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SATELLITE DYNAMICS ABOUT ASTEROIDS

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Abstract

Current space exploration goals for NASA include planned and proposed missions to asteroids. These missions generally call for a period of orbital operations in close proximity to the asteroid, in some cases coming within 2 radii of the asteroid, A challenge for the navigators of these missions is to predict the orbital environment about the asteroid and to derive pre-mission plans for the control of these orbits. This paper investigates the major perturbations asteroid orbiters will encounter over a range of relevant asteroid sizes.

The relevant perturbations acting on the asteroid orbiter are due to the irregular shape of the asteroid, solar radiation pressure and the solar tide. The effects of these perturbations on the satellite osculating elements are discussed in the context of the averaged Lagrange Planetary equations. These averaged equations provide accurate qualitative prediction of the expected satellite motion. Exact solution of these averaged equations are possible in some cases. Additionally, the dynamical environment about an asteroid can be characterized by its shape, size and rotation rate. This characterization identifies regions of orbital stability and instability about the asteroid.

The effects of the solar radiation pressure and of the asteroid shape are investigated in detail, as these dominate over the effect of the solar tide in situations of interest. The solar radiation pressure perturbations are significant for orbiters about small asteroids. The effect of the asteroid shape is significant for all orbiters within a few asteroid radii and dominates over the other effects for larger asteroids. For intermediate sized asteroids, both the solar radiation pressure and the asteroid shape must be taken into account.

The study carried out in this paper is significant as it addresses the non-Keplerian nature of satellite orbits about asteroids. For any potential orbiter mission to an asteroid, these effects must be seriously considered during pre-flight navigation and mission planning. This paper provides an analysis and methodology which allows for qualitatively correct pre-flight planning.

1 Introduction

The investigation of asteroids is of interest to the space science community for several reasons, the most compelling being the relatively unknown properties of asteroids and the insight this information would provide into solar system formation and history. Asteroid orbiters are the most efficient platform with which to investigate asteroids as they allow for a long period of observation and characterization and eventually enable an asteroid landing for direct sampling. The navigation methodology used in approaching, characterizing and orbiting an asteroid is detailed in Reference [10]. The current paper discusses the major perturbations which act on an asteroid orbiter and their effect on the orbit.

By necessity, the discussion in this paper is forced to cover a range of asteroid sizes and heliocentric. orbits, Thus the formulation of these effects is kept general, although specific examples are used throughout. The discussion is limited to the major perturbations acting on the asteroid orbiter. These perturbations may be evaluated given ground based observations of the asteroid. Such an analysis is useful as it describes the basic. motions orbiters will follow. Higher order perturbation theories will begin from these solutions once the asteroi d is encountered and characterized. The validity of these results are limited to less than 100 asteroid radii in general. For radii larger than this, the assumptions made in deriving the results should be verified for applicability.

2 Asteroid and Spacecraft Properties

For a specific mission, , the range of orbits for candidate asteroids is fixed by the mission parameters such as injection energy and mission duration. Still, candidate asteroids may have size differences of an order of magnitude and the shapes of each asteroid will be unique. Also important is the size and mass of the spacecraft, especially in the current age of smaller and lighter spacecraft designs,

2.1 Asteroid Size and Mass

Candidate asteroids for rendezvous and orbital missions have a large variation in size. Stated candidates for planned and potential NASA sponsored missions include Nereus, with a radius < 0.4 kin, Eros with an average radiusof 10 km and Vesta with an average radius of 245 km. Thus the radii of asteroids to consider range over 3 orders of magnitude.

The specific shape of an asteroid is easily specified as a tri-axial ellipsoid. Although the actual shape will deviate from this geometrical figure, it is convenient to use this parameterization in a prior-i studies when details of the asteroid shape are unknown, The tri-axial ellipsoid allows for the inclusion of the major components of the asteroid's non-spherical shape. Using this model the asteroid size and shape is specified by the three major half-dimensions of the body, commonly termed α, β and γ. These parameters are strictly ordered as $\alpha \geq \beta \geq \gamma$.

The density of an asteroid, ρ, is a parameter which cannot be measured until actual encounter with the body occurs and the estimation process begins. For our studies, we assume a density of 3.5 g/cc, although this will be uncertain to a large extent. For icy bodies, this density may be as low as 1 g/cc, for iron bodies this density maybe much larger. This is an important parameter, and given any specific. asteroid the literature should be investigated to see what its mean density is conjectured to be.

Given these parameters, the mass constant μ of the asteroid may be computed:

$$\mu = \frac{4\pi}{3} G\rho\alpha\beta\gamma \tag{1}$$

G is the universal constant of gravitation and has a value of $G = 6.67259\text{x } 10^{-8}\,\mathrm{cm}^3/(\mathrm{gs}^2)$. Another parameter of interest is the average asteroid radius, to be used in μ calculations. 'l'his is defined as the harmonic mean of the three major axes of the asteroid:

$$\bar{\alpha} = (\alpha\beta\gamma)^{1/3} \tag{2}$$

Given average asteroid radii ranging from 0.5 to 250 ktn, the mass parameter will vary over a range:

$$1.2 \text{ x } 10^{-7} \leq \mu \leq 15{,}3 \text{ km}^3/\text{s}^2 \tag{3}$$

Clearly, the relevant forces and their effect will vary over this range. In general, near-Earth asteroids have μ values ranging over $1 \text{ x } 10^{-7} \rightarrow 1 \text{ x } 10^{-3}$ kin^3/s^2. Main belt asteroids of interest have μ values ranging over $1 \text{ x } 10^{-3} \rightarrow 1 \text{ x } 10^{1} \text{km}^3/\text{s}^2$.

2.2 Asteroid Orbits

The heliocentric orbits of asteroids also vary. in general, asteroids of interest range from 1 to 3 AU. This range does not include all asteroids of interest, but should be sufficient for this discussion.

Given the asteroid semi-major axis, a quantity of interest is the rate of change of the asteroid true anomaly, defined as:

$$\dot{N} = \frac{\sqrt{\mu_S A(1 - E^2)}}{R^2} \tag{4}$$

Where μ_S = 1.3272 x $10^{11}\mathrm{km}^3/\mathrm{s}^2$ is the gravitational constant of the sun, A is the semi-major axis of the asteroid, E is the eccentricity of the asteroid and R is the distance of the asteroid from the sun. If the asteroid is in a nearly circular orbit, the expression simplifies to the rate of change of the mean anomaly of the asteroid:

$$\dot{N} = \sqrt{\frac{\mu_S}{R^3}} \tag{5}$$

Using the astronomical unit as a length standard, Equation 5 may be restated as:

$$\dot{N} = \frac{1}{R^{3/2}} \text{ degrees/day} \tag{6}$$

where R is now the semi-major axis of the asteroid expressed in astronomical units.

