Simulated Performance of Diffractive Optical Elements using a Helmholtz Equation Solver

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Outline

1. Electron-Beam Fabrication of Diffractive Optics at JPL
2. Iterative Finite-Difference Helmholtz Equation Solver
3. Grating Simulation
4. Computer Generated Hologram Simulation
5. Cylindrical Lens Simulation
Electron-Beam Fabrication of Diffractive Optics at JPL

- Surface contouring of thin film poly-methyl methacrylate (PMMA, Plexiglas) using JEOL JBX5DII 50 kV electron-beam lithography system

- Can deliver 64 different exposure doses in each pattern (~50 depth levels)

- Proximity effect (backscattered e-beam dose) measured for different substrates and deconvolved out of the desired pattern

- Development in acetone yields exponential depth vs. dose response curve

- Patterns composed of square pixels (typically 0.5 to 2 μm) writing rate ~10^5 pixels/minute

- Quantitative characterization using atomic force microscopy (AFM)

- Fresnel lenses, gratings, computer-generated holograms (collaborators A. Gmitro - Univ. of Arizona, Peter Guilfoyle - Opticomp)

- Wavelength-size pixels and fabrication errors (side-wall etching) require modeling beyond the Fourier optics approximations (flat plate, perfect phase delay over entire pixel)
Iterative Finite-Difference Helmholtz Equation Solver

- Surface relief boundary conditions are properly treated
- Physical structures easily specified as polygons
- Requires minimal storage
- Currently running on Pentium PC, HP workstation, and JPL Cray Y-MP
- Technique is applicable to numerous problems
Helmholtz equation

\[ \nabla^2 U + k_0^2 \varepsilon(x,z) U = 0 \]

Alternating-direction-implicit (ADI) equations for \( n+1 \)st iteration

\[
(\omega_n + \delta_z^2) V = (\omega_n - \delta_x^2) U^n
\]

\[
(\omega_n + \delta_x^2) U^{n+1} = (\omega_n - \delta_z^2) V
\]

\[
\delta_x^2 U_{i,j} = \frac{U_{i-1,j} - 2U_{i,j} + U_{i+1,j}}{(k_0 \Delta x)^2} + \frac{\varepsilon_{i,j}}{2} U_{i,j}
\]

\[
\delta_z^2 V_{i,j} = \frac{V_{i,j-1} - 2V_{i,j} + V_{i,j+1}}{(k_0 \Delta z)^2} + \frac{\varepsilon_{i,j}}{2} V_{i,j}
\]

- \( \omega_n \) are complex acceleration parameters (may use a cyclic sequence)
- Hadley's two-parameter sequence
  \[ \omega_1 = -200 + i0 \]
  \[ \omega_2 = (0.05 + i1.1) \text{Re}\{\varepsilon/2}\]

- Proper node numbering results in tridiagonal matrices
Computational Domain

- Incident field complex amplitude is specified along $z = 0$ boundary
- Field is forced to zero on all other sides
- Absorbers surround region of interest
  - Index-matched at ROI edges
  - Imaginary part increases as $d^p$ with distance $d$ into the absorber
  - Absorber along lower boundary absorbs waves reflected back toward source
Convergence Criteria

1. Field converges to \( n \) significant digits everywhere in region of interest (\( n \) typically 4).

2. Finite difference form of Helmholtz equation converges to some error tolerance (typically 1x1 \( 10^{-10} \)).

Execution Times

- Scale linearly with number of mesh points

- Typical problem with 100,000 mesh points
  - 90 MHz Pentium PC: 1000 sec
  - HP 715/75 workstation:
  - Cray YMP2E/232:
Grating Simulation

- Rectangular groove surface relief grating: period = 2.5 μm, groove depth = 1.0 μm, duty cycle = 50%, refractive index = 1.5, freespace wavelength = 1.0 μm, angle of incidence = 0°, TE polarization

- Simulated using ADI technique and rigorous coupled-wave analysis (RCWA)

- Input wave for ADI technique had unity amplitude (center 30 pm) with raised cosine edges (1 O μm on each side)

- Calculated angular spectrum of plane waves from field slices though ADI results
  - In front of grating: $E_{tot} = E_{inc} + E_{refl}$
  - in back of grating: $E_{tot} = E_{trans}$
    Separate simulation with uniform index was used to find $E_{inc}$

- Calculated the power density flowing in the z direction for each order

- Compared ADI efficiencies to RCWA efficiencies
Grating: INCIDENT Near Field at $z = 0.97$ microns

