EVALUATION OF MODE SHAPE EXPANSION TECHNIQUES ON THE MICRO-PRECISION INTERFEROMETER TRUSS

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ABSTRACT
Several methods for mode shape expansion are investigated. In the first, the dynamic equations of motions are used to obtain direct solutions to the expanded eigenvectors. It is shown that these methods can be interpreted as constrained optimization problems. Previously developed methods using orthogonal projections can also be formulated through constrained optimization. To account for uncertainties in the measurements and in the prediction, new expansion techniques based on least squares minimization techniques with quadratic inequality constraints (LSQI) are proposed. These techniques are evaluated with the full set of experimental data obtained on the MicroPrecision Interferometer testbed, using both the pretest and updated analytical models. The robustness of these methods is verified with respect to measurement noise, model deficiency, number of measured dofs and accelerometer location. It is shown that the proposed LSQI method has the best performance and can reliably predict mode shapes, even in very adverse situations.

NOTATION

a - measured dofs - *aset* dofs
o - non-measured dofs - *oset* dofs
f - $a + o$ - full set of dofs
p - number of modes
$\tilde{}$ - notation used for actual test data
$\hat{}$ - notation used for expanded test data
- no superscript used for analytical model data
ω_i, $\tilde{\omega}_i$ - i^{th} analytical / test modal frequencies
ϕ_{ai}, $\tilde{\phi}_{ai}$ - $(a \times 1)$ i^{th} analytical / test eigenvector at measured dofs
ϕ_{oi}, $\hat{\phi}_{oi}$ - $(o \times 1)$ i^{th} analytical/test eigenvector at non-measured dofs
Φ_{fp}, $\hat{\Phi}_{fp}$ - $(f \times p)$ matrix of p analytical/test eigenvectors at full set
θ_{si} - strain energy for mode i in element s
K_{ff} - $(f \times f)$ full set stiffness matrix
M_{ff} - $(f \times f)$ full set mass matrix
b_{af} - $(a \times f)$ partitioning matrix to select measured dofs from the full set
A_{pp} - $(p \times p)$ unconstrained least-squares projection matrix
P_{pp} - $(p \times p)$ orthogonal Procrustes transformation matrix

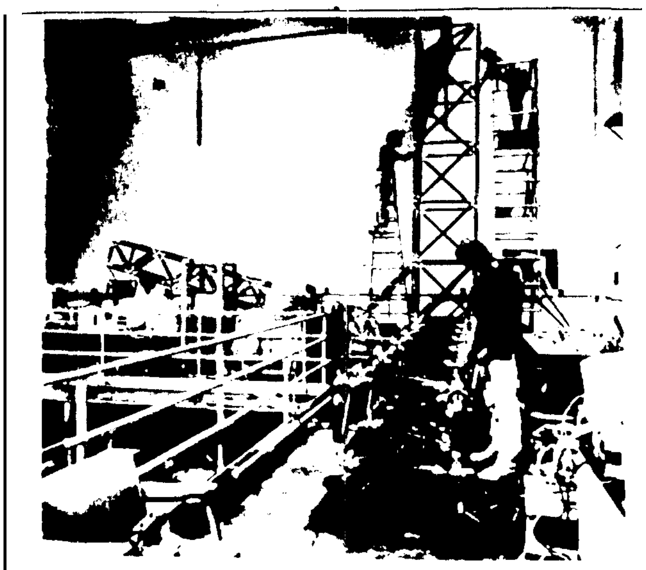
1.0 INTRODUCTION

Physical and financial constraints typically limit the number of degrees of freedom (dofs) monitored during a dynamic structural test. These limitations include laboratory or field restrictions, such as available number of accelerometers and/or data channels, structural constraints, such as inaccessibility of certain parts of the structure, or flight project constraints for on-orbit identification. However, it is often desired to assess the modal response of the full structure at all its dofs. The most common and least demanding reason is for mode shape visualization. Other reasons include correlation of test and analysis results at all the dofs represented in the full Finite Element Method (FEM) model of the structure. Model updating techniques would benefit from the added information provided by mode shape at all dofs. The full mode shape is also useful in predicting the response at unmeasured dofs for structural integrity and reliability assessments to dynamic loads such as earthquakes, impacts or explosions. Control needs include computation of the strain energy distributions for optimal damper and active member placement in vibration attenuation problems. In addition, the tuning of Multiple Input/ Multiple Output (MIMO) control parameters and gains also requires an accurate model at all dofs.

A study is conducted to evaluate the robustness and reliability of several mode shape expansion methods. This includes previously reported, as well as newly proposed methods, among which five methods are retained for comparison of mathematical end structural performance metrics. Sensitivity studies are performed, using actual experimental data. The studies involve taking a subset of the actual set of instrumented dofs, and verifying the accuracy of the expanded prediction. The methods are evaluated as to their sensitivity to combinations of measurement error, distributed and/or localized modelling errors. Sensitivity to modelling error is evaluated by using both the approximate pre-test finite element model and reconciled updated model. The performance of the modal expansion techniques is also estimated with respect to sensor location and quantity. It is shown that a new method based on least-squares minimization techniques with quadratic inequality constraints provides by far the most reliable mode shape estimates, aven in adverse situations.

