Minimum Symbol Transition density on Earth-to-Space Links

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Abstract

This paper summarizes key results on the minimum modulated symbol transition density on Earth-to-Space links required by existing Command Detection Units (CDU's). The results are verified using computer simulations.

1 Introduction

Earth-to-Space links are typically characterized by high symbol signal-to-noise ratio (SNR) and low data rates (7 - 500 symbols per second). The uplink is uncoded and as a result, symbols and bits are identical. Existing transponders employ the Data Transition Tracking Loop (DTTL) in the Command Detection Unit (CDU) to perform symbol synchronization. The latter loop, depicted in Fig. 1, relies on an inphase and a midphase integrator to provide an error signal that is independent of the data polarity. The inphase integrator integrates over a symbol followed by a hard decision on the symbol polarity. By subtracting two successive decisions, a transition detection detector is used to determine whether a no transition (0), a +1 to - 1 transition or a - 1 to +1 transition occured. In the other arm, an integrator estimate of the error in phase (or timing) modulated by the data polarity. Mulltiplying that output by the transition detector output wipes out the polarity and produces an error signal that is filtered by the loop filter and then, used to advance or delay the phase of the local symbol clock. In this paper, let P_t denote the transition probability and p denote the probability of +1. It is well known that $P_t = 2pq$, where $q = 1 - p$ is the probability of - 1.

2 Performance Of the DTTL when $P_t = 1/2$

The performance of the DTTL for random data (that is, $P_t = 1/2$) has been analysed in [1] where it is shown that the variance of the phase error (in units of cycle^2) is given by

$$\sigma_\lambda^2 = \frac{wB_{L0}T}{2R_s \mathrm{erf}(\sqrt{R_s})} \tag{1}$$

where λ denotes the phase error (in cycles), w the midphase window with $w \leq 1$ (unitless), T the symbol duration (in sec), B_L the loop bandwidth (in Hz), R_s the symbol SNR (unitless) and $\mathrm{erf}(x)$ is the error function of x. For $R_s > 6$ dB, $\mathrm{erf}(\sqrt{R_s}) \simeq 1$ and (1) can be approximated by

$$\sigma_\lambda^2 \simeq \frac{wB_LT}{2R_s} \tag{2}$$

Note that the loop SNR (denoted by ρ) is related to σ_λ^2 through

$$\rho = \frac{1}{4\pi^2\sigma_\lambda^2} \tag{3}$$

As an example at 500 sym/sec, $R_s = 10$ dB, and $w = 1$, the minimum operating loop bandwidth required to maintain a 15 dB loop SNR is about xxx Hz.

3 Performance of the DTTL when $P_t \neq 1/2$

The performance of the DTTL for an arbitrary transition density was worked out in [2.] assuming that the noise spectrum at the output of the loop phase detector is independent of the transition density. A technique was presented to select the minimum transition density to achieve a specified degradation in the symbol error rate. More recently, the change in noise spectrum was accounted for in [3], assuming "high" R_s and $w = 1$. in addition, data asymmetry which accounts for unequal rise and fall times in the baseband pulse was included in the analysis. For the purpose of this report, we ignore data asymmetry and focus and transition density. It is shown in [3] that

$$\sigma_\lambda^2 = \frac{h(0) B_L T}{2 R_s g_n'(0)} \tag{4}$$

where $g_n'(0)$ denotes the derivative of the S-curve evaluated at $\lambda = 0$ and is given by

$$g_n'(0) = 2P_t \mathrm{erf}(\sqrt{R_s}) \tag{5}$$

and $h(0)$ denotes the normalized noise spectrum and is given by

$$h(0) = p^2 + q - \mathrm{erf}^2(\sqrt{R_s}) \left\{p^4 + q^2 - 2pq + p^2 q(p+1) + 2pqR_s\right\} + pq(1 + 2R_s) \tag{6}$$

For "high" values of R_s, it can shown that $h(0) \simeq 2P_t$ and as a result, (4) reduces to

$$\sigma_\lambda^2 \simeq \frac{B_L T}{2R_s} \quad \text{independent of } P_t! \tag{7}$$

