DETECTION OF POLAR MOTION EXCITATION BY THE FORTNIGHTLY OCEAN TIDES

by

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• Spectra of polar motion excitation functions exhibit enhanced power in fortnightly tidal band

• Upon subtracting atmospheric wind and pressure effects, fits for periodic terms at the /Wand Mf tidal frequencies can fully account for the observed enhanced power in the fortnightly tidal band

• Ocean tide models predict polar motion excitation effects that are generally two to three times smaller than those observed at fortnightly periods

  • Need improved models for effect of long-period ocean tides on Earth’s rotation
APPROACH

● Polar motion excitation function
  • Use that derived from SPACE94 polar motion values
    ● SPACE94 is a Kalman filter-based combination of space-geodetic Earth rotation measurements
    ● Spans 1976.8–1994 at 1-day intervals

● Remove effects of atmospheric wind and pressure
  • Use atmospheric angular momentum values computed from operational analyses of National Centers for Environmental Prediction (formerly the National Meteorological Center)
  • Pressure term used is that computed assuming oceans respond as inverted barometer to imposed atmospheric pressure changes

● Least-squares fit for periodic terms at tidal frequencies to SPACE94–AAM residual series
  • Fit for mean, trend, and periodic terms at the weekly (M9 and M9'), fortnightly (Mf and Mf'), monthly (Mm), semiannual (Ssa), and annual (Sa) tidal frequencies
  • Fit to entire 18.2-year span of data set
    ● Data set spanning about 18.6 years must be used in order to resolve periodic terms at the M9 and M9' tidal frequencies, or at the Mf and Mf' tidal frequencies, since that is their beat period

● Compare recovered empirical tidal effects with those predicted by ocean tide models
  • Seiler (1991) as analyzed for polar motion effects by Gross (1993) and Brosche & Wunsch (1994)
  • Dickman (1993)
LONG PERIOD LIOUVILLE EQUATION

- Conservation of angular momentum expressed within rotating, body-fixed reference frame

\[ \frac{\partial L}{\partial t} + \omega \times L = \tau \]

where the angular momentum vector \( L = I \cdot \omega + h \)

- Assume rotation is small perturbation from state of uniform rotation at rate \( \Omega \). Keeping terms to first order yields long period Liouville eq.

\[ m(t) + \frac{i}{\sigma_{cw}} \frac{\partial m}{\partial t} = \psi(t) = \chi(t) - \frac{i}{\Omega} \frac{\partial \chi}{\partial t} \]

where: \( m \equiv (\omega_1 + i \omega_2)/\Omega \) (terrestrial location of rotation pole)
\( \psi(t), \chi(t) \) are the polar motion excitation functions
\( \sigma_{cw} \) is complex-valued frequency of Chandler wobble

- Written in terms of reported polar motion parameters \( \sim(\sim) = x_p(t) \cdot i y_p(t) \)
  - In time domain:
    \[ p(t) + \frac{i}{\sigma_{cw}} \frac{\partial p}{\partial t} = \chi(t) = \frac{1.61}{\Omega (C-A)} \left[ h(t) + \frac{\Omega c(t)}{1.44} \right] \]
  - In frequency domain:
    \[ p(\sigma) = \frac{\sigma_{cw}}{\sigma_{cw} - \sigma} \chi(\sigma) \]
POLAR MOTION EXCITATION FUNCTIONS

- In Earth rotation theory, the excitation functions, or z-functions, are the forcing functions that cause changes in the Earth’s rotation (length-of-day) and orientation (polar motion).

- In general, they are functions of changes in:
  - the Earth’s inertia tensor
  - relative angular momentum

- At frequencies far from the Free Core Nutation resonance (that is, at periods long compared to a day), the polar motion excitation functions $\chi(t)$ are related to the polar motion parameters $x_p(t)$ and $y_p(t)$ by:

  $p(t) + \frac{i}{\sigma_{cw}} \frac{\partial p}{\partial t} = \chi(t)$

  where:
  $p(t) = x_p(t) - i y_p(t)$
  $\chi(t) = \chi_1(t) + i \chi_2(t)$

  - This is the equation for simple harmonic motion in the complex plane.
  - The excitation pole is that pole about which the rotation pole instantaneously revolves.
  - Changes in the excitation pole force changes in the polar motion.

