Performance Comparison of Data-Derived Symbol Synchronizers in the Presence of Unbalanced Data Streams

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ABSTRACT

This paper compares the performances of two different types of data-derived symbol synchronizers, namely, Filter and Square (FS) and Digital Data Transition Tracking Loop (DDTTL), in the presence of unbalanced data streams. When the probability of transmitting a +1 pulse, p, deviates from 1/2, the data becomes unbalanced causing potential degradation to the tracking performance of the symbol synchronizers. This paper relates the unbalanced data streams with the minimum transition densities and assess their impacts on the tracking performances of the two symbol synchronizers under investigation. The tracking performances of these two symbol synchronizers are characterized by the variances of the tracking phase jitters (or symbol sync jitters). The results show the symbol sync jitter as a function of symbol Signal-to-Noise Ratio (SNR), loop bandwidth and the probability p.

I. INTRODUCTION

In the past, considerable efforts have been spent [1-11] by various authors to study the performance of "data-derived" symbol synchronizer. Most of these authors [1, 2, 6-8] have assumed 50% transition density, i.e., perfectly balanced data. Several authors [3-4, 9-11] have studied the effects of transition density on the "data-derived" symbol synchronizer such as DTTL. A paper by Simon et al. [5] addressed the effects of the minimum transition density on the tracking performance of these bit synchronizers. It [5] pointed out that the ability of a bit synchronizer to maintain its locked condition depends on the minimum transition density-that is, the minimum number of transitions required in any symbol sequence of specified length. Recently, [9-11] have investigated the effect of minimum transition density on the performance of DTTL.

The purpose of this paper is to assess the impact of the unbalanced data stream on the performance of FS symbol synchronizer and compare the results with those found in [11-12] for DTTL.

II. FS SYMBOL SYNCHRONIZER

A typical FS symbol synchronizer is shown in Figure 1. In this paper the baseband Low Pass Filter (LPF) is assumed to be ideal with bandwidth f_0 and associated transfer function $H(\omega)$. The baseband input signal is described by

$$y(t) = Ad(t) + n(t) \tag{1}$$

where A is the signal amplitude, d(t) is the base band NRZ data symbol sequence with each symbol being statistically independent of each other. Consider now the data d(t) can be expressed as

$$d(t) = \sum_{k=-\infty}^{\infty} d_k m(t-kT) \tag{2}$$

where d_k is a random variable taking on value ± 1 with probability of having +1 is p and -1 is q and is independent from sample to sample, and m(t) is the NRZ symbol pulse with period 'T'. From [6], it can be shown that, for an ideal LPF, the average value of the data at the output of the square-law device is

$$E[d_0(t)]^2 = \sum_{K=-\infty}^{\infty} q^2(t-kT) \tag{3}$$

where q(t) is the filtered symbol waveform and it is given by

$$q(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)M(\omega)e^{j\omega t}d\omega \tag{4}$$

where $M(\omega)$ is the Fourier transform of m(t). Clearly, Eqn (3) is periodic with complex Fourier coefficient is given by [6]

$$C_n=-ij'92(olu=2nti^T \tag{5}$$

where F denotes the Fourier transform operation. Evaluating Eqn (5), we obtain

$$|C_n| = \frac{1}{\pi T}\left|\int_{-\infty}^{\infty}[SI(\alpha_+)-SI(\alpha_-)]^2 e^{-\frac{j2\pi nt}{T}}dt\right| \tag{6}$$

where

$$SI(x) = \int_0^x \frac{\sin(u)}{u}du \tag{7}$$

$$\alpha_\pm = 2\pi f_0(t \pm \frac{T}{2}) \tag{8}$$

The noise, N(t), at the output of the square-law device is given by [6]

$$N(t) = 2Ad_0(t)n_0(t) + [n_0(t)]^2 \tag{9}$$

Note that $d_0(t)$ and $n_0(t)$ are the data and noise at the output of the LPF. The Power Spectral Density (PSD) of f = n/T for the Second term in Eqn (9) can be shown to have the following form

$$S_{n_0 n_0}(\frac{n}{T}) = \frac{2N_0^2 f_0}{\pi}\int_0^{\infty}[\frac{\sin(u)}{u}]^2\cos(\frac{nu}{Tf_0})du \tag{10}$$

