# Global Velocities and Positions Determined From GPS Data Spanning Three Years

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***Abstract.*** Velocities and positions for 41 globally distributed sites have been determined from GPS data spanning three years. A no-fiducial approach has been developed to obtain these results without fixing any individual positions or velocities. Comparison with the ITRF92 velocity field, which was created from a combination of VLBI and SLR solutions, shows WRMS agreement at the level of 4.2 mm/yr, 5.7 mm/yr, and 7.4 mm/yr for geodetic latitude, longitude, and height rates respectively. Positions agree at the level of 6.8 mm, 10.9 mm, and 18.0 mm for latitude, longitude, and height.

## Introduction

Very Long Baseline Interferometry (VLBI) and Satellite Laser Ranging (SLR) have been used to make precise geodetic measurements of position and velocity for over a decade (Ryan et al. 1993 and Smith et al. 1991). The Global Positioning System (GPS) represents a relatively new space–geodetic technique. The first global experiment using GPS took place in 1991. Baseline precision of 2 mm + 4 parts per billion was achieved without the use of fiducial constraints (Heflin et al. 1992) and positions were obtained with centimeter level accuracy (Blewitt et al. 1992), Here we present velocities and positions for 41 sites derived from GPS data spanning three years.

Analysis was carried out in four distinct steps. First, all daily site positions were estimated using weak constraints of 10 m to 1 km for each site component. Details of the analysis strategy are described by Heflin et al. (1992). Days for which GPS P-code encryption was in effect were not used. The number of receiver sites and satellites has increased significantly since 1991. The analysis strategy has also evolved with time. The most significant improvements include estimation of polar motion and UT1 rate along with a stochastic treatment of the satellite forces due to solar radiation pressure (Vigue et al. 1993). Second, all daily solutions were combined using a standard least-squares constant-velocity model to obtain a single set of positions and velocities. Third, internal constraints were applied instead of fiducial constraints The basic idea of internal constraints is to fix several linear combinations of positions and velocities at their estimated values, instead of fixing some number of externally provided

positions and velocities. Finally, the estimated positions and velocities were transformed into the International Terrestrial Reference Frame for 1992 (1 TRF92). The no-fiducial approach used here is applicable to any global geodetic technique and is not specific to GPS. Various aspects of analysis are discussed more thoroughly below.

## Transformation

When determining the positions of a set of points, an observer must choose a frame of reference. A terrestrial reference frame has an origin, a unit of length, and orientations for the three measurement axes. These reference frame parameters may be defined arbitrarily, but comparisons must take place in a particular frame. The origin is usually chosen to be the Earth's center of mass. The standard unit of length is the meter which is indirectly defined by the speed of light and the definition of a second, Approximately speaking, the z-axis points in the direction of the north pole, the x-axis is perpendicular to the z–axis, pointing toward Greenwich, England, and the y-axis is orthogonal to the other two axes in a right-handed sense.

Transformation from one reference frame to another has traditionally been accomplished with a seven–parameter Helmert transformation. Let upper and lower–case letters denote coordinates in two different frames. The Helmert transformation is written as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix} + \begin{vmatrix} 1+S & -R_z & R_y \\ R_z & 1+S & -R_x \\ -R_y & RX & 1+S \end{vmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad (1)$$

where parameters RX, $R_y$, and $R_z$ are small angle rotations, TX, $T_y$, and $T_z$ are translations, and S represents a small change in the unit of length. Differentiating both sides gives the corresponding velocity equation.

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} \dot{T}_x \\ \dot{T}_y \\ \dot{T}_z \end{pmatrix} + \begin{pmatrix} 1+S & -R_z & R_y \\ R_z & 1+S & -R_x \\ -R_y & R_x & 1+S \end{pmatrix} \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} + \begin{pmatrix} \dot{S} & -\dot{R}_z & \dot{R}_y \\ \dot{R}_z & \dot{S} & -\dot{R}_x \\ -\dot{R}_y & \dot{R}_x & \dot{S} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad (2)$$

Each transformation parameter now has a rate. doubling the total number from seven to fourteen (Heflin et' al. 1993). A similar transformation without the scale rate parameter was independently derived by the International Earth Rotation Service (IERS) and used to create the ITRF92 velocity field (Boucher, Altamimi, and Duhem 1993).