- Can be recovered from polar motion observations either by direct numerical differentiation or by deconvolution.

- The SPACE94 polar motion excitation functions have been used here.
SPACE94 EARTH ORIENTATION SERIES

- A combination of space-geodetic Earth rotation measurements
  - LLR (from JPL analysis center)
  - SLR (from University of Texas Center for Space Research analysis center)
  - VLBI [from IRIS “Intensive” (both NOAA & USNO analyses), NASA’s Deep Space Network at JPL, and NASA’s Space Geodesy Program at GSFC]
  - GPS (from S10 and JPL analysis centers)

- Individual series adjusted prior to their combination
  - Leap seconds and tidal terms removed (when necessary) from UT1 values
    - Yoder et al. [1981] model used to remove effect of all long period solid Earth tides
    - Dickman [1993] model used to remove ocean tidal corrections to the Yoder et al. [1981] model values at the $Mf, Mf', Mm,$ and $Ssa$ tidal frequencies
    - Herring [1993] empirical model used to remove effect of semidiurnal and diurnal ocean tides on NOAA’s IRIS “Intensive” UT1 values
  - Bias and rate of each series adjusted to be in agreement with each other
  - Stated uncertainties of each series adjusted so its residual with respect to a combination of all other series has a reduced chi-square of one
  - Outlying data points deleted

- Adjusted series combined using Kalman filter to form SPACE94
  - Consists of values for PMX, PMY, UT1–UTC, their formal uncertainties and correlations spanning October 6.0, 1976 to January 27.0, 1995 at daily intervals
SPACE94 POLAR MOTION EXCITATION FUNCTION

- SPACE94 consists of values for polar motion and UT1–UTC
- Kalman filter used to generate SPACE94
  - Contains a model for the polar motion process
  - Produces estimates of excitation functions as well as polar motion and UT1-UTC
- Polar motion excitation functions used here are those estimated by Kalman filter when generating SPACE94
ATMOSPHERIC ANGULAR MOMENTUM (AAM)

- Angular momentum of atmosphere changes due to:
  - Changes in strength and direction of atmospheric winds
  - Changes in mass distribution of atmosphere (changes in atmospheric pressure)

- Under principle of conservation of angular momentum, the rotation of the solid Earth changes as AAM is exchanged with the solid Earth

- AAM %-functions quantify the atmospheric excitation of Earth rotation
  - AAM pressure term (inertia tensor)
    \[
    \chi_1^p + i \chi_2^p = \frac{-1.00}{(C-A) g} \int p \sin \phi \cos^2 \phi (\cos \lambda + i \sin \lambda) \, d\lambda \, d\phi
    \]
  - AAM wind term (relative angular momentum)
    \[
    \chi_1^w + i \chi_2^w = \frac{-1.43}{\delta^2 (C-A) g} \int (u \sin \phi \cos \phi + i \nu \cos \phi) (\cos \lambda + i \sin \lambda) \, dp \, d\lambda \, d\phi
    \]

- AAM \( \chi \)-functions are computed from operational analyses of:
  - Japan Meteorological Agency (JMA)
  - United Kingdom Met. Office (UKMO)
  - National Centers for Environmental Prediction (NCEP)
  - European Centre for Medium-Range Weather Forecasts (ECMWF)

- AAM \( \chi \)-functions from the NCEP were chosen for use here
  - It is the only series currently available that fully overlaps in time with SPACE94
OCEANIC RESPONSE TO ATMOSPHERIC SURFACE PRESSURE FLUCTUATIONS

- How do oceans transmit atmospheric surface pressure fluctuations to ocean bottom?
  - For AAM pressure term, need pressure evaluated at crustal surface

- Inverted barometer assumption
  - Ocean response to imposed atmospheric surface pressure fluctuations is such that pressure at ocean bottom does not change
  - Generally held to be valid at long periods (> a few days)

- Rigid ocean (no inverted barometer) assumption
  - Atmospheric surface pressure fluctuations fully transmitted (without attenuation) to the ocean bottom

- AAM pressure terms are available that have been computed under each of these assumptions

- AAM pressure term computed under inverted barometer assumption chosen for use here
SPACE94-AAM RESIDUAL CHI FUNCTION (1976.8-1994)
**Observed & Predicted Effects of Long-Period Ocean Tides on the Polar Motion Excitation Function \( \chi(t) \)**