Evaluating Eqn (10), we get

$$\begin{aligned} S_{n_0 n_0}(\frac{n}{T}) = \frac{N_0 f_0}{4R}[&-2signum(\frac{n}{R})n - signum(\beta_-)n \\ &+2signum(\beta_-)R + signum(\beta_+)n \\ &+2signum(\beta_+)R] \end{aligned} \tag{11}$$

where

$$R = f_0 T \tag{12}$$

$$\beta_\pm = \frac{2R \pm n}{R} \tag{13}$$

The PSD of f = n/T for the first term of Eqn (9) is given by

and

$$S_{n_0 s_0}(\frac{n}{T}) = 4A^2\int_{-\infty}^{\infty}S_{d_0}(f)S_{n_0}(\frac{n}{T}-f)df \tag{14}$$

where $S_{d_0}(f)$ and $S_{n_0}(f)$ are the PSDs for the NRZ data and noise at the output of the LPF, respectively. Since the LPF is ideal and the data in unbalanced, Eqn (14) becomes

$$\begin{aligned} S_{n_0 s_0}(f) &= 2A^2N_0(1-2p)^2\int_{\frac{n}{T}-f_0}^{f_0}\delta(f)df \\ &+ \frac{8A^2N_0p(1-p)}{\pi}\int_{\pi(n-R)}^{\pi R}[\frac{\sin(u)}{u}]^2du \end{aligned} \tag{15}$$

Note that the first term of Eqn (15) equals zero when R < n, and it becomes $2A^2N_0(1-2p)2$ when R > n. Let P_t denote the transition probability, then it is well known that $P_t = 2pq = 2p(1-p)$.

Since the tone is generated at f = n/T, n = 1, 2, 3>,.., and hence the Phase-Locked Loop (PLL) can be used to track this tone and synchronize to the bit stream. If we neglect the self noise at low SNR's (< 10 dB), the symbol synchronization timing error variance can be shown to be [6]

$$\frac{\sigma_\tau^2}{T^2} = \frac{T^2[S_{n_0 s_0}(\frac{n}{T}) + S_{n_0 n_0}(\frac{n}{T})](2B_L)}{(2\pi)^2(2A^4|C_n|^2)} \tag{16}$$

where $S_{n_0 n_0}(n/T)$, $S_{n_0 s_0}(n/T)$, and $|C_n|$ are given by Eqns (11), (15) and (6), respectively. Substituting Eqns (11) and (15) into Eqn (16), we obtain

$$\frac{\sigma_\tau^2}{T^2} = \frac{[\alpha_1 + \frac{4}{\pi}\alpha_2 + \frac{R\alpha_3}{\pi R_s}](B_L T)}{2\pi^2 R_s|C_n|^2} \tag{17}$$

where

$$\alpha_1 = \begin{cases} 0, & for\ R<n \\ (1-2p)^2, & for\ R>n \end{cases} \tag{18}$$

$$\alpha_2 = p(1-p)\int_{-\pi(R+n)}^{\pi R}[\frac{\sin(u)}{u}]^2du \tag{19}$$

$$\alpha_3 = \frac{\pi}{8}[-2signum(\frac{n}{R})n - signum(\beta_-)n + 2signum(\beta_-)R + signum(\beta_+)n + 2signum(\beta_+)R] \quad (?0)$$

and

$$R_s = \frac{A^2 T}{N_0} \quad (21)$$

Note that Rs denotes the symbol Signal-to-Noise Ratio (SNR).

III. DTTL SYMBOL SYNCHRONIZER

A typical 1> '1"1'1. symbol synchronizer is shown in Figure 2. The performance of the 1)'1"1'1. for perfect data stream, i.e., $p = 1/?$, has been analyzed in [1]. While the performance of the DTTL for an arbitrary transition density has been investigated in [9-1 1]. Reference [10] shows that the variance of the timing CI101 1s given by

$$\frac{\sigma_\tau^2}{T^2} = \frac{h(0)B_l T}{(2\pi)^2(2R_s g_n'(0))} \quad (22)$$

where $g'_n(0)$ denotes the derivative of the S-curve evaluated at $\tau = 0$ and is given by

$$g_n'(0) = 2P_t erf(\sqrt{R_s}) \quad (23)$$

and h(0) denotes the normalized noise spectrum and is given by

$$h(0) = p^2 + q - erf^2(\sqrt{R_s})[p^4 + q^2 - P_t + p^2q(p+1) + P_t R_s] + \frac{1}{2}P_t + P_t R_s \quad (24)$$

Note that for high SNR h(0) reduces to 21', anti the timing variance becomes independent of P_t [1].