In order to see the meaning of these parameters more clearly, consider two observers measuring the positions and velocities of a single set of points. There are three rotations because the observers' measurement axes may be facing in slightly different direction... The three rotation rates correct for the fact that the axes may be rotating slightly relative to one another, There are three translations because the observers' origins may be offset from one another. The three translation rates correct for any drift of the origins relative to one another. The scale parameter accounts for the fact that the observers maybe using slightly different rulers and the scale rate corrects for a small change in the relative lengths of the rulers. Once the transformation parameters are known, two observers can use the above equations to transform their measurements into a common reference frame for comparison.

The two observers can use their two sets of positions and velocities to estimate the transformation parameters, Let the lower-case letters represent measurements made by one observer and the upper-case letters represent measurements made by the other. When there are many sites, the Helmert transformation equation can be rearranged and written as

$$\begin{pmatrix} x_1 - X_1 \\ y_1 - Y_1 \\ z_1 - Z_1 \\ \vdots \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & X_1 & 0 & Z_1 & -Y_1 \\ 0 & 1 & 0 & Y_1 & -Z_1 & 0 & X_1 & \\ 0 & 0 & 1 & Z_1 & Y_1 & -X_1 & 0 & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{pmatrix} \begin{pmatrix} T_x \\ T_y \\ T_z \\ \\ R_x \\ R_y \\ R_z \end{pmatrix} \quad (3)$$

We can rewrite this equation as simply

$$AX = AX\,\theta_x \quad (4)$$

by defining AX as the vector of coordinate differences, AX as the matrix, and $\theta_t$ as the vector of transformation parameters. Differentiate both sides to get the corresponding velocity equation.

$$\dot{\Delta}_x = \dot{A}_x \theta_t + A_x \dot{\theta}_t \quad (5)$$

Now write both equation.. together.

$$\begin{pmatrix} \Delta_x \\ \dot{\Delta}_x \end{pmatrix} = \begin{pmatrix} A_x & 0 \\ \dot{A}_x & A_x \end{pmatrix} \begin{pmatrix} \theta_t \\ \dot{\theta}_t \end{pmatrix} \tag{6}$$

Simplify the notation once again by defining A as the vector of position and velocity differences, A as the full matrix, $\theta$ as the vector of all fourteen transformation parameters, and adding a measurement noise term.

$$\Delta = A\,\theta + v \tag{7}$$

Notice that the matrix A relating the transformation parameters to the coordinate differences is constructed from the positions and velocities of all sites. Matrix A will come up again during the discussion of internal constraints.

The transformation parameters are estimated using the weighted least--squares solution

$$\hat{\theta} = \left(A^T C_{\Delta}^{-1} A\right)^{-1} A^T C_{\Delta}^{-1} A \tag{8}$$

with covariance matrix

$$C_{\hat{\theta}} = \left(A^T C_{\Delta}^{-1} A\right)^{-1} \tag{9}$$

where

$$C_{\Delta} = C_x + C_X \tag{10}$$

is simply the sum of the covariance matrices from each observer.

## Internal Constraints

The idea of internal constraints is to constrain several linear combinations of positions and velocities at their estimated values instead of fixing some number of position and velocity components at externally provided values. Internal constraints offer two advantages. First, they allow the GPS estimates to remain independent of other techniques and second, they avoid the possibility of systematic effects due to errors in the externally provided information. The fourteen linear combinations being constrained were chosen to have a one–to–one correspondence with the fourteen reference frame parameters, so the effect of internal constraints is to fix the

reference frame parameters at their estimated values. The reader is referred to Vanicek and Krakiwsky (1986) for a complete mathematical derivation of internal constraints. Suffice it to say that we have generalized their formulation to include fourteen constraints and allow those constraints to be non–zero so that the final matrix is fully invertible. The resulting equations are given below.

Consider a set of positions and velocities represented by vector X with covariance matrix C. The matrix A discussed above can be computed from the elements of X and then used to obtain

$$B = \left(A^T A\right)^{-1} AT. \qquad (11)$$

Matrix B specifies which linear combinations will be constrained, Let the diagonal matrix $C_0$ contain non-zero constraints for each of the fourteen parameters, These constraints are applied using

$$C_{constrain} = C - C B^T \left(B C B^T + C_0\right)^{-1} B C . \qquad (12)$$

Although the above equation is valid when $C_0$ is zero, small but non–zero constraints are used so that the new covariance matrix is invertible.

$$C^{-1}_{constrain} = C^{-1} + B^T C_0^{-1} B \qquad (13)$$

The constrained covariance matrix gives reasonable errors for the position and velocity estimates.