For asteroids in the ranges stated, this rate of change varies over:

$$1 \leq \dot{N} \leq 0.188 \text{ degrees/day} \tag{7}$$

2.3 Spacecraft Parameters

The spacecraft parameter of interest is the area to mass ratio for solar radiation pressure computations. Asteroid orbiters will in general be smaller and lighter than spacecraft designed for planetary orbit. For definiteness, we choose a particular value relating to a Discovery class spacecraft of effective area 5 m^2 and mass 160 kg. From this we define the area to mass ratio, B = Area / Mass, or $B = 0.03125\ \mathrm{m}^2/\mathrm{kg}$ for our particular example. This value of the parameter is used throughout the paper for definiteness of discussion.

3 Primary Forces

Other than the attraction of the asteroid gravity, there arc three primary forces acting on an asteroid orbiter: solar tide, solar radiation pressure and asteroid shape.

3.1 Solar Tide

The magnitude and direction of the solar tide force, in an inertial frame centered on the asteroid, can be given approximately as:

$$\mathbf{F}_T = -\dot{N}^2 r \left(1 - 3\cos^2 S\right) \mathbf{r}_s \tag{8}$$

$$\text{Cos} S = \mathbf{r}_s \cdot \mathbf{r}_g \tag{9}$$

where $\dot{N}$ is the true anomaly rate of the asteroid about the sun, r is the radius of the spacecraft from the asteroid center, $\mathbf{r}_s$ is the unit vector from the asteroid center pointing at the spacecraft, $\mathbf{r}_g$ is the unit vector pointing in the anti-solar direction and S is the angle between these two vectors.

Equation 8 is derived implicitly from the Hill equations of motion as stated in Reference [3]. Note that the distance of the asteroid from the sun and the solar mass is present implicitly through the parameter $\dot{N}$.

The equilibrium points associated with this force are at the position where the solar tide and gravitational attraction of the asteroid cancel. Relative to the asteroid their coordinates are:

$$\mathbf{r}_E = \pm \left(\frac{\mu}{3N^2} \right)^{1/3} \mathbf{r}_g \qquad (10)$$

where μ is the asteroid gravitational constant, For an asteroid with density 3.5 g/cc in an orbit at 1.5 All, these points lie at $\approx 300\bar{\alpha}$ from the asteroid. This is further from the asteroid than will in general be considered in this paper. In Reference [7] the effective radius for the sun to capture an initially circular orbit in less than 20 years is given as $450\bar{\alpha}$. Thus, the possibility of solar capture will not be a major consideration for short-term missions to asteroids.

3.2 Solar Radiation Pressure

The magnitude and direction of the solar radiation pressure acting on an asteroid orbiter is expressed in inertial space as:

$$\mathbf{F}_S = g\mathbf{r}_g \qquad (11)$$

$$g = \frac{G_1 B}{R^2} \qquad (12)$$

where G_1 is the solar constant $\approx$ 1 $\times 10^8$ kg $km^3/s^2/m^2$, B is the spacecraft area to mass ratio (defined previously) and R is the asteroid distance from the sun in kilometers. Using the astronomical unit as a measure of length and the nominal value of B = 0.03125, the parameter g can be expressed as:

$$g = 1.389 \times 10^{-10} \frac{1}{R^2} \text{ km/s} \qquad (13)$$

where R is now the asteroid heliocentric distance in astronomical units. This simple relation assumes that the solar radiation pressure force always acts in the anti-solar direction. A more sophisticated model would also include the orientation and the absorption and reflectance properties of the spacecraft.

Similar to the solar tide, the solar radiation pressure force will lead to an equilibrium point on the far side of the asteroid:

$$\mathbf{r}_E = \sqrt{\frac{\mu}{g}} \mathbf{r}_g \qquad (14)$$

Assuming that the asteroid is in a 1.5 AU orbit, this equilibrium point is located at $\approx 125\bar{\alpha}^{3/2}$ km away from the sun. Additionally, this force will modify the solar tide equilibrium point on the sun-ward side of the asteroid. For a 1 km radius asteroid, this point is pushed from 300 km to 1746 km towards the sun. For a 10 km asteroid, this point is pushed from 3000 km to 3700 km. Only for a 100 km asteroid does the effect of the solar radiation pressure become small, with the distance being pushed from 30,000 km to 30,600 km. Note that these numbers are dependent on the spacecraft parameter B, and that for a smaller area to mass ratio, the effect of the solar radiation pressure will decrease accordingly. This simple analysis presages the results discussed in Section 8 where it is seen that the solar tide only becomes significant for large asteroids.

3.3 Asteroid Shape

The force effects due to a non-spherical asteroid shape are quite complex. The most efficient manner in which to characterize them is through a spherical harmonic expansion of the asteroid gravitational field. Although such an approach is generally used in practice, it is not enlightening for pre-mission planning purposes when there is no definitive estimate for the gravitational harmonics. A better approach may be to use closed form expressions of known shapes which approximate the asteroid shape. Examples of these shapes include a tri-axial ellipsoid potential or a collection of point mass potentials rotating in unison (Reference [6]).

The effect of the asteroid shape upon an orbiter can be quite significant, as will be discussed later. The major effects may be discerned by noting the asteroid Type (defined in Section 7.2) and the major gravitational perturbations which can usually be characterized by using a second order gravitational field. in many instances, for characterizing the main effect of the asteroid shape, it is convenient to use the primary oblateness term, J_2.

4 Coordinate Systems

There are two coordinate systems of interest in the study of this problem, one referenced to the asteroid orbital plane, and the other referenced to the asteroid equatorial plane. These are referred to as the Orbital system and the Equatorial system, respectively.

4.1 Orbital System

This system is the preferred coordinate frame for discussing the solar effects: solar radiation pressure and solar tide.

