

**Preliminary TOPEX/POSEIDON
SLR and GPS Station Coordinate Calibration**

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TOPEX/POSEIDON (T/P) is a joint altimetric mission of U.S. NASA and French CNES design launched August 10, 1992. There are a variety of tracking systems on T/P for both operational and precise orbit determination, but the T/P tracking data selected for this study were satellite laser ranging (SLR) data and global positioning system (GPS) receiver data. Each of these tracking systems provides unique information for orbit determination of T/P relative to their respective coordinate reference frames. This paper presents the results of simultaneously processing SLR and GPS tracking data to produce a calibration of the differences between these two coordinate frames. The coordinate reference for SLR processing depends on the adopted station locations while the reference for GPS tracking depends on both the GPS space vehicle (SV) orbits and the locations of GPS ground receivers used for difference observable processing. This paper describes the scheme to determine the preliminary station coordinate calibration and gives the values for the GPS station locations in the SLR frame.

INTRODUCTION

As a preliminary calibration, a 20 day data arc was chosen covering cycles 19 and 20 during late March, early April 1993. A cycle for T/P is one ground track repeat period 127 revolutions long (about ten days long), and the cycles have been numbered sequentially since entering the operational orbit on September 23, 1992. This arc was chosen since there is good GPS and SLR coverage, there is no anti-spoofing to degrade GPS performance, and the GPS SV constellation is free of solar occultations. Since other processing

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results have indicated there are residual modeling errors for solar pressure on the GPS SV's, this last feature improves the SV orbit determination capability which in turn improves the T/P orbit estimates. The orbit accuracy is believed to be within 5 cm radial, rms, and within 15 cm three-dimensional position, rms, based on internal consistency and comparison with T/P project precision orbits.

Two techniques were used to obtain the GPS to SLR frame tie. Both begin with processing the tracking data over one day arcs. A square root information filter was used to produce individual factorized information matrices for the estimated station coordinates and frame tie parameters. The individual one-day information matrices were then combined to produce a global solution for the station coordinates and frame tie parameters.

The first technique uses a seven parameter similarity transformation utility to estimate the frame tie parameters from the global station estimates. In effect the measurements to this weighted least squares adjustment are the GPS station locations estimated in both the GPS and SLR frames. The second technique attempts to estimate the frame tie parameters based on the GPS and SLR observations directly. As is discussed more completely later, the GPS and SLR station coordinates are fixed at their estimated values and the seven parameters are adjusted minimize the observation residuals.

Four test cases used to analyze the GPS to SLR frame tie are summarized in Table 1. The influence of the a priori SLR station locations on the GPS station location determination is analyzed by comparing two orbit and station location solutions; **case one** using GPS data only in the usual fiducial processing mode, and case two using GPS and SLR data with SLR station locations fixed at their a priori values while estimating the same parameters as in case one. **Case three**, a variation on the above test, is to have GPS and SLR data processing as in case two, but estimate all GPS ground station locations (i.e., a fiducial-free approach). A seven parameter frame transformation may be determined by differencing the GPS ground station estimates from cases one and three. **Case four** directly estimates the seven parameter transformation while holding all GPS ground station locations fixed at their values estimated from case one, constraining the T/P and GPS SV orbits in the SLR frame as determined in case three, and using GPS data only.

Table 1. Test Case Description for Analysis of GPS to SLR Frame Tie

Case	Data Type	GPS Station Location Estimate*	7 Parameter Estimate
1	GPS only	3 fixed, 10 estimated	No
2	GPS+SLR	3 fixed, 10 estimated	No
3	GPS+SLR	13 estimated	No
4	GPS+SLR	13 fixed	Yes

* SLR station coordinates are fixed in cases **2,3,4**

SIMULTANEOUS GPS AND SLR OBSERVATIONS

For simultaneous processing of SLR and GPS tracking, tests were performed to select optimal relative data weights. The results of these tests are summarized in Table 2. Tracking from February 2, 1993 of T/P cycle 14 was chosen for the tests, and fits were performed on each data type individually, for completely overlapping 24 hour arcs, and for non-overlapping 12 hour arcs. The orbits from each of these combined fits were compared to reference orbits determined from only SLR or GPS tracking covering the same time period. The resulting rms and mean orbit differences over the 24 hours of the test fit in radial (R), transverse (T), and normal (N) directions are shown in Table 2 for each case. The relative weights for the GPS and SLR data were chosen to make orbit differences from the individual fits to the ten day SLR fit about the same.

