Application of the Coupled Finite Element- Combined Field Integral Equation Technique (FE/CFIE) to Electromagnetic Radiation problems

Vahraz Jamnejad*, Tom Cwik, and Cinzia Zuffada

Jet Prepulsion Laboratory
California Institute of Technology
Pasadena, CA 91109

A coupled finite element-combined field integral equation (FE/CFIE) technique was originally developed for solving scattering problems involving inhomogeneous objects of arbitrary shape and large dimensions in wavelength. The method incorporated an exact integral equation outside an arbitrary surface of revolution (SOR), very near the object and circumscribing it (T. Cwik, V. Jamnejad, and C Zuffada, IEEE-APS Symposium, June 1993). This technique can also be applied to the problems where sources are present, in which case the problem to be solved is that of radiating sources in the presence of the material bodies. This is the classic antenna radiation problem.

The development proceeds in a fashion similar to that for the original coupled technique for scattering problems. Of the three equations for the coupled approach formulation only one, namely the one involving the finite elements is changed. Starting with the Maxwell's Equations with electric and magnetic current sources J_s and M_s, the wave equation for H (or E) field can be written with source terms present. By multiplying the wave equation with a testing function, integrating over the volume of interest that should include the material bodies as well as the sources and some additional manipulation, the finite element equation with source is obtained. This equation is given as

$$j\eta_0 \iiint_V \left[\frac{1}{\varepsilon_r}(\nabla\times\bar{H})\cdot(\nabla\times\bar{T}^*) - k_0^2\mu_r\bar{H}\cdot\bar{T}^*\right]dv + k_0\iint_{\partial v}(\bar{E}\times\hat{n})\cdot\bar{T}^*\,ds =$$
$$j\eta_0 \iiint_V \frac{1}{\varepsilon_r}\bar{J}_s\cdot(\nabla\times\bar{T}^*)\,dv + k_0\iiint_V \bar{M}_s\cdot\bar{T}^*\,dv$$

Following the discretization of the equations by expanding the source terms in finite-element basis functions as is the case for the magnetic or electric field inside the volume, the general coupled approach matrix equation is obtained:

$$\begin{bmatrix} \mathbf{K} & \mathbf{C} & \mathbf{0} \\ \mathbf{C}^{*T} & \mathbf{0} & \mathbf{Z}_0 \\ \mathbf{0} & \mathbf{Z}_M & \mathbf{Z}_J \end{bmatrix} \cdot \begin{bmatrix} \mathbf{H} \\ \mathbf{M} \\ \mathbf{J} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_s \\ \mathbf{0} \\ \mathbf{V}_i \end{bmatrix}$$

The right-hand side vector now includes two terms involving sources: one for electric and magnetic source currents in the finite element volume, V_s, and the other for the outside incident field, V_i. Typically, for scattering problems the V_s term is zero and only an incident field exists which enters the matrix equation via the V_i term, while in antenna problems only the V_s term is present and the V_i term is zero. Of course, in general, both terms can be present and the problems of radiation and scattering can be solved simultaneously. The critical step in the source formulation involves the representation of current sources (volume and/or surface) by the same edge element basis functions over the mesh used in the representation of the H (or E) field inside the volume. Some examples of typical sources such as dipoles, microstrip antenna feeds, etc., are presented which show the suitability of this method for a variety of complicated antenna problems,