Define the $\hat{x}$ axis along the anti-solar unit vector $\mathbf{r}_g$ at the initial epoch, as time progresses the $\mathbf{r}_g$ vector will rotate in this frame. The $\hat{z}$ axis is taken to be normal to the asteroid orbit plane and the y axis follows from the usual construction and lies perpendicular to the sun line and in the orbit plane, pointing in the direction of travel. Assume that the satellite is described at any time by its osculating orbit elements: a, c, i, w, Ω and T. The orbital element $\tilde{\omega}$ = w $\pm\Omega$ is also used where appropriate,

The node of the orbit, Ω, is measured in the x-y plane, increasing in the counter-clockwise direction from the $\hat{x}$ unit vector and is specified by the unit vector:

$$\mathbf{r}_\Omega = \cos\Omega\hat{x} + \sin\Omega\hat{y} \tag{15}$$

The unit vector normal to the orbit plane is specified as:

$$\mathbf{r}_h = \sin\Omega\sin i\hat{x} - \cos\Omega\sin i\hat{y} + \cos i\hat{z} \tag{16}$$

The vector transverse to both the node and normal vectors is defined as:

$$\begin{aligned}\mathbf{r}_T &= \mathbf{r}_h \times \mathbf{r}_\Omega \\ &= -\sin\Omega\cos i\hat{x} + \cos\Omega\cos i\hat{y} + \sin i\hat{z}\end{aligned} \tag{17}$$

Given these vectors, the periapsis unit vector is:

$$\mathbf{r}_p = \cos\omega\mathbf{r}_\Omega + \sin\omega\mathbf{r}_T \tag{18}$$

Introduce the notation $u = w + f$, where f is the true anomaly. Then the satellite is located by the unit vector:

$$\begin{aligned}\mathbf{r}_s &= \cos u\mathbf{r}_\Omega + \sin u\mathbf{r}_T \\ &= [\cos u\cos\Omega - \sin u\sin\Omega\cos i]\,x \\ &\quad + [\cos u\sin\Omega + \sin u\cos\Omega\cos i]\,y + \sin u\sin i\hat{z}\end{aligned} \tag{19}$$

Finally, define the unit vector along the down-track direction of the satellite orbit:

$$\begin{aligned}\mathbf{r}_b &= \mathbf{r}_h \times \mathbf{r}_s \\ &= -[\sin u\cos\Omega +- \cos u\sin\Omega\cos i]\hat{x} \\ &\quad - [\sin u\sin\Omega - \cos u\cos\Omega\cos i]\,y + \cos u\sin i\hat{z}\end{aligned} \tag{20}$$

This completes the specification of the satellite orbit. in this system the anti-solar vector $\mathbf{r}_g$ is specified as:

$$\mathbf{r}_g = \cos(N)\hat{x} + \sin(N)\hat{y} \tag{21}$$

Where N is the true anomaly of the asteroid orhit, and can found by integrating Equation 4.

4.2 Equatorial System

This coordinate system is preferred for describing the effect of the asteroid shape on the spacecraft orbit. The $\hat{z}_e$ axis is defined to lie along the mean asteroid rotation pole w. The $\hat{x}_e$ axis is defined to lie in the asteroid orbital plane, and the $\hat{y}_e$ axis is defined as usual.

$$\hat{x}_e = \frac{\hat{z} \times \hat{z}_e}{|\hat{z} \times \hat{z}_e|} \tag{22}$$

$$\hat{y}_e = \hat{z}_e \times \hat{x}_e \tag{23}$$

The spacecraft position in the equatorial coordinate system is defined with the previously stated vectors in Equations 15 - 20, with $\hat{x}, \hat{y}$, and $\hat{z}$ replaced with $\hat{x}_e, \hat{y}_e$, and $\hat{z}_e$ respectively.

The mean asteroid rotation pole w is located in the orbital coordinate system by its node Ω_w and inclination i_w. Given this, the equatorial coordinate system unit vectors are related to the orbital coordinate system unit vectors as:

$$\hat{x}_e = \cos\Omega_w \hat{x} + \sin\Omega_w \hat{y} \tag{24}$$

$$\hat{y}_e = -\sin\Omega_w \cos i_w \hat{x} + \cos\Omega_w \cos i_w \hat{y} + \sin i_w \hat{z} \tag{25}$$

$$\hat{z}_e = \sin\Omega_w \sin i_w \hat{x} - \cos\Omega_w \sin i_w \hat{y} + \cos i_w \hat{z} \tag{26}$$

5 Solar Tide

Previous investigators have derived the averaged Lagrange equations for this force. A complete derivation of this effect is given in Reference [4]. The effect of the solar tide on the osculating elements is of order $\dot{N}^2/n$. As will be seen, these effects only become significant for spacecraft at large distances from an asteroid.

6 Solar Radiation Pressure

This force is generally small for orbiters and is included in the higher order perturbations. Since asteroids are small, however-, this force grows in importance and is sometimes the dominant force. The main effect is studied by averaging over the satellite orbit, although the averaging removes some effects of interest. In this analysis we assume that the orbiter is not "blown away" by the radiation pressure, which could occur for large B values, very small asteroids or at large distances from the asteroid.

6.1 Center of Orbit Offset

One dynamical effect of the solar radiation pressure which averaging removes is the offset of the orbit plane from the center of the asteroid. This effect is largest when in a solar plane-of-sky orbit. A simple analysis shows the magnitude of this effect.

Assume that the satellite orbit is nominally circular and in the solar plane-of-sky. Then the solar radiation pressure will act normal to the orhit plane with a constant force and push the orbit plane so that the asteroid center no longer lies in this plane.

in inertial coordinates, the potential force for this offset can be expressed as:

$$U(\zeta) = -\frac{\mu}{\sqrt{a^2+\zeta^2}} + g\zeta \tag{27}$$

where ζ is the offset of the orbit center along the sun-line, a is the radius of the circular orbit, g is the solar radiation pressure force and μ is the gravitational constant of the asteroid. The rotation of the sun has been neglected for this simple analysis, The offset coordinate ζ then obeys a second order differential equation:

$$\ddot{\zeta} = \frac{-\mu\zeta}{(a^2+\zeta^2)^{3/2}} + g \tag{28}$$

Due to the small magnitude of the g term, the offset ζ will in genera] be small, and hence allows for the approximation:

$$\ddot{\zeta} = \frac{-\mu\zeta}{a^3} + g \tag{29}$$

This equation describes an oscillation about an equilibrium point,. The equilibrium point characterizes the average offset of the orbit from the asteroid center and is:

$$\zeta_o = \frac{ga^3}{\mu} \tag{30}$$

For an asteroid in a heliocentric circular orbit at a radius of 1.5 All, this becomes:

$$\zeta_o \approx 6.3 \times 10\left(\frac{a}{\bar{\alpha}}\right)^3 \text{ km} \tag{31}$$

in general, this offset distance can be used as a measure of when the averaged equations of motion begin to break down. Taking the spat.ccraft, to an orbit at 25 asteroid radii yields an offset of 1 km. For small asteroids, this effect can become quite large. For example, should the asteroid be only 1 km in radius, the orbit plane will no longer contain the asteroid! Should this offset become large with respect to the asteroid radius, the averaged equations should be reconsidered and compared against the. numerically integrated equations of motion to ascertain their validity.