The T/P GPS carrier phase data was weighted at two cm while the T/P GPS pseudorange was weighted at 2 m. The SLR data was weighted at about two cm (on average, since not all SLR tracking stations are equally weighted). The radial differences for the case with twelve hours of non-overlapping GPS and SLR data are plotted in figure 1. The first twelve hours of the fit contains only GPS data while the second twelve hours of the fit contains only SLR data. Note that the once-per-rev signature evident in the plot shows little variation across the fit, indicating the two data types have appropriate relative weights.

SIMILARITY TRANSFORMATION UTILITY

The seven parameter Helmert similarity transformation parameters are obtained by performing a least squares adjustment of a common set of station solutions with the following relationship:

$$G_i = X_i + T_i + R_i X_i$$

or

$$\begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} + \begin{bmatrix} D & -R_3 & R_2 \\ R_3 & D & -R_1 \\ -R_2 & R_1 & D \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

where: X_i = X,Y,Z = Input Coordinates
 G_i = X_S, Y_S, Z_S = Transformed coordinates
 T_i = T1,T2,T3 = Translations
 R_i = R1,R2,R3 = Rotations
 D = = Scale Parameter

The seven parameter utility program was used to perform a weighted least squares adjustments with a priori constraints as follows:

$$x = [H^T W H + P_0^{-1}]^{-1} H^T W y .$$

where: x = Seven Parameter Estimates (i.e., T1, T2, T3, R1, R2, R3, D)
 H = Observation **Partial**s Matrix ($\partial G / \partial X$)
 W = Weight Matrix
 γ = Observation Residuals Matrix
 P_0 = A priori **Covariance** Matrix

The station coordinate sets were weighted equally first and then with the global estimated uncertainties. A priori for the seven parameters was set to 10^9 cm for translations, 109 arcseconds for rotations, and 109 for the scale parameter.

RESULTS

CASE 1: The GPS processing uses implicit double differencing for calibrated phase and pseudorange tracking from T/P and a network of thirteen globally distributed GPS ground receivers. T/P and GPS SV orbits are simultaneously determined along with selected GPS ground station locations (i.e., a fiducial processing technique in which a subset of ground stations are held fixed, see e.g., Ref. 2). The sensitivity to GPS measurement system biases is determined by estimating a GPS phase center offset for T/P. In this study we used the best estimate available for the phase center offset, and held it fixed at that value.

CASE 2: This case is an extension of case 1 and is primarily intended to produce combined GPS and SLR tracking solutions to verify the relative data weighting selected. Since the SLR tracking data for T/P is sparse, the SLR station locations are not adjusted during the calibration but are held fixed at their **SSC(CSR)92L01** values (Ref. 1).

CASE 3: In order to obtain GPS station coordinates in the SLR frame, all GPS station locations were adjusted in the presence of the combined GPS and SLR observations. Using the combined 20 days of merged station coordinates solutions determined in cases 1 and 3 the seven frame tie parameters can be determined using the utility describe above.

CASE 4: To estimate the frame tie parameters with the GPS and SLR observations, the best fit T/P and GPS SV orbits are first determined as in case 3. Next, the laser observations are removed, the orbits and all other parameters are **held** fixed, and the GPS station locations are replaced with those determined in case 1. Allowing now for frame tie parameters to be estimated, the observation residual errors are forced to be absorbed in the seven parameter estimates.