IV. NUMERICAL RESULTS

The amplitude of the. tone created by the squaring operation can be calculated using Eqn (6). Figure 3 shows the plot of the tone amplitude and this figure shows that the amplitude of the tone is maximum when n = 1 anti R = 1. Using these values for n and R, the plots of the normalized timing variance of the FS synchronizer (Eqn 17) for $B_L T =$ 0.01 are shown in Figures 4 and 5. Figure 4 plots the Normalized timing variance for p = 0.5 (perfect data stream) for various values Of R. 11 is clear that for R = 1, the FS achieves its best performance. Figure 5 show the timing variance for various values of p at R = 1. It is shown that the timing variance of FS synchronizer is not very sensitive to the unbalance data stream for symbol SNR ≥ 4,45. Figure 6 shows the plot of Eqn (17), the normalized timing variance of 1)'1"1'1, synchronizer, for $B_L T = 0.01$ and various values of p. This figure indicates that the performance of the DTTL is independent of p for symbol SNR ≥ 4.54 dB, and (hat the timing variance is also not very sensitive to unbalanced data stream at low symbol SNR, i.e., symbol SNR < 4.45 dB.

V]. CONCLUSIONS

This paper analyzes and compares the performance of the FS and 1) '1 ''1'1, symbol synchronizers under various conditions of unbalanced data stream. Mathematical models for the timing jitter have been derived to characterize the performance of the two synchronizers under investigation. It was found that for symbol SNR ≥ 4.45 dB, both FS and DTTL synchronizers are not sensitive to the unbalanced data stream. 1 low'ever, the performance of the 1)'1'1, synchronizer is always better than that of FS synchronizer.

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Figure Filter-and-Square Symbol Synchronizer

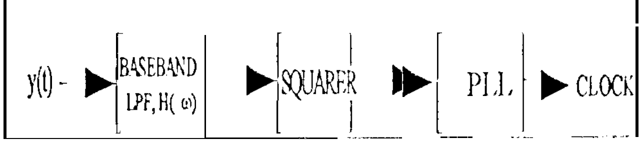

Figure 2. DTTL Symbol Synchronizer

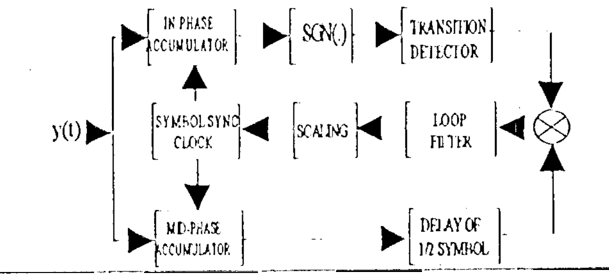

Figure 3. Plot of Tone Amplitude for Various Values of R = fo T

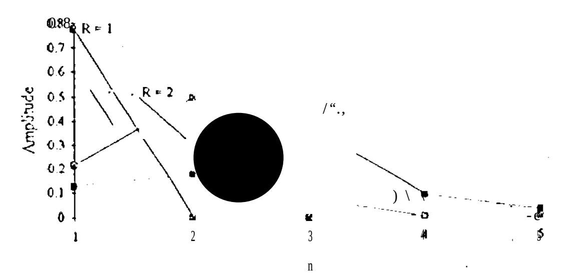

Figure 4. Normalized Timing Variance of FS Synchronizer for $B_L T = 0.01$, p = 0.5

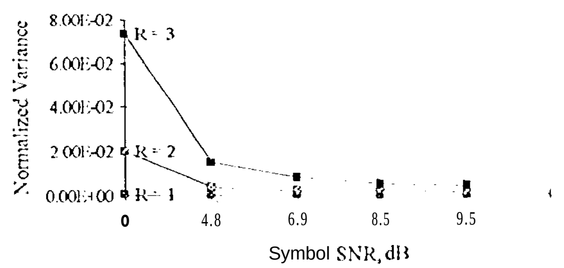

Figure 5. Normalized Timing Variance of FS Synchronizer for $B_L T = 0.01$, R = 1