6.2 Averaged Solar Radiation Pressure

The Lagrange planetary equations describe the variation of the osculating orbit elements of a body when under the influence of additional forces. The solar radiation pressure force may be formulated as a disturbing potential to the central attraction of the asteroid:

$$\mathcal{R} = -r\mathbf{F}_s \cdot \mathbf{r}_s \tag{32}$$

$$= -gr[\cos u\cos(\Omega - N) - \sin u \sin(\Omega - N)\cos i] \tag{33}$$

Define a new element $\lambda = \Omega - N$, This is the node of the orbit measured in the orbital coordinate system rotating with the sun line. This potential is averaged over the mean anomaly to yield the secular potential:

$$\mathcal{R}_s = -\frac{3}{2}gae[\cos\omega\cos\lambda - \sin\omega\sin\lambda\cos i] \tag{34}$$

Using this disturbing potential in the Lagrange equations leads to the equations for the secular effects (see Reference [8]):

$$\frac{d\lambda}{dt} = -\frac{C_g e}{\sqrt{1-e^2}}\sin\omega\sin(\lambda) - \dot{N} \tag{35}$$

$$\frac{di}{dt} = -\frac{C_g e}{\sqrt{1-e^2}} \cos\omega \sin i \sin(\lambda) \tag{36}$$

$$\frac{de}{dt} = -C_g \sqrt{1-e^2}\,[\sin\omega \cos(\lambda) + \cos\omega \cos i \sin(\lambda)] \tag{37}$$

$$\frac{d\omega}{dt} = -\frac{C_g\sqrt{1-e^2}}{e}[\cos w \cos(\lambda) - \sin w \cos i \sin(\lambda)] - \cos i\left(\frac{d\lambda}{dt} + \dot{N}\right) \tag{38}$$

$$\frac{da}{dt} = 0 \tag{39}$$

where the parameter C_g is defined as:

$$C_g = \frac{3g}{2}\sqrt{\frac{a}{\mu}} \tag{40}$$

and N is found by integrating Equation 4.

Note that Equations 35-38 are fully integrable, although the form of their solution cannot be reduced to simple expressions in most cases. The description of the integrable solution is worked out in Reference [8]. In this solution the averaged elements are reduced to a solution with two frequencies on a torus. The solution has degenerate frequencies, i.e. the two fundamental frequencies of the solution have the same period. Thus, all solutions to Equations 35 - 38 are periodic with the same period. The two oscillations do not move uniformly with respect to each other in time, however. Furthermore, as will be discussed later, the period of the solutions is independent of the initial conditions, and only a function of the asteroid heliocentric orbit, the solar radiation pressure force g and the semi-major axis of the asteroid orbiter a (which is conserved in these averaged equations).

It is important to note that these equations are averaged over one orbital period. Hence in the numerical solutions to the full equations of motion there are significant short-period oscillations. Should the amplitude of these oscillations grow large, the applicability of the averaged equations will break-down. This generally occurs when the radius of the orbit about the asteroid grows too large.

The dynamics of an asteroid orbiter under solar radiation pressure alone are now discussed. This discussion in divided into two cases, $\dot{N} = 0$ and $\dot{N} \neq 0$. in the former case, the integrable solution to the eccentricity may be simply stated. In the latter case, there are a number of particular solutions of practical interest.

6.2.1 Dynamics When $\dot{N} = 0$

This case is an idealization and assumes that the asteroid rate about the sun is very small. It is of use as it allows for a simple characterization of the solution for the eccentricity.

Two of the integrals of motion for Equations 35-38 in this case ($\dot{N} = 0$) are:

$$k = \sqrt{1-e^2}\,\sin(\lambda)\,\sin i \tag{41}$$

$$h = e\,[\cos\omega \cos\lambda - \sin w \sin\lambda \cos i] \tag{42}$$

Note that the following bounds and relations may be established:

$$|h| < 1 \tag{43}$$

$$|k| < 1 \tag{44}$$

$$|h| + |k| < 1 \tag{45}$$

$$[\sin w \cos\lambda + \cos w \sin\lambda \cos i]^2 = 1 - \frac{k^2}{1-e^2} - \frac{h^2}{e^2} \tag{46}$$

Using Equation 46, Equation 37 can be written in terms of h, k and e alone:

$$\dot{e} = \mp\frac{C_g}{e}\sqrt{(e^2 - e_l^2)(e_h^2 - e^2)} \tag{47}$$

where

$$e_l^2 = \frac{1}{2}\left[1 + h^2 - k^2\right] - \frac{1}{2}\sqrt{(1 - h^2)^2 + (1 - k^2)^2 - (1 + h^2k^2)} \tag{48}$$

$$e_h^2 = \frac{1}{2}\left[1 + h^2 - k^2\right] + \frac{1}{2}\sqrt{(1 - h^2)^2 + (1 - k^2)^2 - (1 + h^2k^2)} \tag{49}$$

Assuming that $e(0) = e_l$, the equation may be directly integrated to yield the simple solution:

$$e^2 = \frac{1}{2}\left[e_h^2 + e_l^2 - (e_h^2 - e_l^2)\cos(2C_g t)\right] \tag{50}$$

It is clear that $e_l \leq e \leq e_h$. Note that if both $h = 0$ and $k = 1$, then $e_l = e_h = 0$, and the orbit is frozen into a circular orbit in the solar plane-of-sky. Conversely, if $h = 1$ and $k = 0$, then $e_l = e_h = 1$ and the orbit is a degenerate ellipse. Finally, if $h = 0$ and $k \neq 0$, then $e_l = 0$ and $e_h = 1$ and the eccentricity will vary between 0 and 1.

The period of the eccentricity variation is $T_P = \pi/C_g$. Under the condition $\dot{N} \neq 0$, the solution for the eccentricity becomes coupled with the angles, and its period of motion doubles, although for $\dot{N} << 1$ its motion will retain a characteristic double hump and will approach e_h **twice** in every oscillation.

The solution for the angles i, ω and λ cannot be expressed in such a simple form. Bounds on i and λ may be found, however, using the integral k. Define the angle $\alpha_k = \arcsin k$, then these angles will be constrained to lie within the bounds:

$$i \in (\alpha_k, \pi - \alpha_k) \tag{51}$$

$$\lambda \in \begin{cases} (\alpha_k, \pi - \alpha_k) \\ \text{or} \\ (2\pi - \alpha_k, \pi + \alpha_k) \end{cases} \tag{52}$$

Note the interesting result for an orbiter with $k, h \neq 0$: the orbiter will not collide with the point mass ($e \neq 1$) and the angular momentum vector will be bounded away from becoming normal to the sun-line.

6.2.2 Dynamics When $\dot{N} \neq 0$

As noted previously, Equations 35-38 are still integrable. By proper choice of variables, the motion can be reduced to two oscillations, each with the same period. See Reference [8] for further details on this elegant solution.

When orbiting an asteroid, it is desired to keep the eccentricity from becoming large or varying widely, as this may lead to collisions with the asteroid or bring the orbit close to the asteroid where the higher order gravitational effects may perturb the orbit. For $\dot{N} = 0$ the eccentricity was bounded from above by $\sqrt{1 - k^2}$. In the full solution the eccentricity will in general vary over an interval of the same approximate size. This is of concern to the orbiter navigation team as it is an effect which must be controlled and corrected for. Thus it is desired to find any particular solutions to the orbiter equations which will guarantee that the eccentricity remains constant.