At first glance, this result might seem surprising and to a certain extent, erroneous. But a c.loser look at the response of the DTTL indicates that the transition density affects three different fundamental parameters of the loop in opposing ways and the various affects pull the loop in opposite directions and end up cancelling each other. First as the transition density deviates from 50%, the loop bandwidth is reduced by $2P_t$. As an example if the loop was designed to operate with 5 Hz bandwidth with 50% transitions, the operating loop bandwidth with 3% transition would be $5 \times 2 \times 0.03 = .3$ Hz. It is important that the operating bandwidth remains large enough to accomodate the effects of nonideal oscillators such as clock drift and phase noise. To a first order approxima tion, the loop perturbation effect is independent of the SNR or the window size and is depicted in Fig. 2. In Fig. 2.a, the slopes for 40%, 20% and 3.1 % transition probabilities are depicted in normalized form (divided by the slope assuming 50% transition) versus symbol SNR. These curves in effect reflect the change in operating loop bandwidths as a function of P_t. In Fig. 2.b, similar results are presented with the normalization performed with respect to the 20% transition density. In this case, the operating bandwidth is the design bandwidth for a 20% transition and deviates (becomes larger or smaller) from that for different transition probabilities.

The second effect to be discussed is the change in signal power in the loop. Since the slope of the S-curve has changed, the signal power in the loop drops by the square of the slope. Hence, the signal power drops by $(2P_t)^2$.

Third, the noise spectral level in the loop also drops by $2P_t$ since $h(0) \to 2P_t$ at high symbol SNRs. Note that the noise enters the loop mainly through the midphase detector. As the number of transitions decreases, the midphase output is multiplied by more zeros from the transition detector, assuming the latter operates with few errors (which is the case at "high" symbol SNRs). Figure 3 depicts the normalized noise spectral level (normalized to 50% transition in Fig. 3.a and to 20% in Fig. 3.b). Finally, Fig. 4.a depicts the ratio of the tracking variance (with $w = 1$) for various transition densities normalized by the tracking variance for a 50% transition. The tracking variance for a 50% transition density is depicted in Fig. 4 .b. Figure 5 (a and b) depict similar results with $w = 1/4$. The performance is verified through computer simulations which confirm the fact that the variance dots not vary with the transition density at "high" SNR.

4 Conclusion

The goal of this study is to draft a recommendation for minimum transition densities required for Earth-to-Space links. The performance of the DTTL as a function of the transition density is understood for "high" symbol SNR and with window equal to unity. In this case, the tracking jitter (and thus, the degradation on telemetry) of the DTTL is unaffected by the transition density. However, the loop bandwidth varies with the transition density and it is important to guarantee that the operating bandwidth at all times is adequate to handle the nonideal effects of the oscillators found in practice. Equally important is the effect of the transition density on the acquisition performance of the DTTL, which is not quantified in the literature to the best of the authors knowledge. It is suggested that further studies be performed by extending the results in [2] and [3] before any recommendation is drafted. Specifically,

the jiiter should be characterized as a function of the transition density for "low" symbol SNR's and various windows, as future systems might have coded uplinks,

the acquisition of the DTTL should be characterized as a function of the symbol transition density,

the results should be verified through simulations and measurements with the Advanced Receiver,