ORBIT COMPARISONS: Smaller differences between the official T/P Precision Orbit Ephemeris (POE) and the combined GPS-SLR solution were expected and obtained as shown in Figures 2-6. The RMS Radial and three-dimensional differences for the various cases are presented in figures 2 and

3. Mean differences of the Earth fixed coordinates are also provided in figures 4-6.

FRAME TIE PARAMETERS: A moving 10 day merging of the information arrays gives an indication of the minimum number of arcs required to obtain with adequate uncertainties, the seven parameter frame tie **parameters**. Figures 7 and 8 show the translation and rotation estimates based on subsets of the 20 day global solutions. The 20 day global solutions are given in Table 3.

Table 3. GPS-SLR Global Seven Parameter Frame Tie Solutions

T1	T2	T3	R1	R2	R3	D	Notes
cm	cm	cm	.001"	.001"	.001"	10⁻⁸	
2.0	1.5	0.6	0.0	0.1	-1.7	-0.1	Case 1 vs 3 Equal Weights
2*2	2.0	0.7	0.2	0.0	-1.6	-0.2	Case 1 vs 3 Unequal Weights
0.6	2.7	0.6	0.0	0.1	-6	0.0	Case 4

FUTURE WORK

There are, of course, other measurement system factors *which influence* the estimates of a SLR-GPS frame tie, including the effect of T/P orbit error and arc length, geometric ties between GPS and SLR phase centers (both ground based and on T/P spacecraft), and the **level** of fiducial processing used for GPS. Using a converged GPS solution from the process described above, sensitivity to the GPS phase center offset can be tested by holding it at a fixed value when adding SLR data and comparing this result to that obtained by simultaneously estimating a SLR phase center offset. Testing the sensitivity/observability of the seven parameter transformation can also be performed by estimating a subset of the transformation (e. g., the three position offset parameters) and then comparing these to values obtained from the full seven parameter solution. The sensitivity of the combined estimate to our particular data set is determined by combining subsets of SRIF arrays from different arcs and comparing to individual arc solutions and the global solution. Each of these effects will be considered in future work.

CONCLUSIONS

The most notable conclusion of this work is that the frame tie parameters are very small. This conclusion is supported by the small orbit differences and the results from the two rather different techniques used to obtain the **seven** parameter frame ties.

ACKNOWLEDGEMENTS

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REFERENCES

1. IERS Technical Note #11, Central Bureau of IERS-Observatoire de Paris, June 1992, p 75.
2. **Guinn**, J. R. and P. J. Wolff, "TOPEX/Poseidon Operational Orbit Determination Results Using Global Positioning Satellites," Paper AAS 93-573 presented at the **AA S/AIAA Astrodynamics** Specialist Conference, Victoria, B. C., Canada, August 16-19, 1993.

TABLE 2.

TOPEX GPS/LASER Study
MIRAGE GPS & Laser Dynamic Orbits (DOY 033 - Cycle 14)

CASE	Obs Residuals (cm)		Orbit Differences (cm)						RMS / Mean					
	GPS	Laser	R	I	N	R	I	N						
1) GPS Only	0.55	--	--	--	--	4.1	.01	18.0	4.1	13.9	0.1			
2) Laser Only	--	2.1	4.	/.01	18.0	/-4.1	13.9	/-0.1	--	--	--			
3) GPS(24hr) + Laser (24hr)	0.62	/12.1	4.5	/-0.2	10.7	/-1.8	13.4	/0.1	5.4	/-0.2	15.7	/2.2	2.8	/-0.
4) GPS(12hr) + Laser (12hr)*	0.59	/12.1	4.5	/-0.2	10.8	/-1.8	13.7	/0.1	5.2	/0.2	21.6	/14.9	24.1	/0.3
5) Cases3 w/CM Estimation†	0.59	/12.1	4.5	/-0.2	10.7	/-1.8	13.4	/0.1	5.4	/-0.2	15.7	/2.2	2.8	/-0.