There are four such orbits where the eccentricity and the other osculating elements remain constant on average in the presence of the solar radiation pressure. These solutions are frozen orbits. Two of these orbits lie in the asteroid orbital plane, the other two lie in the solar plane-of-sky.

First, an important parameter to define is:

$$\Lambda = \frac{C_g}{\dot{N}} \tag{53}$$

Note that, in terms of the elementary constants of this problem, this parameter may be expressed as:

$$A = \frac{3G_1 B}{\sqrt{\mu_S A(1 - E^2)}}\sqrt{\frac{a}{\mu}} \tag{54}$$

All these parameters are constant for any given spacecraft, spacecraft orbit and asteroid orbit. Significantly, this is true even if the asteroid is in an eccentric orbit about the sun.

For an asteroid with an average radius of α in a heliocentric. circular orbit at 1.5 AU and an orbiter with semi-major axis a, the parameter may be expressed as:

$$\Lambda \approx 1.72\sqrt{\frac{a}{\alpha^3}} \tag{55}$$

'1'bus, for large orbits A becomes large and for large asteroids A becomes small.

The solar plane-of-sky solutions have their angles specified as $i =$ 7r/2 and sin $\omega \sin(\lambda) = -1$. This directly implies that $\omega = \pm\pi/2$ and $\lambda = \mp\pi/2$. Thus the orbit angular momentum vector either points towards or away from the sun and the periapsis lies along the z-axis below or above the asteroid, respectively. in other words, the periapsis is chosen so that the orbiter velocity points along the asteroid's orbital motion about the sun.

Substitute the above angles into Equations 35 - 38 to find that all the. equations for the osculating elements are zero except for λ:

$$\dot{\lambda} = \frac{C_g e}{\sqrt{1 - e^2}} - \dot{N} \tag{56}$$

The eccentricity which zeroes out this equation is:

$$e_1 = \frac{1}{\sqrt{1 + \Lambda^2}} \tag{57}$$

Note that this eccentricity is constant even if the asteroid is in an elliptic orbit about the SUU.

For larger asteroids, this eccentricity approaches 1, whereas for larger orbits about an asteroid, the eccentricity approaches O.

The asteroid orbital plane solutions have an inclination of either $i =$ O, π. If i = O, then choose w $+ \lambda = \pi$, if $i = \pi$, then choose w — λ = O where the quantity $\tilde{\omega} = \omega \pm \lambda$ is the orbit periapsis in the orbit al reference frame measured from the anti-solar vector. in other words, the orbit periapsis is chosen so that the orbiter velocity points against the asteroid orbital motion.

Substitute the above angles into Equations 35- 38 to find that all the equations for the osculating elements are zero expect for $\tilde{\omega}$:

$$\dot{\tilde{\omega}} = \frac{C_g\sqrt{1 - e^2}}{e} - \dot{N} \tag{58}$$

The eccentricity which zeroes out this equation is:

$$e_2 = \frac{\Lambda}{\sqrt{1 + \Lambda^2}} \tag{59}$$

'1'bus, for larger orbits, this eccentricity tends to 1, whereas for larger asteroids, this eccentricity tends to O.

For either solution to be of practical interest requires that the eccentricity not be too large. Note that $e_1 = e_2 = 1/\sqrt{2}$ when A = 1. From Equation 55, the size of the orbit necessary for this equality is:

$$a \approx \frac{\alpha^3}{3} \tag{60}$$

For orbits a larger semi-major axis, the solar plane-of-sky solution with e_1 is preferred (as it is more circular), whereas for orbits with a smaller semi-major axis, the orbital plane solution with e_2 is preferred.

6.2.3 Period of Oscillations

An item of practical interest is the period of the integrable motion, as this indicates the frequency of correction which must be applied to the orbit. It also indicates the severity of the effect if not in a frozen orbit: the longer the period, the longer until a correction must be made.

From Reference [8] the period of the integrable motion is:

$$T_P = \frac{2\pi}{\dot{N}} \frac{1}{\sqrt{1+\Lambda^2}} \tag{61}$$

For an orbit at altitude a about an asteroid of uniform radius α at 1.5 AU from the sun, the value of this period is:

$$T_P \approx \frac{668}{\sqrt{1+3a/\alpha^3}} \text{ days} \tag{62}$$

As indicated earlier, the general eccentricity variation has a characteristic '{double hump" where the eccentricity becomes large twice every period. Thus, the eccentricity may change from O to 0.5 over a quarter of this period. For an asteroid of radius 1 km and an orbit altitude of 20 km, this characteristic time is 21 days. For an asteroid of radius 10 km and an orbit altitude of 50 km, this characteristic time is 155 days. Thus, for a small asteroid, this is an important effect which must be controlled frequently, while for a larger asteroid, this effect must not be controlled as frequently.

7 Asteroid Shape Effects

The generic shape of an asteroid is non-spheroid in genera] and can usually be approximated by a tri-axial ellipsoid. The distorted shape of an asteroid can have severe consequences for the dynamics of a satellite orbit within a few radii of the body ant] will have significant consequences for the dynamics of a satellite within $\approx$ 10 radii of its body. In this section we give a brief overview of how an asteroid may be characterized in terms of its shape and what the major effects of its shape on the orbit dynamics are.

7.1 Asteroid Characterization

Assume, for simplicity, that the asteroid may be characterized as a constant density tri-axial ellipsoid. This is a general model, as by varying the three semi-major axes of the ellipsoid the asteroid shape can be specified as a sphere, pancake, cigar or any intermediate shape. Thus, the tri-axial ellipsoid model allows the analyst to approximate the major shape distortions of the body in terms of a simple model.

Given the three semi-major axes of the asteroid: α, β and γ, given in order of their size, the total mass of the asteroid may be computed from Equation 2. The dimensions α, β and γ of the asteroid may usually be measured, or bounded, from Earth-based observations. The mass is still an uncertain parameter, however, as the uncertainty in the density of the asteroid may often be quite large, although estimates on its nominal value do exist (2.6 g/cc for C-type asteroids and 3.5 g/cc for S-type asteroids, Reference [1 O]). Another parameter of importance is the rotation rate of the asteroid, ω_A, or the period of one rotation, $2\pi/\omega_A$. This parameter may also be measured using ground-based observations, Given these parameters, the generic. dynamics which an asteroid orbiter will encounter can be described. This allows for pre-mission planning based on a more realistic model of the body, although mission plans cannot be finalized until the actual body parameters are estimated upon rendezvous with the asteroid.