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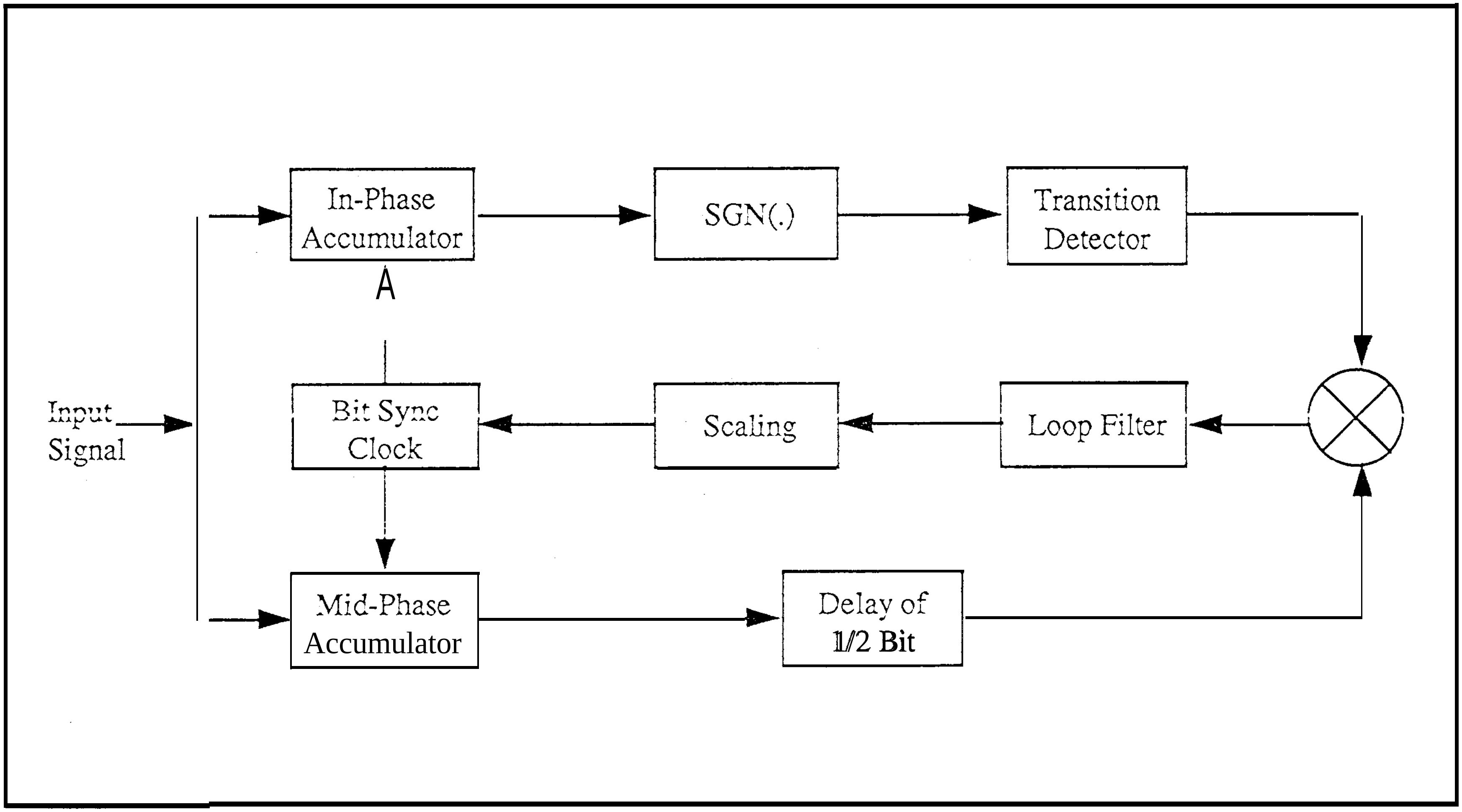


Figure 1 Block Diagram for the Digital Data Transition Tracking Loop (DTTL)