## Results

In summary, daily estimates were computed, combined, internally constrained, and then transformed into ITRF92. The estimated translation and scale parameters obtained were

$$\begin{aligned} S &= -1.3 \pm 0.4 \text{ ppb}, \dot{S} = -0.3 \pm .2 \text{ ppb/yr} \\ TX &= -1.4 \pm 0.3 \text{ cm}, \dot{TX} = 2.3 \pm 0.2 \text{ cm/yr} \\ T_y &= 4.2 \pm 0.3 \text{ cm}, \dot{T}_y = -5.4 \pm 0.2 \text{ cm/yr} \\ T_z &= -1.6 \pm 0.2 \text{ cm}, \dot{T}_z = -4.0 \pm 0.1 \text{ cm/yr} \end{aligned} \qquad (14)$$

The unit of length as defined by GPS differs by about 1 ppb from that used for ITRF92 and the estimated scale rate is not significant, The GPS determination of the center of mass is offset from the ITRF92 value by several cm and is drifting by several cm/year. The GPS determination of the center of mass

is currently limited by the ability to model forces on the satellites such as those due to solar radiation pressure. The error bars above only reflect the contribution from ITRF92 because the internal constraints fix the GPS scale and origin at their estimated values.

The resulting global GPS velocity field is shown in Figure 1. Tectonic motion is clearly visible on the map. Spreading between the North American and Eurasian plates, slip between the Pacific and North American plates, convergence of the Pacific and Eurasian plates, and the northward drift of the Australian plate are clearly evident. More detailed interpretation by fitting a plate motion model is left as future work. The numerical estimates of position and velocity are given for July 1, 1992 in Table 1. Three sigma formal errors are used for all tables and figures. The sites GOLO, JPLO, and PINO, represent positions and velocities before the 92JUN28 Landers earthquake. The positions for BRMU, CANB, FORT, GOLO, GOLD, HOBA, KOUR, MADR, METS, and WETB refer to the top of the antenna. Positions for all other sites refer to the GPS monument location.

In addition to the qualitative agreement with known tectonic motion, a rigorous test is provided by comparison with the ITRF92 velocity field, Figure 2 shows comparisons of the GPS estimates for geodetic latitude, longitude, and height rate with those from ITRF92 which represent a combination of VLBI and SLR solutions. The WRMS differences are 4.2 mm/yr, 5.7 mm/yr, and 7.4 mm/yr for the latitude, longitude, and height rates respectively. A similar comparison for positions gives WRMS differences of 6.8 mm, 10.9 mm, and 18.0 mm. Of course, many sites are not collocated with VLBI or SLR, so their estimates represent new information which can be used to test models of plate motion and plate rigidity, as well as provide a global framework in which regional experiments can take place.

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Figure 1. Global GPS Velocity Field

