A Parallel Discrete Surface Integral Equation Method For the Analysis of Three-Dimensional Microwave Circuit Devices with Planar Symmetry

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I. Background

It has been found that the Discrete, Integral Equation (DSI) technique is a highly effective technique for the analysis of microwave circuits and devices [1,2], The DSI is much more robust than the traditional Finite Difference Time-Domain (FDTD) method in a number of ways. Principally, since it is based on a volume discretization using unstructured dual grids, circuit devices with non-separable geometries are accurately modeled. Furthermore, structures with fine details can be modeled using locally refined grids, rather than having to refine the grid globally. A disadvantage of the DSI algorithm is that the numerical grid and the sparse matrices describing the field updates must be stored, However, this can be greatly relaxed by exploiting symmetries in the model. In this paper, a technique that exploits planar symmetries is described, namely, planar symmetry is recognized for three-dimensional geometries which can be uniquely described by a projection onto a two-dimensional plane. To this end, the three-dimensiontil model is described by a two-dimensional grid. Furthermore, due to regularity along the third-dimension the sparse matrices used for the field updates need only be computed for the two-dimensional grid. This results in a significant savings in storage, increasing potential problem sizes that can be solved by orders of magnitude. Based on the planar DSI algorithm, a highly scalable parallel algorithm is presented. The parallel algorithm is based on a spatial discretization of the two-dimensional mesh. By treating the update matrices as a subassembly of matrices, the matrix-vector multiplications are easily done in parallel requiring only minimal interprocessor communication. Finally, the robustness and efficiency of the algorithm is illustrated through a number of' examples.

II. The Planar DSI Algorithm

The DSI algorithm is based on the discretization of Ampere's and Faraday's laws in their integral form [1-3]. The vector fields are projected onto the edges of a dual, staggered grid. The grid is assumed to be unstructured and irregular in a two-dimensional plane, and regular in the third-dimension, assumed here to be the z-direction. Therefore, the unstructured two-dimensional grid maintains its form for all values of z. A small section of the two-dimensional grid composed of quadrilateral cells is illustrated in Fig. 1, The solid lines represent the primary grid edges in the plane of the grid. The dotted lines represent the secondary grid edges, which joins the centers of the primary grid cells and are actually a half a cell above or below the transverse plane, At each node of the two-dimensional grid is a vertical edge of length dz.

In the plane of the two-dimensional grid, the transverse electric field vectors $\vec{e}_t$ are assumed parallel to the primary grid edges (solid lines) and constant along

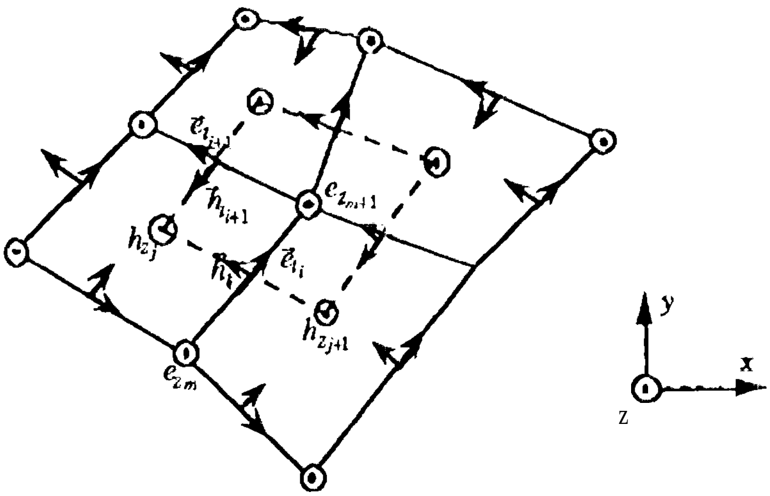

Fig. 1 Two-dimensional grid describing the three-dimensional geometry.

their length. The vertical electric field vectors $\hat{z}e_z$ are assumed to be z-directed and constant along the length of the vertical edges. These vertical edges can be viewed as posts connecting stratified layers of the two-dimensional grid. The normal magnetic field vectors $\hat{z}h_z$ are z-directed and assumed constant along the vertical secondary grid edges which are bisected by the center of the primary grid cells. Finally, the transverse magnetic field intensity vectors are assumed parallel to the secondary grid edges (dotted lines) and are also constant along their length. The transverse magnetic fields are actually at the center of the faces bound by two vertical edges and two transverse edges in the two planes connected by the vertical edges. Finally, each vector field component is assumed to be constant over the entire support of the dual face through which it passes.

Faraday's and Ampere's laws are discretized based on the above approximation of the fields. Approximating the time-derivatives using a central difference, and staggering the fields in time leads to an explicit time-marching solution. However, it is realized that the vector fields updated are normal to the faces bounded by the grid edges, Since the face normals are not always parallel to the dual grid edges prosing through the face, the normal fields must be projected onto the dual grid edges prior to updating the dual field [1-3]. The updates are then expressed in discrete form as

$$[H_z^{n+1}]_k = [H_z^n]_k + \bar{\bar{A}}_{h_z}[E_t^{n+1/2}]_k \tag{1}$$

$$[B_t^{n+1}]_{k+1/2} = [B_t^n]_{k+1/2} + \bar{\bar{A}}_{h_t}\begin{bmatrix} E_{z_{k+1/2}}^{n+1/2} \\ E_{t_{k,k+1}}^{n+1/2} \end{bmatrix} \tag{2}$$