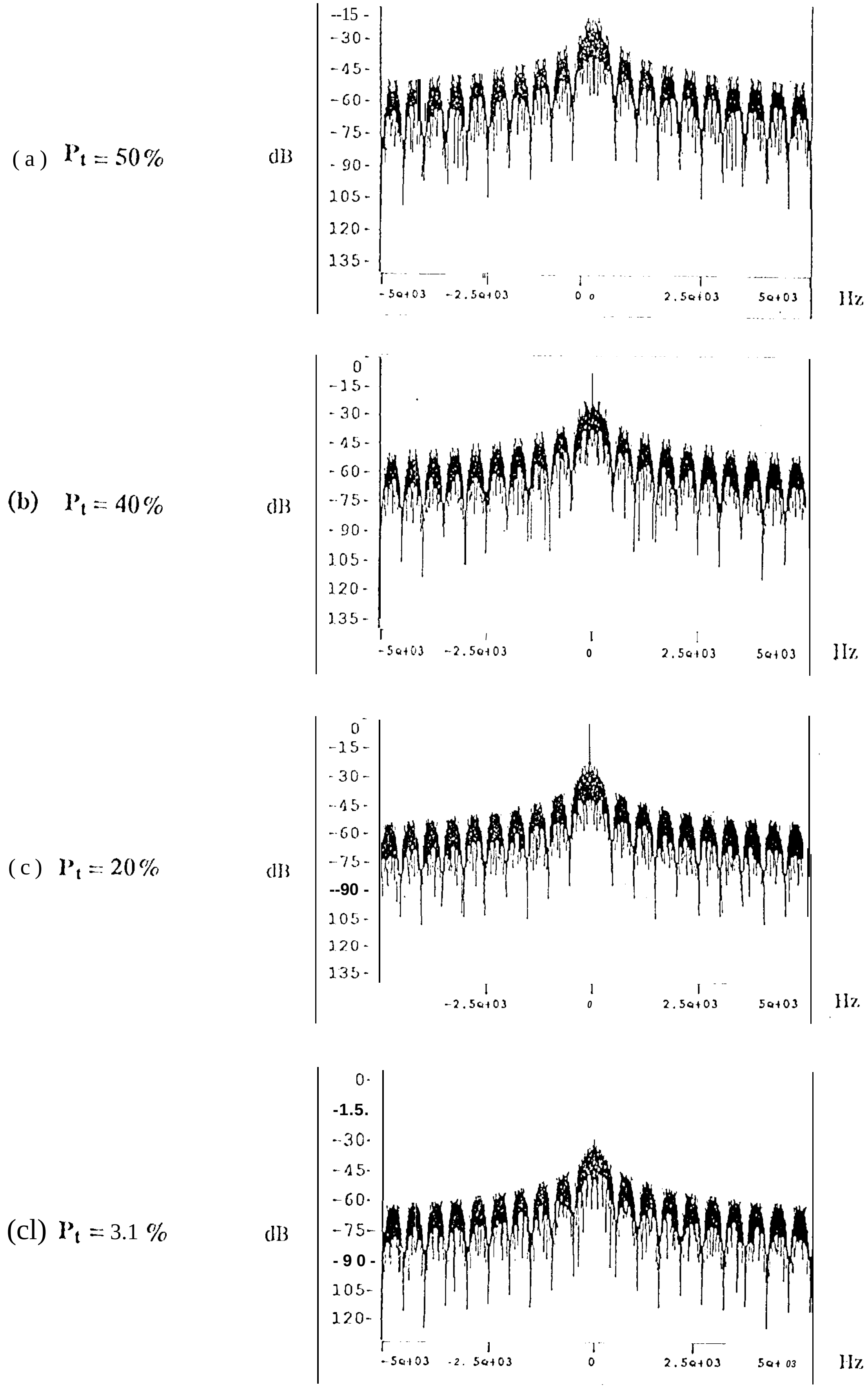


Figure 2 Data Spectrum for Various Transition Densities