2.0 MICRO-PRECISION INTERFEROMETER (MPI) TESTBED

The Micro- Precision Interferometer (MPI) testbed at the Jet Propulsion laboratory (JPL) is a lightly-damped truss-structure comprised of two booms and a vertical tower with dimensions of $7m \times 6.3m \times 5.5m$, and weighing 210 *kg* (Fig. 1). It is composed of 250 aluminum struts connected to 60 node balls. The careful design of the strut to node assembly ensures linearity in the response [6]. The primary objective of the MPI is to perform system integration of Control-Structure Interaction (CSI) technologies to demonstrate the end-to-end operation of a space-based optical interferometer [14] The high imaging resolution of future space missions will require a 15*nm* RMS control of the optical pathlength ovar the 7*m* baseline of the structure. Accurate modelling and response prediction are essential for the successful implementation of these control methodologies. Detailed modal testing and model updating were performed on the MPI and a high fidelity model was achieved for the first fifteen structural modes up to 60 Hz [6,7]. For the purpose of this analysis, only the first nine structural modes up to 50 Hz will be considered



gure 1 The MicroPrecision Interferometer Testbed.

The accuracy of the experimental procedures is substantiated by two independent sets of modal tests, carried out with distinct equipment, processors and personnel. The accuracy of the identified modal frequencies is of the order of 0.5% and the Modal Assurance Criteria (MAC, Eq. 18) between the two sets of mode shapes greater than 0.98 for most of the modes. However, the accuracy of the identified mode shapes is only of the order of 15% (Eq. 16). This infers that a high degree of uncertainty is associated with mode shape values, even with precise test procedures, excellent frequency repeatability, and better than average MACs.

Compared to the experimental data, the analytical pretest model has frequency errors of the order of 5%, and mode shape errors of the order of 25%, with the largest errors in the higher modes. The model was later improved by a combination of sub-component testing and full model Bayesian estimation [7]. The modal frequencies and mode shape errors was reduced to approximately 1 % and 10% respectively, and are within the accuracy expected from the experimental procedure.

Because of the rather large uncertainties in the mode shapes, it is unreasonable to assume that there is an exact closed-form expansion solution. New optimal methods are proposed which use constraint equations to that take mto account existing uncertainties in the measured mode shapes. In the process, it will be shown that most expansion methods can be mathematically formulated in terms of a constrained optimization problem.

3.0 DESCRIPTION OF MODE SHAPE EXPANSION METHODS

3.1. Guyan Static Expansion

This method is based on the assumption that the inertial forces acting on the non-measured dofs can be neglected with respect to the elastic forces [2]. This leads to an exact analytical relationship between the mode shapes at the measured and unmeasured dofs. Using the experimental mode shape data obtained at the instrumented dofs, $\bar{\phi}_{ai}$ the predicted mode shapes at the full set of dofs, $\hat{\phi}_{fi}$, can thus be inferred from:

$$\hat{\Phi}_{fi} = b_{ff}^T \begin{pmatrix} \bar{\Phi}_{ai} \\ -K_{oo}^{-1} K_{oa} \bar{\Phi}_{ai} \end{pmatrix} \tag{1}$$

where b_{ff} is the partitioning matrix between measured and unmeasured dofs

$$b_{ff} = \begin{pmatrix} b_{af} \\ b_{of} \end{pmatrix} \tag{2}$$

An alternate and equivalent formulation results in solving the following constrained minimization problem,

$$\min_{\hat{\Phi}_{fi}} \left[\frac{1}{2} \hat{\Phi}_{fi}^T K_{ff} \hat{\Phi}_{fi} \right] \quad subject\ to\ \hat{\Phi}_{ai} = \bar{\Phi}_{ai} \tag{3}$$

Eq. 3 can be interpreted as finding the expanded mode shape $\hat{\phi}_{fi}$ which minimizes the total strain energy of mode i, such that the predicted mode shape equals the test values at the measured dofs. The solution to Eq. 3 can be shown to be:

$$\hat{\Phi}_{fi} = K_{ff}^{-1} b_{af}^T (b_{af} K_{ff}^{-1} b_{af}^T)^{-1} \bar{\Phi}_{ai} \tag{4}$$

It is demonstrated that Eq. 1 and Eq4 yield equivalent solutions [15]. However, Eq. 1 only requires one matrix inversion, where as Eq. 4 requires two. Eq. 1 is thus preferred for its computational efficiency.

3.2. Kidder Dynamic Expansion

This method was proposed by Kidder [3], and later used by Berman [5] to update structural models. The inertial forces are no longer assumed to be negligible, leading to an exact solution of the mode shapes at the unmeasured dofs, $\hat{\phi}_{oi}$ as a function of the test modal frequency, $\bar{\omega}_i$, and the test mode shapes at the measured dofs, $\bar{\phi}_{ai}$.

$$\hat{\Phi}_{fi} = b_{ff}^T \begin{pmatrix} \bar{\Phi}_{ai} \\ [K_{oo} - \bar{\omega}_i^2 M_{oo}]^{-1} [K_{oa} - \bar{\omega}_i^2 M_{oa}] \bar{\Phi}_{ai} \end{pmatrix} \tag{5}$$