* Relative Weights: GPS = 2cm, Laser = ~2cm

† Frame Tie CM Estimates: X = 2.6cm, Y = 12.5 cm, Z = -10.3 cm

FIGURE 1.
GPS+LASER 12/ 2 (1cm GPS) v s SLR 10-day

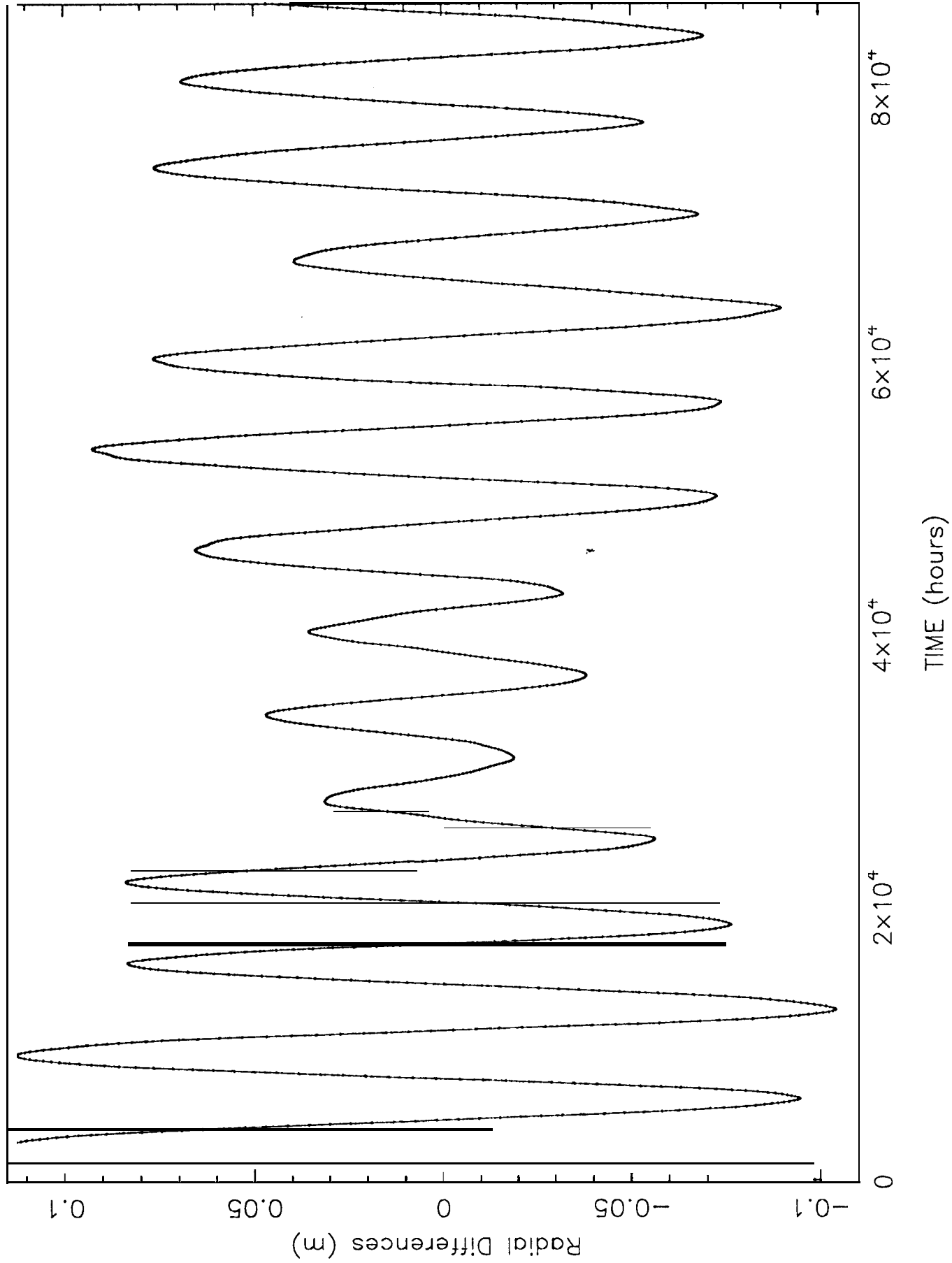