Figure 3a g'(0) /g'(0) @Pt=50 % vs Symbol SNR

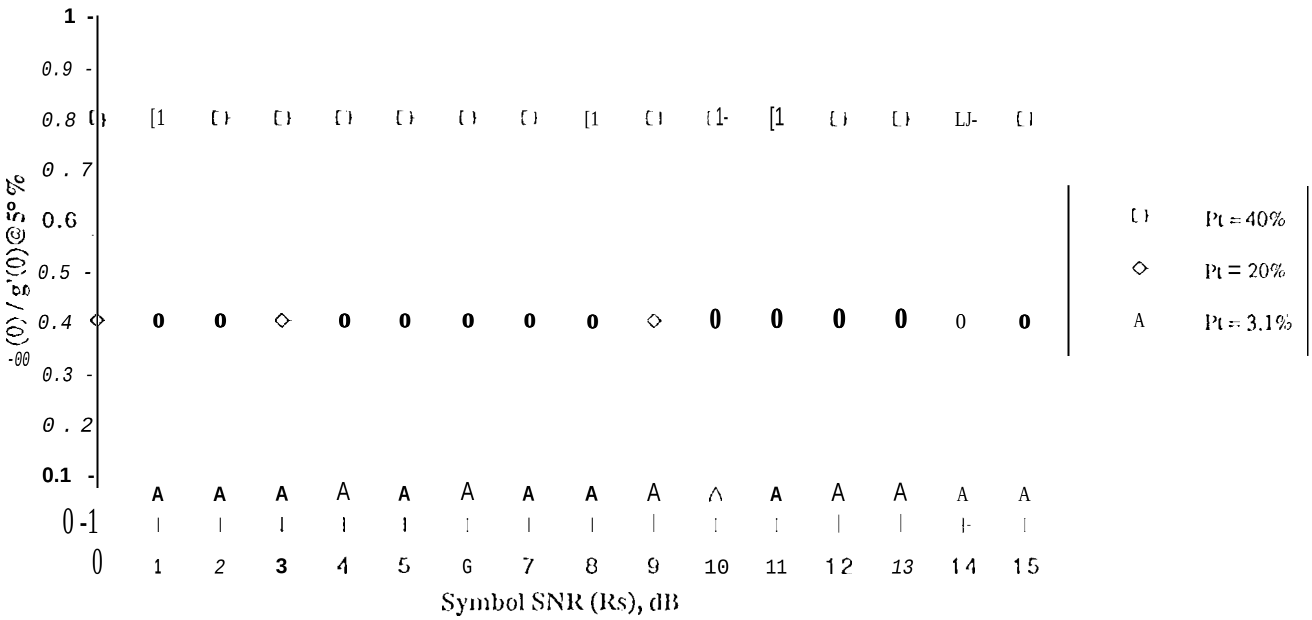
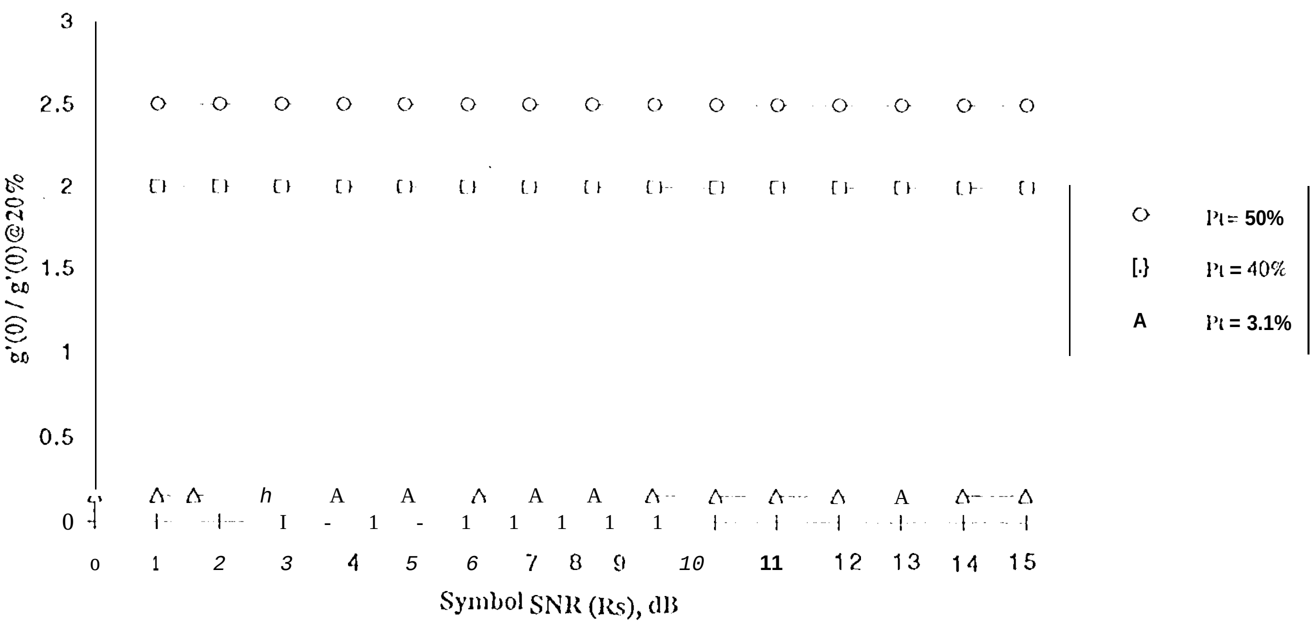