Again, the Kidder dynamic reduction can be reformulated in terms of a constrained optimization problem expressed as:

$$\min_{\hat{\Phi}_{fi}} \left[\frac{1}{2} \hat{\Phi}_{fi}^T K_{ff} \hat{\Phi}_{fi} - \frac{1}{2} \bar{\omega}_i^2 \hat{\Phi}_{fi}^T M_{ff} \hat{\Phi}_{fi} \right] \quad subject\ to\ \hat{\Phi}_{ai} = \bar{\Phi}_{ai} \tag{6}$$

Physically, Eq. 6 finds the expanded mode shape $\hat{\phi}_{fi}$ which minimizes the difference between the strain energy and kinetic energy for mode i, such that the predicted mode shape equals the test values at the measured dofs. Equivalently, Eq. 6 is the minimization of the loss of modal energy resulting from damping. The solution to Eq. 6 can be shown to be:

$$\hat{\Phi}_{fi} = (K_{ff} - \bar{\omega}_i^2 M_{ff})^{-1} b_{af}^T \left[b_{af} (K_{ff} - \bar{\omega}_i^2 M_{ff})^{-1} b_{af}^T \right]^{-1} \bar{\Phi}_{ai} \tag{7}$$

Again the direct solution in Eq. 5 can be demonstrated to be equivalent to the solution in Eq. 7. However, Eq. 5 is preferred since it only involves one inverse and avoids the ill-conditioning problem associated with inverting near-singular matrices for $\bar{\omega}_i = \omega_i$.

The "Kidder Dynamic Expansion' method described herein, is not to be confused with the "Dynamic Expansion Method" proposed by O'Callaghan [10]. The latter adds a dynamic force correction term to the Guyan expanded result (Eq. 1), expressed in terms of both the full FEM model and the statically reduced FEM model for the observed dofs, m_a and k_a :

$$\hat{\Phi}_{fi} = b_{ff}^{T} \begin{pmatrix} \tilde{\Phi}_{ai} \\ K_{oo}^{-1} K_{oa} + K_{oo}^{-1}\left[M_{oa} - M_{oo} K_{oo}^{-1} K_{oa}\right] m_a^{-1} k_a \tilde{\Phi}_{ai} \end{pmatrix} \quad (8)$$

The dynamic expansion method has been shown to produce reasonable results on actual test cases [1 1]. However, it is based on a series of equation manipulation which cannot be directly traced back to an actual physical interpretation. Hence, it cannot be viewed in the perspective of a minimization algorithm, and will not be considered herein. The same remark applies to the "Hybrid Expansion" method proposed by Kammer [131.

3.3 LeastSquares Projection

3.3.a. Unconstrained least. Squares Expansion Mathod

Both the Guyan and Kidder expansion methods require an a-priori knowledge of the FEM mass and stiffness matrices to predict mode shapes at unmeasured dofs. Kammer [B], and later O'Callaghan [9] and Lallement [121, have proposed methods which only require a-priori knowledge of the analytical mode shapes. These methods identify the least-squares projection A_{pp} that minimizes the Frobenius norm or quadratic error between the experimental and analytical mode shapes at the measured dofs. The method is also known as the "Modal Expansion" method. It is expressed as the unconstrained minimization problem:

$$\min_{A_{pp}} \left\| \tilde{\Phi}_{ap} - \Phi_{ap} A_{pp} \right\|_2 \quad (9)$$

Eq. 9 has a unique solution only if the number of measured dofs exceeds the number of modes, and Φ_{ap} has full column rank p. Under those conditions, the projection matrix A_{pp} is obtained from the least squares solution of the Moore-Penrose pseudo-inverse, and is then used to compute the expanded mode shapes from the p measured modes and paired analytical modes. A variation of this method is also possible, where the expanded dof are constrained to match the experimental values at the measured dofs [12].

3.3.b. Procrustes Expansion Method

Smith proposes an expansion method expressed as a constrained least-squares minimization problem [4]. This method expands the mode shapes by orthogonal Procrustes transformation of the experimental eigenvectors into the space spanned by the predicted analytical eigenvectors at the measured dofs:

$$\min_{P_{kk}} \left\| \Phi_{ap} - \Phi_{ap}\, {}_{PP} \right\| \quad subject\ to \quad P_{pp}^{T} P_{pp} = I \quad (10)$$

The Procrustes transformation is a mathematical technique which rotates two sets of same dimension into each other. The orthogonal transformation matrix P_{pp} is computed for the p experimental and paired analytical mode shapes at the a measured dofs through a singular value decomposition [4]. The Procrustes transformation preserves mass orthogonality and is numerically efficient. However, it requires correct pairing between the analytical and experimental eigenvectors, end selection of a set of measurement locations which fully spans the space of the *p modes* used for the expansion. Furthermore, the expanded mods shapes at the instrumented dofs no longer equal the measured values. Smith has attempted a variation of the Procrustes orthogonal expansion method by retaining the measured dof values and transforming only the unmeasured dofs. This modification was not recommended since it resulted in a loss of orthogonality in the eigenvectors, and a more jagged appearance in the mode shapes (*e.g.*, "fess of smoothing").