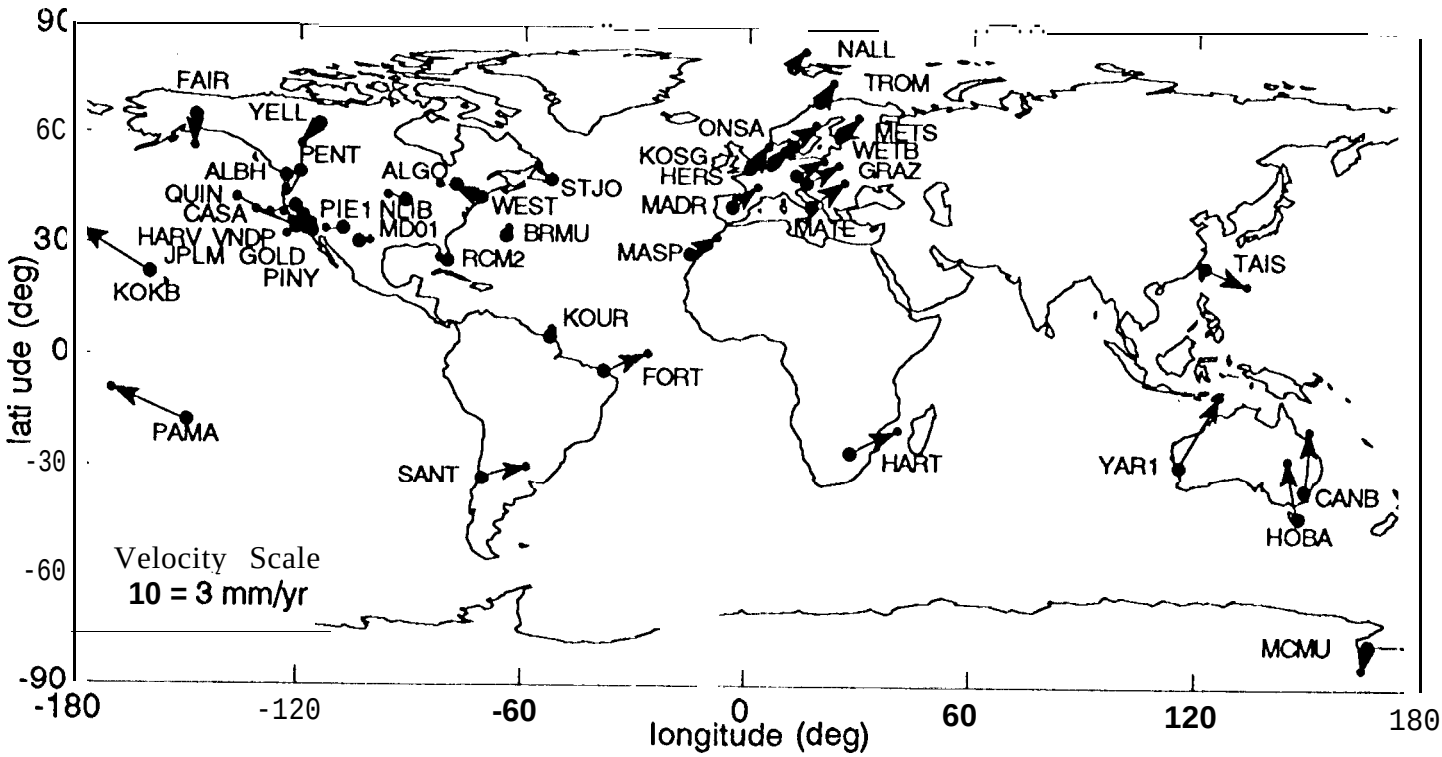