Figure 3b g'(0) / g'(0) @Pt=20 % vs Symbol SNR



A

Figure 4a h(0) / h(0) @Pt=50% vs Symbol SNR

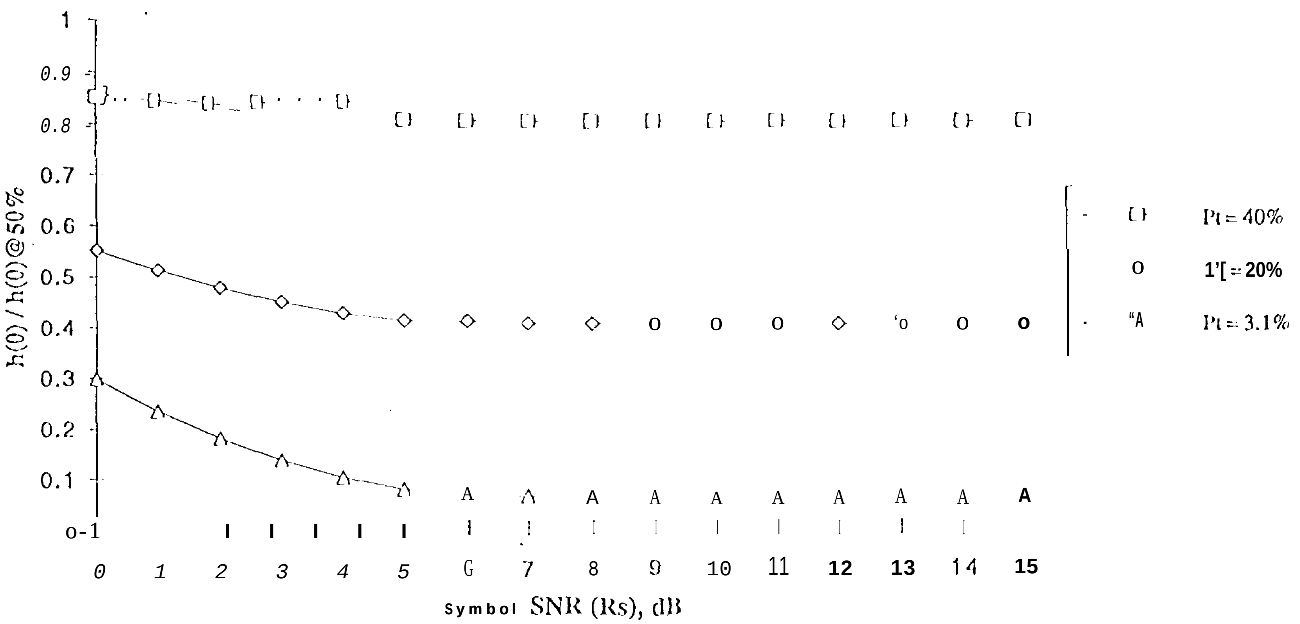


Figure 4b h(0) / h(0) @Pt=20% vs Symbol SNR