3.4. Least. Squares with Quadratic Inequality Constraints

The constrained minimization versions of the Guyan end Kidder expansion methods impose that the value of the expanded mode shape at the measured dofs, $\hat{\phi}_{ai}$ identically equals tha measured values $\tilde{\phi}_{ai}$ (Eqs. 1,5). Existing errors in the experimental values $\tilde{\phi}_{ai}$ propagate errors in the estimates of the mods shape at the unmeasured dofs $\hat{\phi}_{oi}$. Furthermore, ordinary experimental errors may impede optimization problems with equality constraints; when the equality between the measured data and the optimized data cannot be met *individually at every dofs*, the constrained optimization problem may either have an impossible solution or the wrong solution. Although penalty methods or generalized least-squares methods could be formulated to incorporate uncertainties resulting from experimental or analytical errors, the solution is dependent on the value of the relative weighting parameter Γ. While Γ is theoretically related to the covariance of the measurement and model errors, its correct value is difficult to assess.

To circumvent these weaknesses, the modal expansion problem can be reformulated as a quadratic function minimization with theunderstanding that error in the expanded mode shape exists, and that it is bounded by the expected measurement error. Mathematically, the expansion problem could be viewed as being a least-squares minimization problem with quadratic inequality constraints (LSQI) of the general form:

$$\min_{\hat{\phi}} \| A\hat{\phi} - b \|_2 \qquad subject\ to\ \| B\hat{\phi} - d \|_2 \le \alpha \| \tilde{\phi} \|_2 \quad (11)$$

The immediate advantage of the LSQI formulation is to allow convergence within a domain of probable solutions, while taking into account uncertainties associated with experimental errors. Mathematical techniques for solving this problem have been published and are easily implemented [11. Three modal expansion methods are proposed using three different 1S01 formulations.

3.4.a. Least-Squares Strain Energy Minimization with Quadratic Measurement Error Inequality Constraint

The first LSQI formulation, is the counterpart of the constrained optimization form of the Guyan method (Eq. 3). It finds the expanded mode shapes which minimize the modal strain energy, under the provision that the *quadratic mode* shape error at the measured dofs is of the order of the experimental uncertainty, Since the stiffness matrix K is a symmetric positive definite matrix, there exists a unique upper triangular matrix $G \in R^{f \times f}$ with positive coefficients m the diagonal, such that:

$$K = G^{T} * G \quad (12)$$

This is known as the Cholesky factorization. It is used to express the modal strain energy as a quadratic function. The first formulation of the LSQI problem (*LSQI1*) is then expressed as:

$$\min_{\hat{\phi}_{fi}} \| G\hat{\phi}_{fi} \|_2 \qquad subject\ to\ \| \hat{\phi}_{ai} - \tilde{\phi}_{ai} \|_2 \le \alpha \| \tilde{\phi}_{ai} \|_2 \quad (13)$$

As $\alpha \to 0$, *LSQI1* converges to the Guyan static expansion (Eq. 1). Based on the results obtained from the two independent sets of experimental data mentioned previously, a nominal value of 15% is assumed for the expected mode shape error parameter α used in the 1S01 expansion methods.

3.4.b. Least Squares Strain Energy Minimization with Quadratic Expansion Error Inequality Constraint

A second LSQI formulation (*LSQI2*) is proposed which minimizes the modal strain energy, subject to the constraint that the quadratic error between the optimally expanded mode shape, $\hat{\phi}_{fi}$ and the mode shape obtained from direct expansion $\tilde{\phi}^{d}_{fi}$, is less than the expected experimental error. Direct expansion methods are those that have a closed-form solution, such as the Guyan static and the Kidder dynamic methods (Eqs. 1,5). *LSQI2* is then formulated as:

$$\min_{\hat{\phi}_{fi}} \| G \hat{\phi}_{fi} \|_2 \qquad \text{subject to} \quad \| \hat{\phi}_{fi} - \hat{\phi}^d_{fi} \|_2 \leq \alpha \, \| \tilde{\phi}_{ai} \|_2 \tag{14}$$

As $\alpha \rightarrow 0$, *LSQI2* converges to the direct expansion solution $\hat{\phi}^d_{fi}$.

3.4.c. Least Squares Dynamic Residual Force Minimization with Quadratic Measurement Error Inequality Constraint

An LSQI formulation could be proposed, analogous to the Kidder method which minimizes the loss of modal energy (Eq. 6). However, when the experimental modal frequencies are almost identical to the predicted analytical frequencies, quasi-singularities of the objective function would create ill-conditioning of the Choleski factorization. To circumvent this problem, the objective function is redefined as the quadratic norm of the modal residual force, With this new formulation, the LSQI problem is now to find the optimal $\hat{\phi}_{fi}$ that minimizes the modal residual force such that the quadratic error between the expanded mode shape and the experimental mode shape at the measured dofs is within the bounds expected from experimental error.

$$\min_{\hat{\phi}_{fi}} \| (K - \tilde{\omega}_i M) \hat{\phi}_{fi} \|_2 \qquad \text{subject to} \quad \| \hat{\phi}_{ai} - \tilde{\phi}_{ai} \|_2 \leq \alpha \, \| \tilde{\phi}_{ai} \|_2 \tag{15}$$

As $\alpha \rightarrow 0$, *LSQI3* indirectly solves the eigenvalue problem for the given experimental modal frequency and mode shape data at the measured dofs.