The dynamics fan asteroid orbiter can be discussed completely, with the tri-axial ellipsoid formulation, using three dimensionless parameters:

$$\delta = \frac{\mu}{\omega_A^2 \alpha^3} \tag{63}$$

$$\hat{\beta} = \frac{\beta}{\alpha} \quad (64)$$

$$\hat{\gamma} = \frac{\gamma}{\alpha} \quad (65)$$

Note that $\hat{\gamma} \leq \hat{\beta} \leq 1$ in general. The parameter δ is the ratio of the gravitational attraction at the long end of the ellipsoid to the centripetal acceleration at the long end, assuming the bod y's mass is concentrated at the ellipsoid center. The parameters $\hat{\beta}$ and $\hat{\gamma}$ are a measure of the ellipticity of the tri-axial ellipsoid,

The equations of motion for the orbiter about a tri-axial ellipsoid model of an asteroid are most easily stated in a coordinate frame rotating with the asteroid. Note that this assumes that the asteroid isin principal axis rotation. The expression of the potential of the tri-axial ellipsoid is in terms of elliptic integrals. See Reference [9] for a complete derivation of these equations and a description of their proper-ties.

Orbiter motion about an steroid can be characterized in two basic ways. The first way is an evaluation of near-synchronous motion about the asteroid. This investigates the satellite dynamics when in or near a 1:1 resonance with the asteroid rotation rate. The characteristics of motion in this regime can vary markedly depending on the asteroid parameters. The second way is an evaluation of motion when not in near-synchronous motion shout the asteroid. in these cases, the satellite dynamics can be understood by approximating the asteroid with an oblate spheroid. This case is of importance when designing the nominal orbital phases about an asteroid.

Another way in which to characterize the gravitational effects of the asteroid shape is hy specifying the 2nd order gravitational harmonics, $C_{20} = -J_2$ and C_{22}. The J_2 coefficient characterizes the oblateness of the asteroid shape and, assuming a tri-axial ellipsoid model, is computed as:

$$J_2 = \frac{1}{10\alpha^2}(\alpha^2 -1 \beta^2 -- 27') \quad (66)$$

The parameter is bounded in general as $J_2 \leq 0.2$, a more realistic upper bound would be to take $J_2 \leq 0.1$. The C_{22} coefficient characterizes the ellipticity of the asteroid equator and for a tri-axial ellipsoid is computed as:

$$C_{22} = \frac{1}{20\alpha^2}(\alpha^2 - \beta^2) \quad (67)$$

In general, $C_{22} \leq 0.05$.

7.2 Near-synchronous Orbits

The first characterization of an asteroid is the determination of the stability of synchronous orbits in the asteroid's equator. For larger bodies, such as the Earth or the major planets, where the body shape **is nearly** spheroid, there are 4 synchronous orbits in the equator of the body, two of these orbits are stable and two arc unstable. In the planetary case, motion near the stable synchronous orbits will oscillate about the orbit, whereas motion near the unstable synchronous orbits will tend to drift away in longitude only, the motion being stable in the radial direction in general (Reference [1]).

When considering orbits about an asteroid, these 4 synchronous orbits still exist, but their stability prop erties may have changed. in particular, if the asteroid shape is distorted enough, the 2 stable orbits become unstable. This signals an important effect on the general dynamics of near-synchronous orbits about the asteroid, as the instability in these two synchronous orbits acts in the radial direction and may cause an orbiting satellite to crash on the surface in short time spans. Given the non-dimensional parameters of au asteroid, the synchronous radius will be close to:

$$a_{\text{sync}} \approx \bar{\alpha}\delta^{1/3} \quad (68)$$

As described in Reference [9], the dynamics about asteroids which have 4 unstable synchronous orbits are different than the dynamics about asteroids which have 2 unst able and 2 stable orbits.

Given the three parameters $\delta, \hat{\gamma}$ and $\hat{\beta}$, it is possible to determine the asteroid's "Type". The designation used here is that an asteroid with two stable and two unstable synchronous orbits is of Type 1, while an asteroid with four unstable synchronous orbits is of Type II. 'l'he computation of an asteroid's type involves the evaluation of au algebraic inequality involving elliptic integrals. Scc Reference [9] for a detailed discussion of the evaluation of the asteroid type. Figure 1 is a plot giving sufficiency conditions for an asteroid to be of Type II. If the values for δ and $\hat{\beta}$ lie under the curve, then the asteroid is of Type II for all possible values of the smallest dimension $\hat{\gamma}$. If the values lie above this curve, then the asteroid with $\gamma = \beta$ is of Type 1. However, there may be values of $\hat{\gamma}$ which are still small enough for the asteroid to be of Type 11.

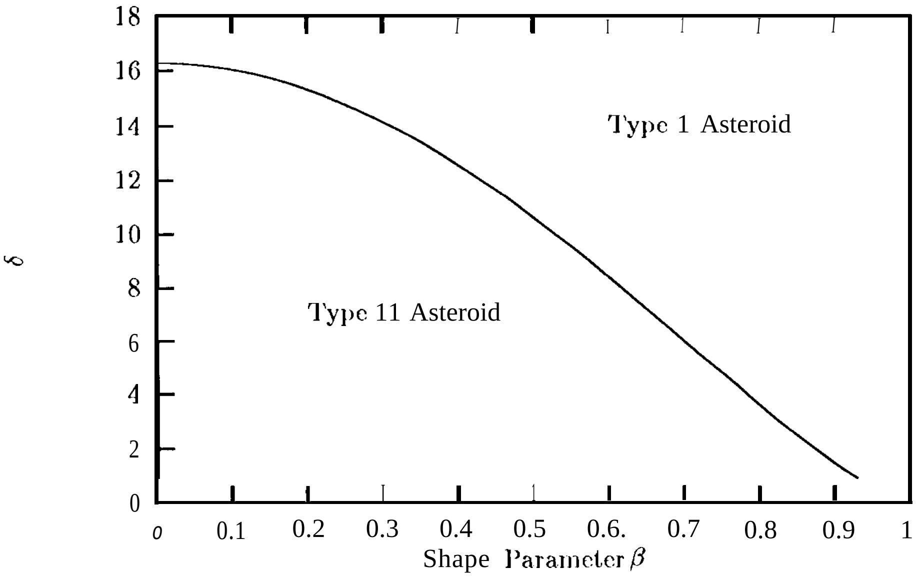

Figure 1: Sufficiency Conditions For Type II Asteroids

Given the asteroid type, the stability and safety of near-synchronous orbits may be evaluated. If the asteroid is definitely of Type 1, then synchronous orbits may be of scientific interest as they allow for long dwell periods over certain locations on the asteroid surface. Due to the perturbations of the actual gravity field, additional design and control must be performed to ensure the stability of the orbit. However, there will be no major instability effects which must be fought.