Figure 2 - Radial Orbit Comparisons
MIRAGE vs NASA POE

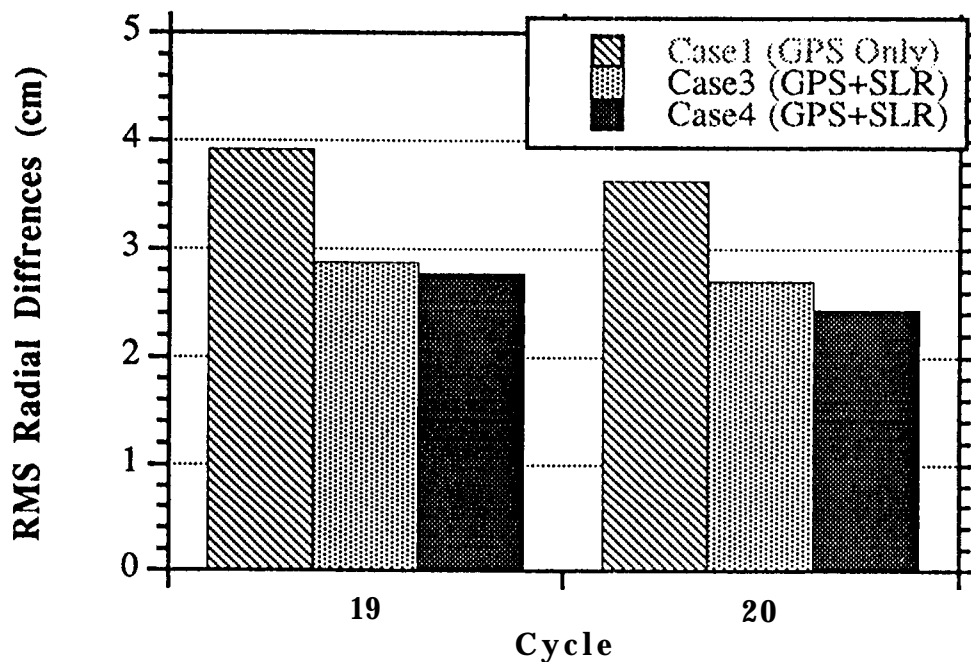


Figure 3 - 3D Orbit Comparisons
MIRAGE vs NASA POE

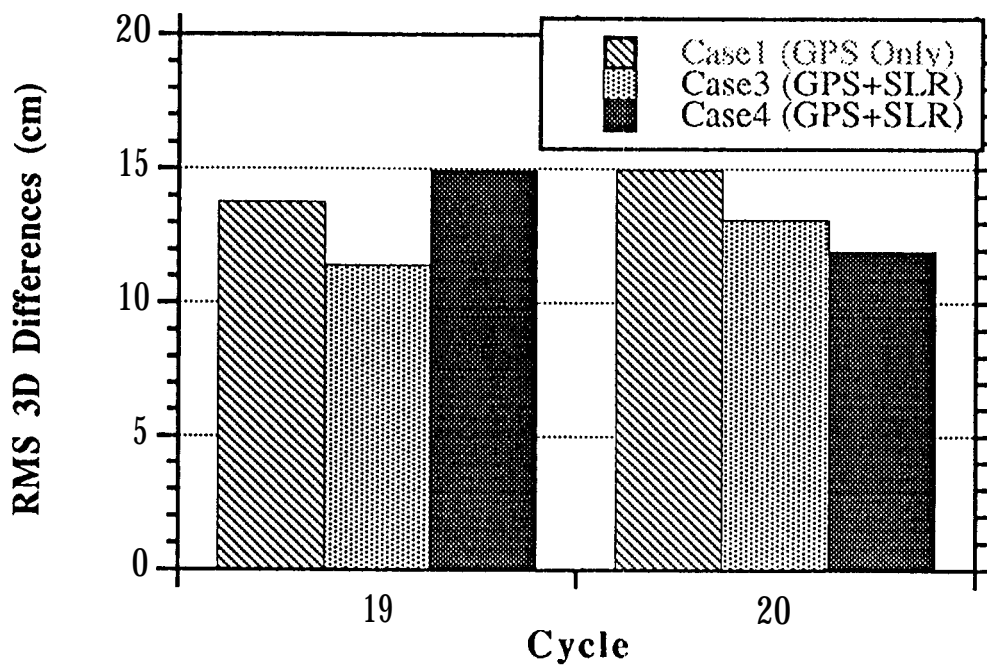