4.0 PERFORMANCE METRICS

A first error metric is proposed to evaluate the relative quadratic point-to-point error at each dof between the-predicted expanded mode shape $\hat{\phi}_{fi}$ and the actual measured mode shape ϕ_{fi}, for each mode *i*:

$$\Delta = \frac{\| \psi - \bar{\psi} \|_2}{\| \bar{\psi} \|_2} \tag{16}$$

In comparing mode shapes at each point, normalization of the eigenvectors is achieved by least squares fit of the expanded mode shape to the reference mode shape. Alternatively, the mean cumulative error in the mode shape as a function of the mode *n* can be used to determine the modal number at which the expansion methods start to break down :

$$\epsilon(n) = \frac{1}{n} \sum_{i=1}^{n} \Delta(i) \tag{17}$$

The orthogonality properties of eigenvectors, as inferrad in the Modal Assurance Criteria (MAC), can also be used as a performance metric. The MAC matrix between two eigenvectors, ϕ_i and $\phi_j \in R'$, is defined as:

$$MAC_{i,j} = \frac{| \phi_i^T \phi_j |^2}{| \phi_i^T \phi_i | \, | \phi_j^T \phi_j |} \tag{18}$$

The MAC is used hereto verify the orthogonality between the axpanded mode shapes and tha actual mode shapes measured at all dofs. The mass cross-orthogonality condition (MX) is also a potential performance metric. Howaver, when applied to this case study, MX provided the same information as the MAC, and therefore will not be used herein.

The last three performance metrics are global metrics describing the total error throughout the whole set of dofs. Errors can also be evaluated at the local structural element level by the strain energy distribution associated with each element *s* and with each mods *i*. Analogous to MX which measures the accuracy of the expanded mode shape $\hat{\phi}_{fi}$ with respect to the FEM mass matrix M_{ff} the element strain energy verifies the fit of $\hat{\phi}_{fi}$ with respect to the FEM stiffness matrix K_{ff}. The element strain energy error between the analytical θ_s and the expanded $\hat{\theta}_s$ identifies the discrete dofs where the expansion does not agree with the model. Such errors typically result from localized modelling errors or actual structural damage.

5.0. SENSITIVITY STUDY OF EXPANSION METHODS

5.1. Performance to Nominal Data Sat and Updatad Model

The expansion methods are first investigated for their reliability and intrinsic performance when all experimental and analytical conditions are favorable. The expansion is executed with the updated (i.e., "ideal") analytical FEM model and mode shapes, from a subset of the high quality experimental data measured on the MPL The measured data is not corrupted by additional noise. Here, twelve locations have been retained as the "measured set", and are expanded to the full 240 dofs recorded during the actual test. The final 240 dofs locations represent 3 dofs at each of the 80 node balls forming the truss structure. The location of the 12 dofs are optimally selected as to give, for a Guyan reduced model, the best MAC with respect to the predicted analytical modes [6]. This particular set of instrument location is referred to as "*aset5*". As will be demonstrated through the test cases, *aset5* provides enough information to identify tha first nine modes, with the exception of mode 6 which is not properly represented. The expansion of missing mods 6 will thus provide a measure of the methods' robustness to unmeasured modal information. This survey will compare the following methods: Guyan, Kidder, Procrustes, LSQI strain energy minimization with measurement error (1 S011), and with expansion error (LSQI2), and LSQI residual dynamic force minimization with measurement error (LSQI3).

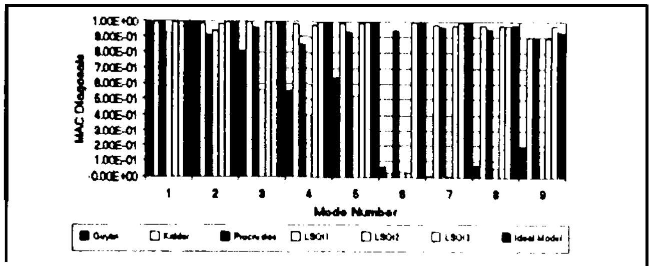


Figure 2 MAC Diagonal (measured vs expanded) - Expansion from 12 dofa (aset5) to 240 dofs with ideal model and no additional measurement error.

The MAC of the mods shapes expanded from experimental *aset5* data (12 dofs) with respect to the actual full measurements (240 dofs) is shown in Fig. 2 for all five expansion methods. The MAC of the ideal analytical model with respect to the full 240 dofs measurement set is also included in Fig, 2 for reference. Tha Guyan method can only expands the first 2 modes properly, with MAC's greater than 0.66. LSQI1 produces mode shape estimates which are slightly worse than the Guyan method, especially in the lower modes.The Kidder method generates expanded mode shapes which have MAC's greater than 0.97 for seven of the nine modes, but could not be identify mode. LSQI2 yield the same level of accuracy as the Kidder expansion. The Procrustes expansion method can predict mode 6, but only modes 1 and 3 are greater than 0.95. The mediocre results are explained by the fact that all nine modes ara expanded simultaneously from the initial 12 dofs subset, It was observed that the Procrustes method is vary sensitive to the number of simultaneously expanded modes and to the set of measurement locations.

LSQI3 is the expansion method that performs the best across all modes. It is capable of predicting unmeasured mods 6 better than the Procrustes method. Foremost, it is the only expansion method investigated so far which results in better MAC diagonals with respect to the measured data at all dofs than the analytical model used to expand the modes. These observations are consistent with the Performance demonstrated in Fig. 3 through the mean cumulative mode shape "error of the first nine modes (e.g., $\epsilon(9)$ Eq. 17).