If the target asteroid is determined to be of Type 11, then it becomes imperative to avoid the strong instability associated with near-synchronous orbits. The strength of this instability is due to the nature of the two, new unstable synchronous orbits. Unlike the previous unstable synchronous orbits, which have real characteristic exponents and hence arc hyperbolically unstable, the new unstable orbits have complex characteristic exponents and hence the unstable manifold spirals away from the orbit in phase space. This spiral motion leads to a large variation in the orbit radius and quickly brings the orbiter to a close flyby or crashing trajectory. If the satellite orbits arc far away from the asteroid (usually further thau 3 or 4 radii), the orbit will not suffer the instability effects.

If the mission plan calls for the spacecraft to orbit within 3 or 4 radii of a Type II asteroid, which is in general close to the near-synchronous orbit radii, a strategy of orbiting the asteroid in retrograde orbits must be followed. Such orbits allow the spacecraft to achieve low altitude orbits without becoming synchronous with the asteroid rotation rate. Reference [9] presents an example of a family of stable periodic. orbits which are retrograde about a Type II asteroid. This family of orbits exist at arbitrarily low altitudes about the asteroid.

Note that the instability of near-synchronous orbits affect all direct orbits shout a Type 11 asteroid, and not just the equatorial orbits. ‘1’bus, near-po]ar orbits should also be treated with care. Although they may technically be “retrograde”, the instability effects may still affect them, especially if they are close to a 1:1 resonance with the asteroid rotation rate.

See l’able 1 for a brief list of characteristics of some asteroids of current interest,, Observe that most of these asteroids arc of Type II.

Name	α (km)	β (km)	γ (km)	$2\pi/\omega$ (hours)	$\hat{\beta}$ (-)	γ (-)	δ (-)	Type	J_2	C_{22}
Vesta	265	250	220	5.3	0.94	0.83	7.06	1	.051	.006
Ida	26.5	11.5	9.0	5	0,43	0.34	1.18	II	.095	.041
Eros	20	7	7	5.27	0.35	0.35	1.00	II	.088	.044
Gaspra	9.5	6	5.5	7	0.63	0.58	5.75	11	.072	.030

‘J’able 1: Parameters and Shape Characterizations for Some Asteroids of Interest

It should be cautioned that for either Type I or II asteroids, synchronous, near-circularpolar orbits arc unstable in general. ‘] ’his instability acts on the orbit eccentricity. For Type I asteroids, the region of instability tends to be small and easily avoided.

7.3 Asteroid Oblateness Effects

If the asteroid is of Type I or if a retrograde orbit strategy is taken about a Type II asteroid or if the orbit radius is far from the asteroid, then the main effect of the asteroid shape on the orbit becomes the oblateness effect. For Type I asteroids, this is because these asteroids tend to be less distorted in the equatorial plane and oblateness is the largest effect. For Type 11 asteroids, which may be quite distorted in the equatorial plane (have large C_{22} values), this is only true if the satellite is in a retrograde orbit about the asteroid or is far from the asteroid. For Type II asteroids, following such orbits effectively performs a spatial average over the asteroid equator, reducing the dynamical effect of the asteroid’s shape to an oblateness effect.

Numerical simulations of such orbits show that oblateness is a significant effect, and that retrograde orbits shout Type II asteroids can be reliably predicted using the theory of orbits about an oblate spheroid. Properly, to introduce such an approximation, the asteroid should be replaced with an oblate asteroid with the same mass and an average equatorial radius between the largest and smallest equatorial radii. Reference [2] investigates orbits about an oblate spheroid.

A further simplification replaces the effect of the oblate spheroid by the J_2 oblateness term in the asteroid gravitational potential. For motion in a potential with a J_2 term, the averaged Lagrange Planetary equations may be computed as (Reference [5]):

$$\frac{d\Omega}{dt} = -\frac{C_s}{(1-e^2)^2}\cos i \tag{69}$$

$$\frac{d\omega}{dt} = -\frac{C_s}{(1-e^2)}\left[\frac{5}{2}\sin^2 i - 2\right] \tag{70}$$

(71)

where i, c ancl a suffer no secular effects and

$$C_s = \frac{3nJ_2\alpha^2}{2a^2} \tag{72}$$

The osculating elements are referenced to the Equatorial coordinate system, as described previously. As is classically known, the secular effect of the oblateness acts primarily on the orbit node and periapsis. Although the orbit semi-major axis, eccentricity and inclination remain constant on average, there are short period variations in all of these elements.

Using a tri-axial ellipsoid model of the asteroid, the J_2 value is computed using Equation 66. As mentioned earlier, the maximum limit on J_2 is 0.2, although a more practical limit is to take $J_2 \leq 0.1$. The size of the J_2 term for an asteroid is in general quite large, as compared to this term for planets. See Table 1 for J_2 values for the asteroids listed there. Compare these to the J_2 value for Saturn, 0.016, which is the largest among all the planets.

Taking a maximum value of $J_2 = 0.1$, an asteroid with average radius $\tilde{\alpha}$ and a circular orbit with semi-major axis a, the secular rate of precession will be on the order of

$$C_s \approx 734 \left(\frac{\tilde{\alpha}}{a}\right)^{7/2} \text{ degrees/day} \tag{73}$$

Note that this is a maximum value, as the precession rate is multiplied by a reducing trigonometric factor. For a 10 km orbit about a 1 km asteroid this rate is 0.23 degrees/day, for a 50 km orbit about a 10 km asteroid this rate is 2.6 degrees/day, and for a 200 km orbit about a 100 km asteroid, this rate is 65 degrees/day. Thus, for orbiters at a few asteroid radii, the effect of the precession can become very large.

Due to the large dynamical effect of the asteroid oblateness, it must be accounted for in the pre-mission planning of the orbital phase, It also places restrictions on which orbits are feasible to maintain and control. As an example, under such large precession rates, it may not be feasible to keep au asteroid orbiter close to the Earth plane-of-sky, as the orbit will be moved out of this plane under the precession effects. To maintain such an orbit geometry may require maneuvers every few days. It is important to note that the precession of the orbit occurs about the mean rotation pole of the asteroid. The orientation of the mean rotation pole of the asteroid will in genera] point in an arbitrary direction for any given asteroid. It is essential that the rotation pole orientation be known prior to rendezvous. Lack of this information would complicate the pre-mission planning of the orbital phase.

8 Combined Effects

In this section, the relative strength of the three major force perturbations acting on an asteroid orbiter are compared. The strength of these forces are measured by their coefficients in the averaged Lagrange planetary equation. Comparison of their ratios indicates which forces dominate in which regions.