Figure 4- MEAN Orbit Comparisons (Earth-Fixed X)
MIRAGE vs NASA POE

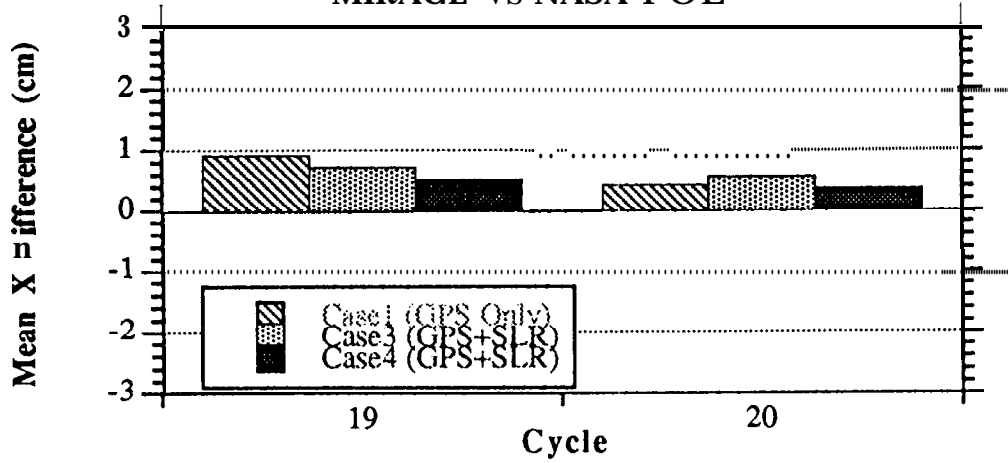


Figure 5- MEAN Orbit Comparisons (Earth-Fixed Y)
MIRAGE vs NASA POE

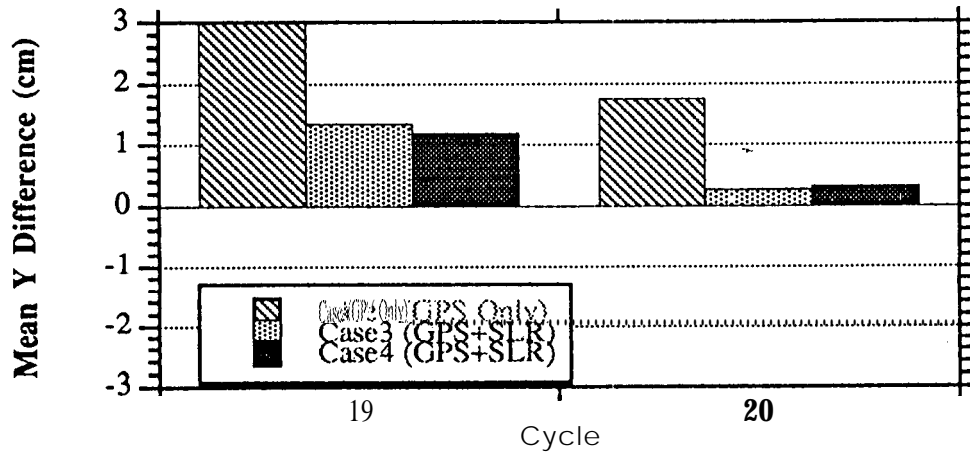