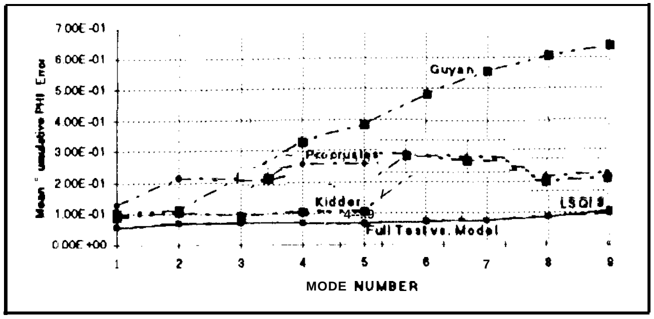


Figure 3 Mean cumulative mode shape error . Expansion from 12 dofs (*aset5)* to 240 dofs with updated FEM model and no additional measurement noise.

5.2. Sensitivity to Mode Shape Measurement Error

There are many sources of noise in the processing of mode shapes: accelerometer accuracy, wire mass and damping, shaker coupling, method of excitation... An additional error can be introduced by the modal identification method. It suffices to say that the measured mode is never pristine. It could be desirable, therefore, to have a mode shape extrapolation procedure that is not only insensitive to noise, but that can also filter it out too.

A sensitivity analysis is performed herein to evaluate the performance of the mode shape extrapolation methods with respect to distributed measurement noise. For lack of a better model, the noise is represented as an additive random error superimposed upon the true mode shape. Future work should investigate the effect of non-Gaussian measurement errors representative of a defective sensor or a consistent operator error. Spatially localized errors will be considered later in the context of isolated modelling errors and damage.

The quadratic difference bet ween the mode shapes obtained from the two independent tests were of the order of 15%. This level serves es a basis for the following error analysis, and effects of additive mode shape errors of the order 15%, 25% and 50% are investigated. To infer the mean prediction, Monte Carlo simulations are performed with 30 averages. The analysis reveals that LSQI1 and LSQI2 methods which are based on strain energy minimization provide, at a very high computational cost, only a minor improvement in the predicted expansion compared to the Guyan or Kidder methods. The following evaluation is thus limited to the Guyan, Kidder, Procrustes and LSQI3 methods.

The mean quadratic error between the expanded and fully measured modes shapes is compared for the first nine modes of the MPI as a function of noise level for each of the expansion methods (Fig. 4). The expansion is from *aset5* with 12 dofs up to the full 240 dofs, and is achieved with the updated "ideal" FEM model and eigenproperties. The error between the "ideal" analytical mode shapes and the measured mods shapes at all dofs with and without added measurement noise are also included for comparison.

As expected, the performance of the expansion methods grows worse as the noise in the measured data increases. As before, the Guyan method has the overall worse performance, followed by the Kidder and the Procrustes methods. The Kidder method is the most sensitive, for which the error in the mode shape increases linearly with the measurement error. The Guyan, Procrustes and LSQI methods are equally sensitive to measurement noise.

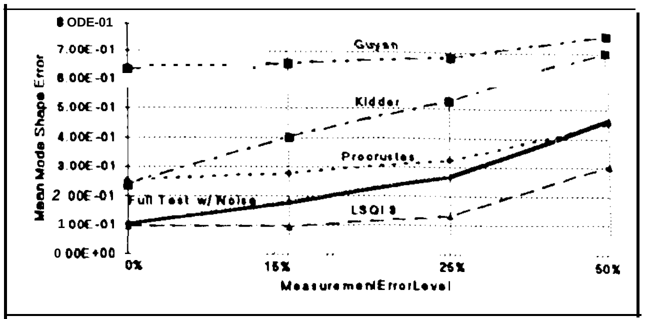


Figure 4 Mean mode shape error as a function of measurement noise - Expansion of nine modes from 12 dofs (aset5) to 240 dofs with updated FEM model.

As in the ideal situation, the error in the modes expanded with the Guyan, Kidder, and Procrustes method are greater or equal than the error in the' measurement. Only the LSQI3 method is capable of expanding mode shapes to a greater level of accuracy than the measured data, even when the original data is corrupted by significant amounts of noise. In fact, for moderate amounts of measurement noise, *e.g.*, less than 25%, the first nine modes expanded with the LSQI 3 method from only 12 instrument locations are almost as accurate as the noise-free mode shapes measured at all dofs.

5.3. Sensitivity to Selection of DOF Location and Quantity.

Five different sets of instrument locations and quantities are considered, *i.e.*, *aset's*, as summarized in Table i. The data measured at the *aset* location is expanded to the full 240 dofs, representing an expansion ratio 1 to 80 for *aset 1*, 1 to 40 for *aset* 2 and 3, and 1 to 20 for *aset* 4 and 5. The instrument location is selected either according to engineering judgement at the dofs of highest deformation, r%., the tip of the booms, or according to optimal criteria such as best MAC fit from a static reduction or best mode shape fit over multiple modes.

ASH' #	NUMBER OF INSTRUMENTS	LOCATION CRITERIA
I	3	1 per boom tip
2	6	best mode 1&2
3	6	2 per boom tips
4	12	triax @ boom tips
s	12	optimal MAC

Table V Summary of Instrument Location Cases.