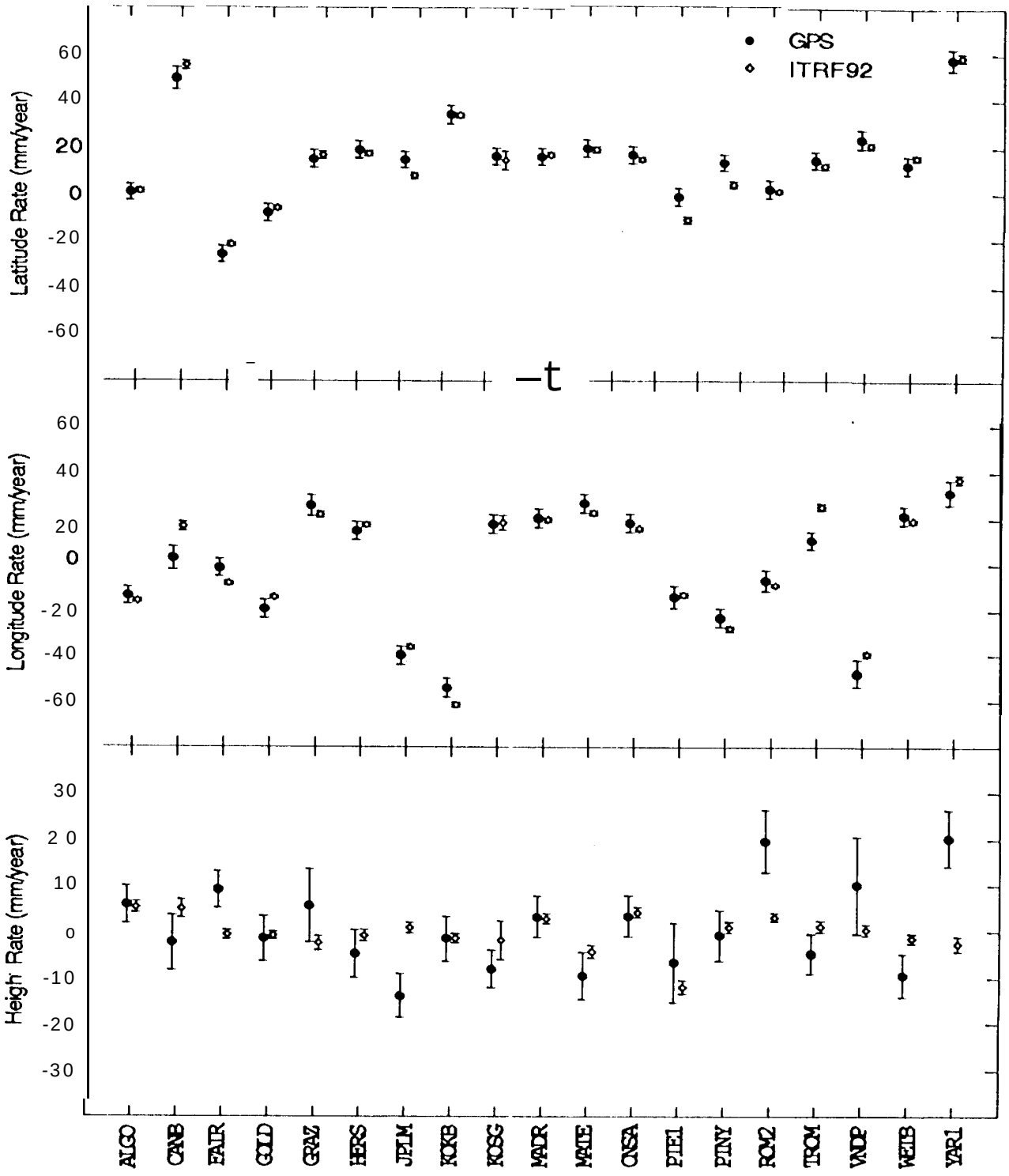