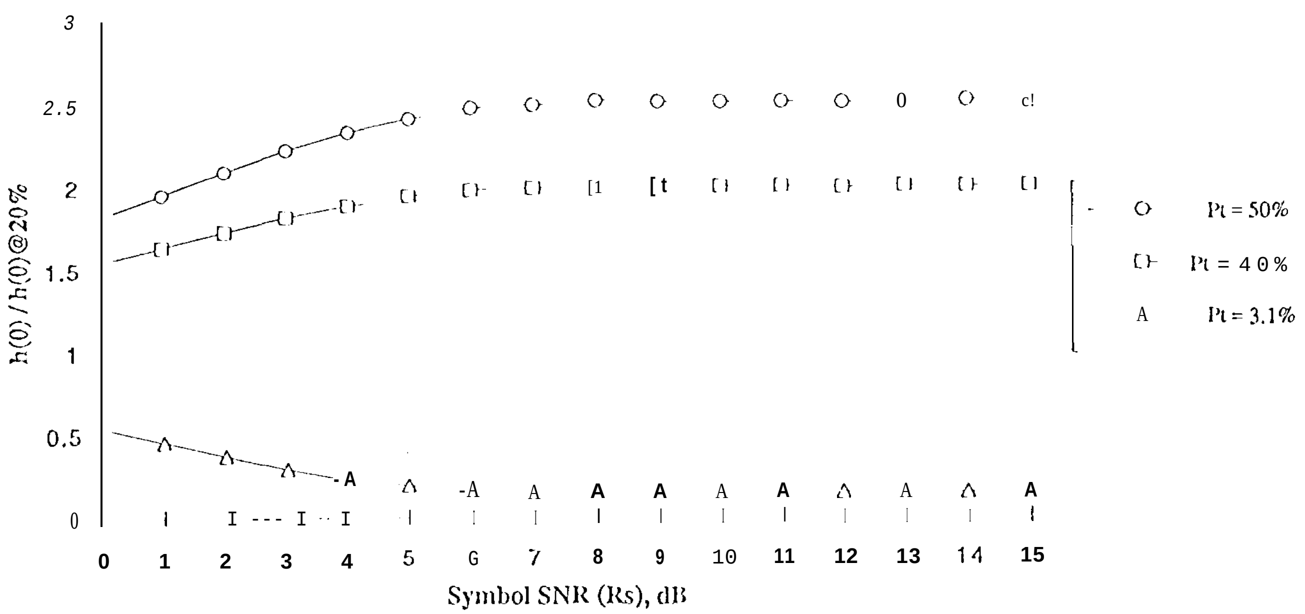


Figure 5a $\sigma_\lambda^2 / \sigma_\lambda^2$ @50% vs Symbol SNR for w=1

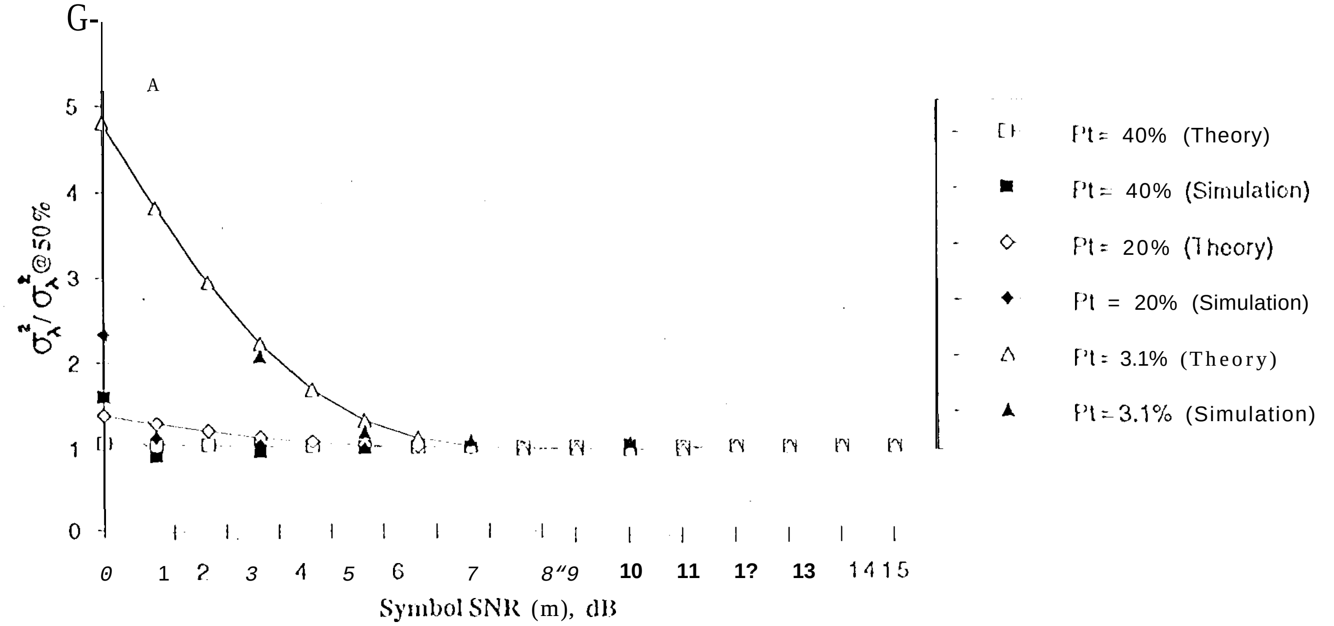


Figure 5b σ_λ^2 vs Symbol SNR for Pt=50% and w=1