- **Magnitude**
  - Y-axis: $0.05, 0.1, 0.15$
  - X-axis: Position, $x$ (microns) $0, 10, 20, 30, 40, 50, 60$

- **Phase (waves)**
  - Y-axis: $-2, 0$
  - X-axis: Position, $x$ (microns) $0, 10, 20, 30, 40, 50, 60$

- **Plane Wave Amplitude**
  - Y-axis: $0, 2, 4, 6$
  - X-axis: Normalized Spatial Frequency, $u\lambda$ $-2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2$
Grating: REFLECTED Near Field at z = 0.97 microns

- **Magnitude**
  - X-axis: Position, x (microns)
  - Y-axis: Magnitude

- **Phase (waves)**
  - X-axis: Position, x (microns)
  - Y-axis: Phase (waves)

- **Plane Wave Amplitude**
  - X-axis: Normalized Spatial Frequency, $u \lambda$
  - Y-axis: Plane Wave Amplitude
Grating: TRANSMITTED Near Field at z = 2.03 microns

- **Magnitude**
  - Y-axis: 0 to 0.3
  - X-axis: Position, x (microns) from 0 to 60

- **Phase (waves)**
  - Y-axis: -2 to 2
  - X-axis: Position, x (microns) from 0 to 60

- **Plane Wave Amplitude**
  - Y-axis: 0 to 5
  - X-axis: Normalized Spatial Frequency, \( u*\lambda \) from -1.5 to 2
## RCWA-ADI Comparison

Rectangular groove grating: period = 2.5 pm, depth = 1.0 pm
grating index = 1.5, cover index = 1.0

<table>
<thead>
<tr>
<th>Order</th>
<th>RCWA Backward DE</th>
<th>ADI Backward DE</th>
<th>Error</th>
<th>RCWA Backward Rel. DE</th>
<th>ADI Backward Rel. DE</th>
<th>Error</th>
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<tbody>
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<td>0.0104</td>
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<td>0.16476</td>
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Total: 0.059535

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Total: 0.940465

Grand Total: 1

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RCWA Backward Rel. DE

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ADI Backward Rel. DE

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RCWA Forward Rel. DE

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ADI Forward Rel. DE

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Error
Computer Generated Hologram Simulations

- Used Gerchberg-Saxton iterative algorithm to design one-dimensional CGHS with pixel widths of 0.5, 1, 2, 3, and 4 μm
- Calculated near field using ADI technique
- Calculated Fourier optics near field from pixel depths (quantized to match ADI grid)
- Extracted single-cell CGH near fields and arrayed them 8 times (isolates far field spots)
- Propagated both solutions to the far-field using Fraunhofer approximation and compared intensity patterns
Computer Generated Hologram: Instantaneous Electric Field
Pixel Size = 4 microns: Far Field (z = 100 mm) from ADI Simulation

Far Field from Fourier Optics: RMS Error = 0.0196
Pixel Size = 0.5 microns: Far Field \((z = 100 \text{ mm})\) from ADI Simulation

Far Field from Fourier Optics: RMS Error = 0.4069
Flat Phase Plate Model: Far Field Intensity Error

RMS Error vs CGH Pixel Width/Freespace Wavelength
Cylindrical Lens Simulation

- Focal length $\infty \mu m$, width 50 $\mu m$, lens index 1.5, focal medium index 1.0
- Calculated near field using ADI technique
- Propagate a field by convolving with impulse response of free space

$$U(x,z-z_0) = U(x,z_0) * h(z-z_0)$$

$$= F.T.^{-1}(F.T\{U(x,z_0)\} F.T\{h(z-z_0)\})$$

$$F.T.\ h(z-z_0) = \begin{cases} 
\exp\left(\frac{i(z-z_0)}{\lambda} \sqrt{1-(u\lambda)^2}\right), & (u\lambda)^2 \leq 1 \\
\exp\left(-\frac{(z-z_0)}{\lambda} \sqrt{(u\lambda)^2 - 1}\right), & (u\lambda)^2 > 1
\end{cases}$$
Cylindrical Lens: Instantaneous Electric Field
Summary

- Extended Hadley's ADI Helmholtz solver to allow accurate simulations of diffractive optical elements
- Grating - reasonable agreement with RCWA diffraction efficiencies
- Computer generated holograms - quantitative calculation showing breakdown of Fourier optics model for wavelength-size features
- Cylindrical lens - calculation of fields and focal plane intensity