Figure 2, Comparison of GPS and ITRF92 Velocity Estimates




WRMS differences 4.2 mm/yr, 5.7 mm/yr, and 7.4 mm/yr for latitude, longitude, and height.

'T'able 1. GPS Positions and Velocities

| Name | x mm | Y mm | Z mm | Vx mm/yr | VY mm/yr | VZ mm/yr |
|---|---|---|---|---|---|---|
| ALBH | -2341332832* 2 | -3539049508* 2 | 4745791399* 2 | -7± 4 | -11± 4 | -6* 4 |
| ALGO | 918129591* 2 | -4346071222* 2 | 4561977802* 2 | -12* 4 | -7* 4 | 4* 4 |
| BRMU | 2304703701* 9 | -4874817264*I4 | 3395186916*1O | -4*1O | 14*14 | -2*1O |
| CANB | -4609961O1* 4 | 2682557159* 3 | -3674444003± 3 | -24± 6 | 11± 5 | 41* 5 |
| CASA | -2444430120* 8 | -442868764(1*13 | 3875747386*1O | -27* 9 | -31± 13 | 26± 11 |
| FAIR | -2281621325* 1 | -1453595777* 2 | 5756961958* 2 | -24* 4 | -14* 4 | -3± 4 |
| FORT | 4985387181*29 | -3954999002*27 | -42$426612* 7 | 68*27 | -9*26 | 10* 8 |
| GOLO | -2353614103* 3 | -4641385468* 5 | 3676976528* 4 | -16* 5 | 5* 6 | -7* 6 |
| GOLD | -235361411O* 2 | -4641385476* 2 | 3676976518* 2 | -19* 4 | 5* 4 | -7* 4 |
| GRAZ | 4194424051* 5 | 1162702475* 3 | 4647245270* 5 | -14* 6 | 23± 5 | 14* 6 |
| HART | 5084625459* 6 | 2670366538* 5 | -2768494025± 3 | -8± 8 | 39* 7 | 16* 5 |
| HARV | -2686069144* 4 | -4527084488± 5 | 3589502240* 4 | -35± 5 | 35* 6 | 19* 5 |
| HERS | 4033470300* 3 | 23672687* 2 | 4924301170* 3 | -18* 4 | 15* 4 | 8* 5 |
| HOBA | -3950183991*12 | 2522364466* 9 | -4311588435*12 | -22*12 | 25*1O | 35*12 |
| JPL0 | -2493304060* 3 | -4655215570* 5 | 3565497343* 3 | -34* 5 | 29* 6 | 3± 6 |
| JPLM | -2493304062* 2 | -4655215563* 3 | 3565497347* 2 | -26* 4 | 36* 4 | 4* 4 |
| KOKB | -5543838081± 3 | -2054587551* 3 | 2387809576* 2 | -6* 5 | 55± 4 | 31* 4 |
| KOSG | 3899225332* 2 | 396731747* 2 | 5015078299± 2 | -19* 4 | 16± 4 | 4± 4 |
| KOUR | 3839591575* 6 | -5059567676* 7 | 579956877* 2 | 6* 8 | -6* 8 | 7* 4 |
| MADR | 4849202566* 2 | -360329201* 2 | 4114913076* 2 | -6* 4 | 21* 4 | 14* 4 |
| MASP | 5439189171* 4 | -1522054857* 3 | 2953464202* 2 | -1* 5 | 23± 4 | 12± 4 |
| MATE | 4641949817* 3 | 1393045201* 2 | 4133287271* 2 | -27* 4 | 20* 4 | 9* 4 |
| MCMU | -1310695213± 3 | 310468877* 3 | -6213363449* 5 | 19* 5 | 1* 5 | -7* 7 |
| MDO1 | -1329998628*15 | -5328393405± 28 | 3236504221*17 | 14*14 | 22*26 | -14± 16 |
| METS | 2892571046* 2 | 1311843291* 2 | 5512634067* 2 | -20* 4 | 8* 4 | 4* 4 |
| NALL | 1202430727* 1 | 252626619* 1 | 6237767487* 3 | -12* 4 | 7* 4 | 13* 5 |
| NLIB | -130934391* 4 | -4762291736* 8 | 4226854664* 7 | -14* 6 | -4* 9 | 10* 8 |
| ONSA | 3370658748* 2 | 711876969* 2 | 5349786824* 2 | -16± 4 | 15* 4 | 12± 4 |
| PAMA | -5245195125* 5 | -3080472391± 5 | -1912825513* 3 | -41* 8 | 46± 7 | 22* 5 |
| PENT | -2059164600* 1 | -3621108398* 2 | 4814432419* 2 | -16* 4 | -5* 4 | -8± 4 |
| PIE1 | -164091671O* 4 | -5014781197* 6 | 3575447153* 4 | -12* 5 | 9* 7 | -4* 6 |
| PI NO | -2369510386* 3 | -4761207218* 5 | 3511396099* 3 | -22± 5 | 19 ± 6 | 4* 6 |
| PINY | -2369510359* 2 | -4761207212* 3 | 3511396144* 3 | -17* 4 | 18± 5 | 11* 4 |
| QUIN | -251 7230895* 6 | -4198595193* 9 | 4076531262* 8 | -24* 7 | -1* 10 | 5± 9 |
| RCM2 | 961318983* 3 | -5674090965± 5 | 2740489584* 3 | -4* 5 | -17* 6 | 11± 5 |
| SANT | 1769693271* 4 | -5044574138* 5 | -3468321 143* 4 | 24* 6 | 38* 7 | 30* 5 |
| STJO | 2612631332* 2 | -3426807014* 2 | 4686757756* 2 | -12* 4 | -2* 4 | 12* 4 |
| TAIS | -3024781857* 5 | 492$936895* 5 | 2681234518* 3 | -30* 6 | -16± 7 | -15± 5 |
| TROM | 210294O436* 2 | 721569344* 2 | 5958192076* 2 | -18* 4 | 6* 4 | 1± 4 |
| VNDP | -2678089764* 6 | -4525437807* 8 | 3597431477* 6 | -38± 6 | 29* 8 | 25* 7 |
| WEST | 1492233464* 5 | -4458089422* 8 | 4296045927* 7 | -17* 6 | -15* 9 | 20± 8 |
| WETB | 4075578713* 2 | 931852621* 2 | 4801570051* 2 | -20* 4 | 18* 4 | 1* 4 |
| YAR1 | -2389025328± 4 | 5043316830± 4 | -3078530950* 3 | -48* 5 | 28± 6 | 41* 5 |
| YELL | -1 224452389* 1 | -268921609]* 1 | 5633638285* 2 | -21* 3 | -11* 3 | o* 4 |