As expected the expansion error decreases as the number of instrumented location increase (Fig. 5). Again, the Guyan method has the worst performance over all cases. Procrustes is the most sensitive to the *aset* selection, as shown by the 75% decrease in error from *aset1* to *aset5*. Expansion with the Kidder method only benefits slightly from thsr increase in the number of dofs. As before, onfy the LSQI3 method is capable of expanding the mode shape to the same degree of accuracy as the measured data, regardless of the selected *aset*. Fig. 5 also shows the sensitivity of the expansion error to dof location. *Aset2* and *aset3* include the same number of dofs, but located at different points on the structure. Whereas, the performance of the Kidder method improves from *aset2* to *aset3*, the Procrustes method worsens. This implies that for optimal performance, each expansion method should have its own dof selection criteria.

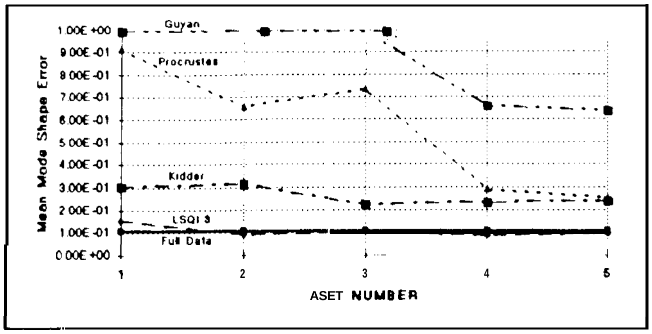


Figure 5 Mean mode shape error as a function of *aset* selection - Expansion of nine modes to 240 dofs with updated FEM model and no additional measurement noise.

A separate analysis demonstrated that tha Procrustes method is not only sensitive to the *aset* selection, but also to the number of modes used in the simultaneous expansion and to the pairing between the analytical and experimental modes. This is a disadvantage compared to the Guyan, Kidder and LSQI3 methods which expand the modes *individually* without the need for mode pairing. In these latter methods, the mode pairing is indirectly accomplished through the FEM model and does not require any user input or engineering judgement.

5.4. Sensitivity to Model Error

All the modal expansion methods proposed herein use an analytical model to predict the mode shapes at the unmeasured dofs. The FEM model plays an important role in the regularization of spurious information, the filtering out of the measurement error, and the prediction in the event of insufficient information. This is especially true of the Kidder and the LSQI3 methods which rely heavily on the full dynamic equations. In the following, both distributed (i.e., global), and localized errors are investigated.

Distributed errors in the analytical mesa or stiffness matrix, such as errors resulting from the uniform structural properties (*e.g.*, mass density or modulus of elasticity), only scale the eigenvalue problem by a multiplicative constant, and have little influence on the modal expansion prediction. Another form of global model error can be introduced by deficiencies in the model form, as would typically occur in a pretest model. To this effect the actual pretest model of the MPI is used for demonstration. It is composed uniquely of rod elements, and can only predict the first 4 modes. The "ideal" updated modal used in the previous expansion analyses is constructed uniquely of bar elements, and can accurately predict the first nine modes. As shown in Fig. 6, the mean cumulative mode shape *error* over nksa modes is 20% for the pre-test model, and is only 10% for the updated modal. The errors in the Guyan, the Kidder, the Procrustes, arrd the LSQI3 methods expandad with the pretest model from *aset5* to the full 240 dofs are also shown in Fig. 6. As expected, the level of error is slightly worse when the eigenvectors are expandad with the pre-test model than with the updated model, especially in the higher modes where the pretest and updated modal start to diverge (Fig. 3). The Guyan and the Procrustes methods display little sensitivity to model form error. The Kidder method is the most sensitive, as is shown by the sharp increase in the error beyond mode 3 resulting in a mean error which is twice as high than that obtained with the updated model expansion. Although the performance of the LSQI3 has also worsened, it is still the best by a factor of two relative to the other expansion methods, and it remains the only method which is capable of expanding mode shapes to a higher degree of accuracy than the model,

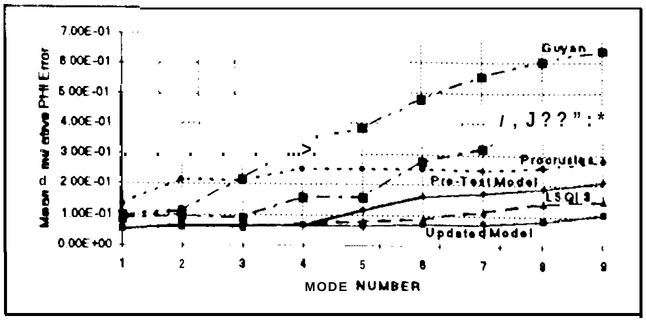


Figure 6 Mean cumulative mode shape error - Expansion from 12 dofs (*aset5)* to 240 dofs with pre-test FEM model and no additional measurement noise.

Spatially localized model error, such as would occur from local errors in the model form or properties, or from changes m the actual structure resulting from fatigue or damage are also expected to affect the predictability of the expanded mode shapes. To simulate this situation the stiffness of the longest strut in the pretest model, connecting the tower to the optics boom, is decreased by half. This only changes the pre-test frequencies of modes 5 and 6 by less then 3%, while keeping all other frequencies almost the same. However, the effect of this localized error on the analytical mode shapes is significant, as shown in Fig. 7 where major jump for mode 5 and 6 correspond to a 300% increase in the mode shape error relative to the "undamaged" pretest model. Although the effect on the analytical modes is extreme, none of the expended mode shapes are affected, and the expansion errors for each method is almost the same as those obtained previously with the "undamaged" pretest model (Fig. 6). The fact that the expansion error does not increase from the "undamaged" pretest model case implies that the expansion errors are more sensitive to global modal form errors than to localized element errors.