<table>
<thead>
<tr>
<th>Tidal Component</th>
<th>Amplitude (mas)</th>
<th>Phase (degrees)</th>
<th>Amplitude (mas)</th>
<th>Phase (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prograde</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M9' (9.12-day)</td>
<td>0.54 ± 0.45</td>
<td>38 ± 48</td>
<td>0.21 ± 0.45</td>
<td>79 ± 126</td>
</tr>
<tr>
<td>SPACE94–AAM</td>
<td>0.13</td>
<td>73</td>
<td>0.21</td>
<td>15</td>
</tr>
<tr>
<td>Dickman</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M9 (9.13-day)</td>
<td>0.47 ± 0.45-30</td>
<td>55</td>
<td>0.41 ± 0.45</td>
<td>-95 ± 63</td>
</tr>
<tr>
<td>SPACE94–AAM</td>
<td>0.32</td>
<td>73</td>
<td>0.52</td>
<td>15</td>
</tr>
<tr>
<td>Dickman</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seiler/Gross</td>
<td>0.72</td>
<td>55</td>
<td>0.59</td>
<td>72</td>
</tr>
<tr>
<td><strong>Retrograde</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mf' (13.63-day)</td>
<td>1.61 ± 0.45</td>
<td>56 ± 16</td>
<td>2.01 ± 0.45</td>
<td>87 ± 13</td>
</tr>
<tr>
<td>SPACE94–AAM</td>
<td>0.52</td>
<td>100</td>
<td>0.71</td>
<td>8</td>
</tr>
<tr>
<td>Dickman</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seiler/Gross</td>
<td>0.72</td>
<td>55</td>
<td>0.59</td>
<td>72</td>
</tr>
<tr>
<td>Mf (13.66-day)</td>
<td>0.86 ± 0.45</td>
<td>93 ± 30</td>
<td>2.73 ± 0.45</td>
<td>14 ± 10</td>
</tr>
<tr>
<td>SPACE94–AAM</td>
<td>1.26</td>
<td>100</td>
<td>1.72</td>
<td>8</td>
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<tr>
<td>Dickman</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seiler/Gross</td>
<td>1.72</td>
<td>55</td>
<td>1.44</td>
<td>72</td>
</tr>
<tr>
<td>Mm (27.55-day)</td>
<td>0.75 ± 0.45</td>
<td>49 ± 35</td>
<td>0.82 ± 0.45</td>
<td>-59 ± 32</td>
</tr>
<tr>
<td>SPACE94–AAM</td>
<td>0.47</td>
<td>136</td>
<td>0.28</td>
<td>-7</td>
</tr>
<tr>
<td>Dickman</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seiler/Gross</td>
<td>0.78</td>
<td>74</td>
<td>0.92</td>
<td>28</td>
</tr>
</tbody>
</table>

Quoted uncertainties are 1-sigma formal errors.

Prograde and retrograde amplitudes \( A \) and phases \( \phi \) defined by:

\[
\chi(t) = A_p e^{i \alpha_p} e^{i \phi(t)} + A_r e^{i \alpha_r} e^{-i \phi(t)}
\]

where the subscript \( p \) denotes prograde and \( r \) denotes retrograde.
RESULTS

• At $Mf$ tidal frequency, observations agree best with Dickman model predictions
  • Prograde phases differ by 7"
  • Retrograde phases differ by 6°
  • Dickman predicted prograde amplitude 47% too large
  • Dickman predicted retrograde amplitude 37% too small

• At $Mf'$ tidal frequency, observations agree best with Seiler model predictions
  • Prograde phases differ by 1"
  • Retrograde phases differ by 15"
  • Seiler predicted prograde amplitude $< 1/2$ that observed
  • Seiler predicted retrograde amplitude $< 1/3$ that observed

• At the $M9$ and $M9'$ tidal frequencies, spectrum shows no enhanced power, and the recovered amplitudes are at level of formal error
  • Observations provide upper limit for effect at these frequencies

• At the $Mm$ tidal frequency, spectrum shows no enhanced power, but the recovered amplitudes are somewhat larger than the formal error
  • Formal error may be underestimated by about factor of 2
  • Observations provide upper limit for effect at this frequency

• Discrepancies between the 2 model predictions, and between predictions and observations, illustrate the need for improved models for effect of long-period ocean tides on the Earth’s rotation