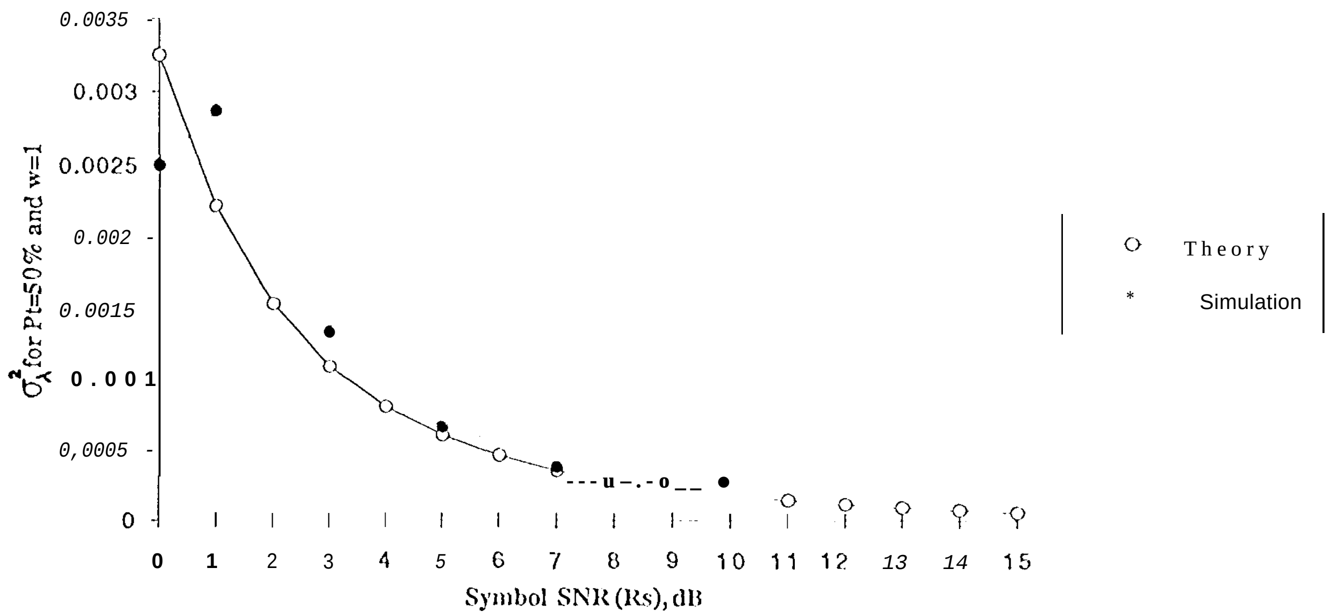


Figure 6a Simulation σ_λ^2 / σ_λ^2 @50% vs Symbol SNR for w=0,25

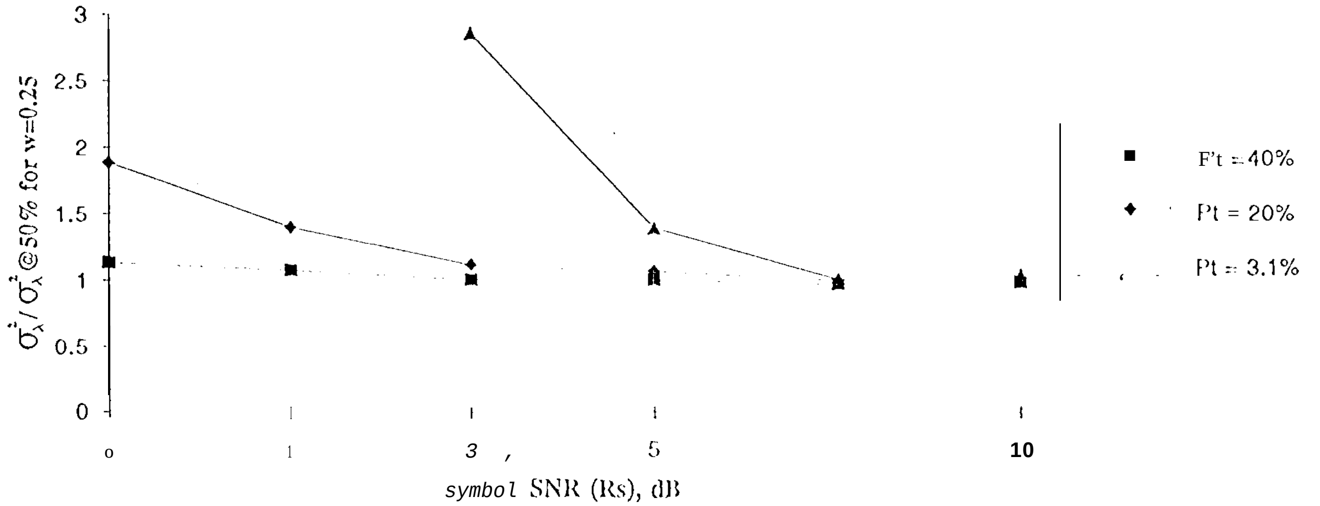


Figure 6b σ_λ^2 vs Symbol SNR for Pt=50% and w=0.25