5.5 Sensitivity To Measurement *And* Model Error

Finally the performance of the expansion methods is assessed for the previous combination of global and local modelling error with an additional 25% error in the measured mode shape values. The results ara summarized in Fig. 8. The solid lines represent the accuracy of the different forms of the MPI model with respect to the tnsa test data at aff dofs, and the dashed lines represent the expended mode shapes from *aset5* to the full 240 dofs using the damaged pre-test model end noise corrupted measurements.

As expected, adding measurement noise to the damaged pretest model worsens the performance of all the expansion methods by approximately 50% (Figs. 7). In the presence of both modal error and measurement noise, the Guyan and the Kidder method perform equally poorly, and generate mode shapes which ara worse than predicted by the darnaged pretest model. The Procrustes method performs better than the Guyan and the Kidder methods, especially at the higher modes, and can predict the lower modes to the same level of accuracy as the noise contaminated data. Once again, the LSQI3 method performs exceptionally well. It generates mods shapes which are only off by 15%, although the data used is contaminated by 25% noise and the model has the wrong form and a damaged member. Furthermore, comparison between the element strain energies of the damaged pretest modal and the

LSQI3 expanded mode shapes show that the largest differences occur at the "damaged" strut location. Thus, the mode shapes errpanded with the LSQI3 method are also capable of identifying damaged members or localized model error, even in the presence of measurement noise.

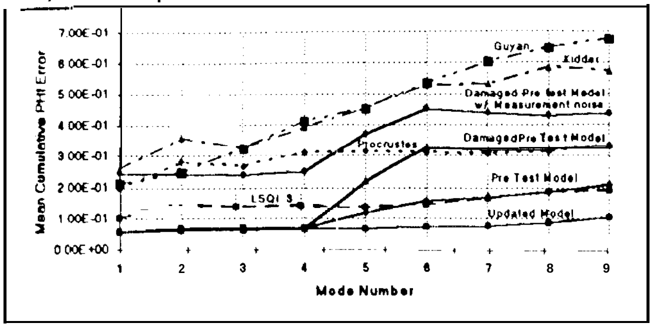


Figure 7 Mean cumulative mode shape error . Expansion from 12 dofs *(aset5)* to 240 dofs with damaged pre-test FEM modal and 25% additional measurement noise.

6.0. CONCLUSION

Several mode shape expansion methods have been proposed and investigated. These expansion techniques fall into three main categories. The first one uses direct solutions of the static and dynamic equations to obtain a closed-form equation. This category includes tha Guyan and the Kidder methods. It is shown that these direct methods can also be written in terms of an unconstrained minimization problem. The second category uses least-squares method to minimize the error between the measured and modelled eigenvectors. Within this category, the Procrustes method imposes orthogonality of the mode shapes. The third category formulates the expansion as a least-squares minimization problem, in which measurement or expansion error is incorporated as a quadratic inequality constraint.

The trade study demonstrated that the 1S01 method based on minimization of the dynamic force equation and subject to bounds imposed by measurement noise has the best performance. The Procrustes method has an average performance, whereas the direct methods are the worst. Tha LSQI methods based on strain energy minimization yield results comparable to the Guyan or Kidder methods, even in the presence of large measurement noise, and without any computational advantage.

It was shown that the Guyan method can only properly expand the first faw modes. To get suitable expansion with the Guyan method, a minimum ratio of 3 to 4 accelerometer per mode is required . as commonly practiced experimentally. lower ratios of instrumented dofa to modes and bettar performance can be achieved with the Procrustes, the Kidder and the LSQI3. Under ideal experimental and analytical conditions, the Kidder method cars correctly expand all modes rapresented in the data sat. This method is not sensitive to the *aset* selection, but is extremely sensitive to noise and modal deficiencies. It was shown that the 1S01 methods based on strain energy minimization did not improve on the accuracy of the direct methods, while imposing a significant computational cost. Computationally, the most efficient expansion method is the Procrustes method. Along with the LSQI3 method, it is the only method which can properly expand mods shapes which are not completely represented in the selected instrument locations. However, the Procrustes method can only achieve this if the analytical and experimental modes are properly paired. Pairing is automatically guaranteed in the other methods through the FEM modal and the measured modal frequencies. Furthermore, the Procrustes method is very sensitive to measurement dof location and selection, as well as to the number of simultaneously expanded modes, In an actual situation this is a big disadvantage as the real solution is not known, and the variation in the error can be great.

The LSQI expansion method with dynamic force minimization has the best all. around performance. It is insensitive to moderate amounts of measurement error, and is capable of predicting eigenvectors at unmeasured dofs with greater accuracy than the noise-corrupted data measured at those locations. LSQI3 is the only method which is capable of regularizing global and local model errors, resulting in mode shapes of higher accuracy than the model originally predicted, even in the presence of experimental noise, This makes the LSQI expansion method with Dynamic Force Minimization ideally suited for recursive model updating, damage detection and response prediction technique. Its biggest disadvantage is in its computational requirements, however, with the advent of faster and more powerful computers this is no longer an issue.

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