Figure 6- MEAN Orbit Comparisons (Earth-Fixed Z)
MIRAGE vs NASA POE

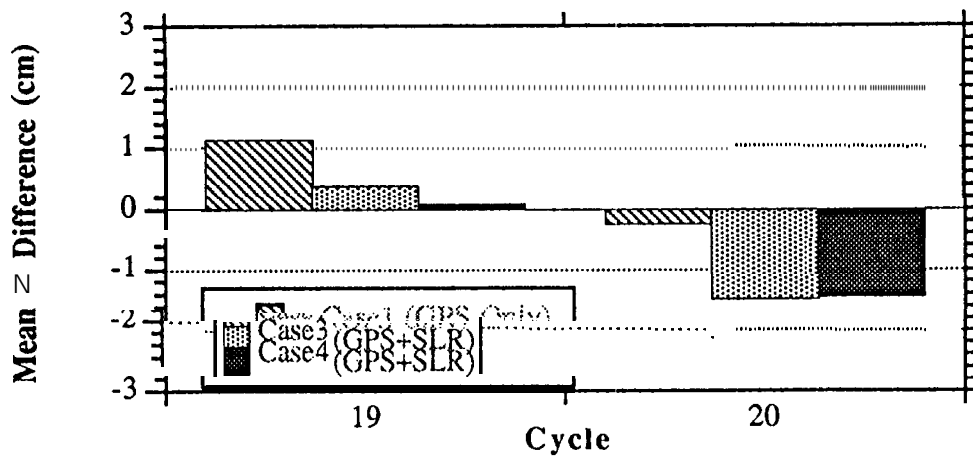


Figure 7- Frame Tie Translations

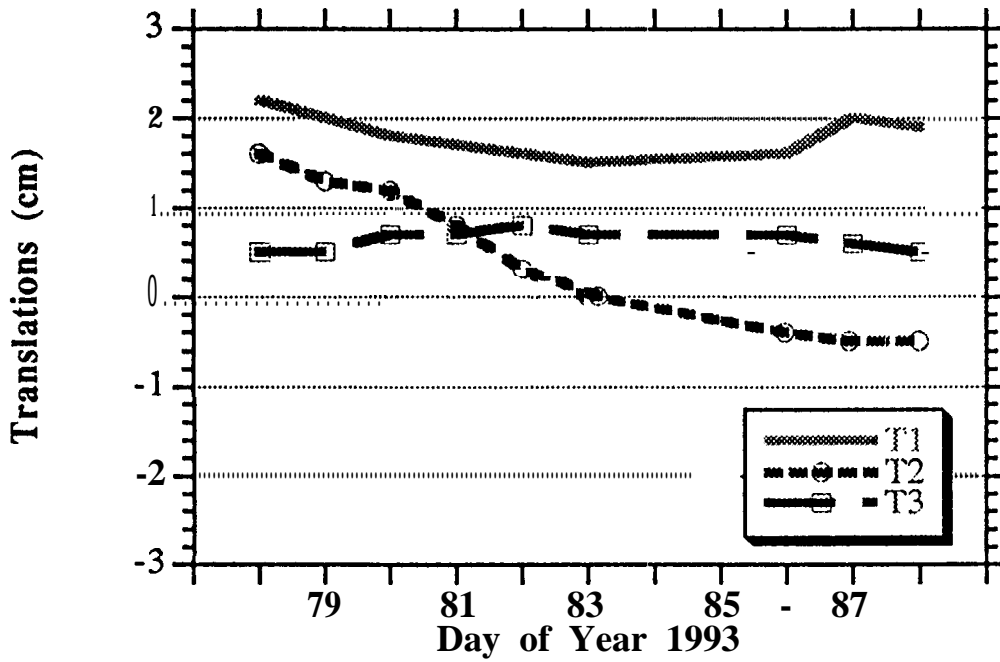


Figure 8 - Frame Tie Rotations