Define the following characteristic "frequencies" for the solar tide (C_t), solar radiation pressure (C_g) and asteroid shape (C_s):

$$C_t = \frac{\dot{N}^2}{n} \tag{74}$$

$$C_g = \frac{3gna^2}{2\mu} \tag{75}$$

$$C_s = \frac{3nJ_2\alpha^2}{2a^2} \tag{76}$$

These are the frequencies identified with the averaged Lagrange planetary equations for each of the major effects considered in this paper. Their size represents the magnitude of the effect they have on the osculating elements.

For comparison purposes, assume a circular asteroid orbit at 1.5 AU, the nominal value of the spacecraft parameter $B = 0.03125$ and $J_2 = 0.1$. Also, assume that the mass of the asteroid is computed using an average radius $\tilde{\alpha}$ and that the spacecraft is in a circular orbit with semi-major axis a.

Under these assumptions, the frequencies take on the characteristic values:

$$C_t = 5.94 \times 10^{-}\left(\frac{a}{\bar{\alpha}}\right)^{3/2} \text{ degrees/day} \tag{77}$$

$$C_g = 0.46 \frac{a}{\bar{\alpha}^3}\sqrt{\frac{a}{\bar{\alpha}^3}} \text{ degrees/day} \tag{78}$$

$$C_s = 734 \left(\frac{\bar{\alpha}}{a}\right)^{7/2} \text{ degrees/day} \tag{79}$$

The relative ratios of these parameters are then:

$$\frac{C_g}{C_s} \approx 6.3 \times 10^{-4} \frac{a^4}{\bar{\alpha}^5} \tag{80}$$

$$\frac{C_t}{C_g} \approx 1.3 \times 10^{-4} a \tag{81}$$

$$\frac{C_t}{C_s} \approx 8.1 \times 10^{-5} \left(\frac{a}{\bar{\alpha}}\right)^5 \tag{82}$$

The regimes of dominance of the different effects are found by identifying the point where the ratios are equal to 1, For ratios less than 1, the effect in the denominator dominates; for ratios greater than 1, the effect in the numerator dominates.

For the relative strength between the solar tide and solar radiation pressure, note that the solar radiation pressure formally dominates when:

$$r < 7700 \text{ km} \tag{83}$$

This is well within the orbit radii assumed for this paper. Further, at such a distance from an asteroid, the notion of an orbit becomes clouded due to the weakness of the gravitational attraction of the asteroid. Thus, within the assumptions made for the current analysis, it is clear that the solar tide effects are subservient to the solar radiation pressure forces. When orbiting a large planet, however, it is clear that the tidal forces may compete with the solar radiation pressure effects.

For the relative strength of the solar radiation pressure to be larger than the asteroid shape effect, the orbit semi-major axis must be:

$$a > 6.3\bar{\alpha}^{5/4} \tag{84}$$

For small asteroids (α <2 km) the solar radiation pressure effect is clearly dominant for all but the closest of orbits. Thus, the implementation of the frozen orbits about a small asteroid is feasible and may be the desired approach.

For intermediate sized asteroids ($2 < \alpha < 10$ km) the solar radiation pressure effect is of the same order as the asteroid shape effect for orbits in the range of interest. Thus, design of orbital missions should take this into account. The major effects which should be considered are that the frozen orbit design, which is feasible for the smaller asteroids, cannot be maintained without correction. The general effect of the asteroid shape will be to precess the angular momentum about the asteroid rotation pole and to cause secular change to the argument of periapsis. Thus, in order to take advantage of the constant eccentricity property of the frozen orbits, correction maneuvers must be performed periodically to reset the node and periapsis of the orbit. Conversely, if a frozen orbit design is not implemented, the eccentricity of the orbit will have a secular increase and must be corrected whenever it becomes too large. Note that, for a constant semi-major axis, as the eccentricity increases so do the secular rates of change of the orbit node and periapsis in the equatorial plane. Thus, there is additional impetus to control the eccentricity to smaller values.

For large asteroids ($10 < \alpha < 100$ km) the effect of the solar radiation pressure is clearly dominated by the J_2 effect. Thus, except for large orbits about the asteroid, the orbital phase strategy may be planned considering the asteroid shape effects only.

Note that for the solar tide effect to dominate over the asteroid shape effect requires an orbital radius greater than:

$$a > 26\bar{\alpha} \tag{85}$$

Thus, as expected, the magnitude of the solar tide effect is the smallest of the three, except when in very large orbits about the asteroid.

9 Conclusion

Based on the analysis in this paper, it is possible to characterize the major force effects an asteroid orbiter will encounter during its nominal mission phase. Of the three candidate perturbations to the asteroid's central attraction, only the solar radiation pressure and the asteroid shape are seen to be important. The solar tide has a relatively small effect as compared to these forces, and must only be considered once precision trajectory and orbit determination work is pa-formed. The solar radiation pressure is important when orbiting small and intermediate sized asteroids, or orbiting any asteroid at a large distance. It can be a significant effect as it drives the eccentricity towards larger values. The asteroid shape is important when orbiting any asteroid within a few radii, although its effect remains significant out to 10 asteroid radii. Furthermore, when orbiting an asteroid within a few radii, it is crucial to evaluate the stability of synchronous orbits. This can be done by computing the asteroid type. If it is of Type 1, then near-synchronous orbits may be flown if proper control and prediction is possible. If it is of Type 11, then near-synchronous orbits will be too unstable to realistically control, unless a robust on-board autonomous navigation and control strategy is implemented.

For orbiters at small asteroids, the solar radiation pressure will dominate, except when very close to the asteroid. Thus the shape and tidal effects may be treated as small perturbations and not accounted for in the primary mission plans. It may be feasible to implement frozen orbits designed to cancel the secular effects of the solar radiation pressure in this situation.

For orbiters at intermediate sized asteroids, both the solar radiation pressure and the shape effects will be of the same order of magnitude for orbit radii of interest. Frozen orbits designed for the solar radiation pressure will be affected by the precession of the orbit about the asteroid rotation pole. Generally, the asteroid rotation pole will lie in an orientation which does not allow the precession effects to be incorporated into the frozen orbit design. Additionally, the secular motion in the argument of perigee will move the frozen orbit out of its preferred orientation. Thus, it may not be feasible to implement the frozen orbit design. Coupling between the solar radiation pressure and the J_2 effect must also be modeled and corrected. Furthermore, if the asteroid is of Type 11, there will be restrictions on the orbit placement due to the instability of near-synchronous orbits.

For orbiters at large asteroids, the solar radiation pressure effect is small and the asteroid shape effects dominate the environment. If the asteroid is of Type 1, then there are no stringent restrictions on the placement of orbits about the asteroid. If the asteroid is of Type II, then orbits within 4 asteroid radii should be retrograde. For both these cases, the effect of the asteroid oblateness is large and will drive the design of the orbital phase of the mission. The presence of this effect renders inertially fixed orbits unfeasible except for circular orbits in the asteroid equator or circular orbits in polar orbits about the asteroid.

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