$$[H_t^{n+1}]_{k+1/2} = \bar{\bar{A}}_{h_p}[B_t^{n+1}]_{k+1/2} \tag{3}$$

$$[E_z^{n+3/2}]_{k+1/2} = \bar{\bar{D}}_{e_z}[E_z^{n+1/2}]_{k+1/2} + \bar{\bar{A}}_{e_z}[H_t^{n+1}]_{k+1/2} \tag{4}$$

$$\left[D_t^{n+3/2}\right]_k = \overline{\overline{D_{\epsilon_t}}}\left[D_t^{n+1/2}\right]_k + \overline{\overline{A}}_{\epsilon_t}\begin{bmatrix} H^{n+1}_{t_{k,k+1}} \\ H^{n+1}_{t_{k+1/2}} \end{bmatrix} \tag{5}$$

$$\left[E_t^{n+3/2}\right]_k = \overline{\overline{A_{\epsilon_r}}}\left[D_t^{n+3/2}\right]_k \tag{6}$$

where the subscript k refers to the discrete height along the z-direction, D_t and B_t are the flux densities, the $\overline{\overline{D}}$'s are diagonal matrices, and the $\overline{\overline{A}}$'s are sparse matrices, Note that these matrices are only associated with the two-dimensional grid since they are the same for all values of k (Note that inhomogeneities in material parameters can easily be built into these expressions),

111. The Parallel DSI Algorithm

The parallel DSI algorithm is based on a spatial decomposition of the two-dimensional grid. To this end, the grid is decomposed into contiguous, non-overlapping subdomains. The decomposition is currently being performed using two-different techniques: 1) The Recursive Inertia Partitioning (RIP) algorithm, which is a power of two algorithm and ideal for hypercube computers, and 2) the Greedy algorithm. Both techniques offer excellent load balance. Once the mesh is decomposed into subdomains, each subdomain is assigned to a different processor. The matrices in (l)-(6) are then expressed as a subassembly of matrices, where each submatrix represents the updates of the fields within each subdomain. Subsequentl y, the matrix vector products are simpl y expressed as

$$\overline{\overline{A}}x = \sum_{i=1}^{P} \overline{\overline{A}}_i x_i$$

where P is the total number of processors. Initially, each matrix-vector product is computed for all k concurrently on each processor, Subsequently, the elements of x_i shared by adjacent processors are fully updated using interprocessor communication. This approach maximizes the ratio of computation to communication leading, to a highly scalable algorithm.

IV. Numerical Examples

Based on the above technique, a number numerical examples will be presented. As an example, consider the Gysel power divider illustrated in Fig. 2. This is a planar device printed on a 10 mil Alumina substrate. The device has five ports, three are fed by 50 Ω, and two by 100 Ω lines. The two 100 Ω ports are terminated through 50 Ω chip resistors connected to ground. The S-parameters for this device are illustrated in Fig, 3. The two-dimensional grid representing, the device was composed of roughly 20,000 quadrilaterals, and the height of the three-dimensional grid was 24 cells. The solution was obtained on an 8-node iPSC/860 hypercube, requiring 5000 time iterations, and was performed in roughly 2000 seconds.

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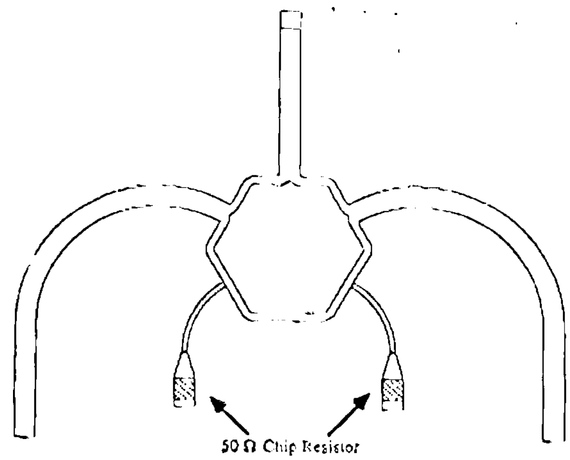

Fig 2. Gysel Power Divider

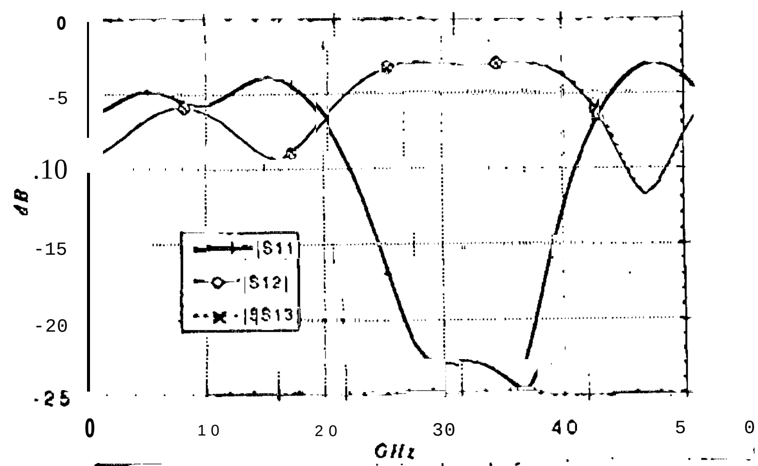

Fig. 3 S-parameters of Gysel power divider using the planar and parallel-DSI algorithm