'}{r_2^2} - \frac{3(\vec{x}_2 \cdot \vec{x}')^2 - r_2^2 r'^2}{2r_2^4} \right], \quad (3)$$

to order  $(r'/r_2)^2$ . Using this expression in Equation (2), some significant cancellations occur, leaving a "tidal" frequency shift given by

$$(f_2 - f_1)/f_1 = - \left( \frac{GM}{c^2 r_2} \right) \frac{3(\vec{x}_2 \cdot \vec{x}')^2 - r_2^2 r'^2}{2r_2^4}. \quad (4)$$

Transformation of the coordinates appearing in Equation (4) to the local reference frame of the second clock leaves this result unchanged. Equation (4) agrees with the results of detailed derivations of the metric for an observer in a local freely-falling frame.<sup>16</sup>

In order to determine how a violation of the UGR would affect Equation (4), we will focus upon possible long-range couplings which are directly proportional to the Newtonian potential. Two parameters  $\alpha_1$  and  $\alpha_2$ , one for each clock, should then be inserted into Equation (2) before each of the respective Newtonian potentials. The derivation proceeds as described above, but Equation (4) is found to be modified accordingly:

$$(f_2 - f_1)/f_1 = (\alpha_2 - \alpha_1) \frac{GM}{c^2 r_2} - (1 - \alpha_1) \frac{GM(\vec{x}_2 \cdot \vec{x}')}{c^2 r_2^3} - \alpha_1 \left( \frac{GM}{c^2 r_2} \right) \frac{3(\vec{x}_2 \cdot \vec{x}')^2 - r_2^2 r'^2}{2r_2^4}. \quad (5)$$

In addition to the "tidal" term, which is now multiplied by  $\alpha_1$ , two additional terms have appeared in Equation (5). The first term is the well-known contribution for two different types of clocks, which persists even if the clocks are unseparated ( $\vec{x}' = 0$ ). We will refer to this as the "zeroth-order" null redshift. A second contribution is seen to arise from the second term if the clocks are separated, which we will refer to as the "first-order" null redshift. This represents a second "null" redshift effect, which is expected to be absent if the UGR is valid.<sup>17</sup> Because this term is larger than the "tidal" term by a factor of  $V/T$ , it can be tested at an interesting level of accuracy. In fact, it may be possible to perform a more precise test of this effect, rather than the zeroth-order null effect, by taking advantage of the geometrical dependence. Furthermore, only a single type of frequency standard is necessary, which alleviates the need for two different types of clocks of comparable stability.<sup>18</sup>

Let us consider tests in the scalar gravitational potential with Earth-orbiting clocks. The null frequency shift is of order  $|\Delta f/f| \sim 7 \times 10^{-17}(1 - \alpha_1)(r'/\text{km})$ , where  $r'$  is the geocentric separation. For the Vessot-NASA redshift experiment conducted in 1976,<sup>6,19</sup> in which a hydrogen maser oscillator was launched on a Scout rocket to an altitude of  $10^4$  km,  $|\Delta f/f| \sim 10^{-12}(1 - \alpha_1)$ . Between apogee and a transmitter failure prior to splashdown, fractional frequency residuals of order  $2 \times 10^{-14}$  were obtained during the flight (see Ref. [19]). Because of the geometrical dependence of the null effect on the angular position of the spaceborne maser relative to the Sun, it is not clear how strong a null test was provided by the flight. It would be best to fit the new model directly to the data. Nevertheless, the residuals suggest that  $1 - \alpha_1$  was possibly tested to an accuracy approaching 2%. It might also be possible to obtain a test at the 1% level with the atomic clocks flown on the satellites of the Global Positioning System (GPS), which orbit the Earth at an altitude of 20,000 km with satellite speed  $v \sim 3.9$  km/s or orbit with

a large apogee, and with a more stable atomic frequency standard, could improve upon these accuracies by two or three orders of magnitude.<sup>21</sup>

A particularly interesting experiment would be a test of the first-order null redshift in the gravitational potential of the galaxy (for a test of the zeroth-order null redshift in the gravitational potential of the Virgo cluster, see Ref. [22]). Solutions to the long-standing problem of how to explain galactic rotational velocities have invoked a dark matter halo,<sup>23</sup> mollifications of Newtonian gravity (discussed in Ref. [23]), or covariant theoretical alternatives to general relativity.<sup>24</sup> The dark matter itself could generate a new long-range field or gravitational coupling.<sup>25</sup> Recently, UFF experiments have been analyzed to search for any anomalous acceleration of the test masses that is directed towards the galactic center.<sup>26,27</sup> The results provide limits at the level of 0.1% on the ratio of any anomalous differential acceleration to the Newtonian acceleration expected from the total mass of the galaxy (dark+seen), or from only the dark matter component. These limits are much looser (by several orders of magnitude), than the limits on UFF violations due to the Sun. Again, it is still possible that a new galactic interaction could preserve the UFF, but lead to a detectable violation of the UGR.

If the total mass of the galaxy interior to the solar orbit is involved in the UGR violation, then in the solar system the magnitude of the null effect is of order  $|\Delta f|/f \sim 3 \times 10^{-16}(r'/\text{AU})$ , where  $r'$  is the heliocentric separation. This should be reduced by (25 - 50)% if only the dark component is involved (see Ref. [26]). The direction to the galactic center is inclined only ~ 5.5 arcdegrees to the ecliptic plane, so nearly the full magnitude of the null effect can be easily achieved with a space mission in the ecliptic plane. At a distance of 30 AU (the orbit of Neptune), the null frequency shift would reach 1 part in  $10^4$ . An atomic frequency standard stable to

1 part in  $10^{16}$  over the several years required to reach this distance or beyond could provide a 1% test. This is now feasible with a new type of trapped-ion frequency standard, which is currently being developed at JPL.<sup>28</sup> The trapped-ion technique offers important advantages for reducing long-term frequency errors. Recent tests have demonstrated that a fractional frequency stability of at least 1 part in  $10^{16}$  is possible over long periods with sufficient regulation of operating parameters and environmental influences. Equally important for use on a spacecraft, the standard can be designed to have a small mass and low power consumption.<sup>29</sup> At present, both NASA and the Russian Space Agency (IKI) are developing possible missions to Pluto. There are no plans at this point, however, to include a trapped-ion standard on the spacecraft.

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